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Symmetric and Positive Definite Broyden Update for Unconstrained Optimization

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Abstract:

Broyden update is one of the one-rank updates which solves the unconstrained optimization problem but this update does not guarantee the positive definite and the symmetric property of Hessian matrix.

In this paper the guarantee of positive definite and symmetric property for the Hessian matrix will be established by updating the vector y_k which represents the difference between the next gradient and the current gradient of the objective function assumed to be twice continuous and differentiable .Numerical results are reported to compare the proposed method with the Broyden method under standard problems.

Key words: Broyden update, Hessian matrix, Quasi Newton condition .

Introduction:

Many methods of the unconstrained optimization generate a Hessian matrix satisfying each of the principal properties of this matrix (positive definite and symmetric property). One of these methods is Boyden method, where the Hessian matrix is given in the following form

$$B_{k+1} = B_k + \frac{(y_k - B_k s_k) s_k^T}{\|s_k\|^2} \quad \ldots(1)$$

Where

B_{k+1} is the next approximation of Hessian matrix,
B_k is the current approximation of Hessian matrix ,
$y_k = F(x_{k+1}) - F(x_k)$[5], … (2)
$s_k = x_{k+1} - x_k$[5], … (3)
$f: R^n \rightarrow R$ is continuous and differentiable function
x_{k+1} is the next sollution,
x_k is the current solution . $F(x_k)$ be the gradient of f at x_k .

The positive definite property is very important to guarantee the existence of the minimizer of the objective function, because the Hessian matrix is symmetric (f is continuous), so the symmetric property of matrix B_k is very important to guarantee the convergence of B_{k+1} to the orginal Hessian matrix .

Symmetric Positive Definite Broyden Rank One Update (SPBR1 update)

Given f:D ⊆R^n→R f is assummed to be continuous and differentiable on D, Based on Zhang Xu condition

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$$y_k^* = \lambda_k \, y_k \quad \ldots \quad (4)$$

where λ_k is a matrix can be determined to guarantee the symmetric and positive definite properties of B_{k+1}, from the Broyden update and (4)

$$B_{k+1} = B_k + \frac{(y_k^* - B_k s_k) s_k^T}{s_k^T s_k} \quad \ldots(5)$$

Now to guarantee the symmetric property of the next approximation of Hessian matrix, suppose that

$$s_k = y_k^* - B_k s_k \quad \ldots \quad (6)$$

By substituting (6)in (5) we get

$$B_{k+1} = B_k + \frac{s_k s_k^T}{s_k^T s_k} \quad \ldots(7)$$

From (6) $\lambda_k \; y_k = B_k s_k + s_k$ … (8)

The solution of the above equation for α_k is

$$\lambda_k = \frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k} \in R^{nxn} \quad \ldots \quad (9)$$

And hence

$$B_{k+1(SPBR1)} = B_k + \frac{\left(\left(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}\right) y_k - B_k s_k\right) s_k^T}{s_k^T s_k} \quad \ldots(10)$$

Theorem (1)

The Hessian matrix B_{k+1} in **SPBR1** update satisfies the extended Q-N condition. (Zhang xu condition [7]) $B_{k+1} s_k = y_k^*$

Proof:

$$B_{k+1} s_k = (B_k + \frac{s_k s_k^T}{s_k^T s_k}) s_k$$

$$= B_k s_k + \frac{s_k s_k^T}{s_k^T s_k} s_k$$

$$= B_k s_k + s_k$$

By equation (6) the proof is completed .

Theorem (2)
The Hessian matrix B_{k+1} produced by **SPBR1** update $B_{k+1(SPBR1)} = B_k + \frac{s_k s_k^T}{s_k^T s_k}$ is symmetric.
Proof:

$$B_{k+1(SPBR1)} = B_k + \frac{s_k s_k^T}{s_k^T s_k}$$

$$B_{k+1(SPBR1)}^T = B_k^T + \frac{s_k^{T^T} s_k^T}{s_k^T s_k}$$

Since $B_k^T = B_k$ and $s_k^{T^T} = s_k$, we obtain $B_{k+1(SPBR1)}^T = B_{k+1(SPBR1)}$

Theorem (3)
The Hessian matrix produced by SPBR1 update is positively definite .
Proof:
By taking the equation (5)
let $0 \neq z \in R^n$, then

$$z^T B_{k+1} z = z^T (B_k + \frac{(y_k^* - B_k s_k) s_k^T}{s_k^T s_k}) z$$

$$z^T B_{k+1} z = z^T B_k z + \frac{z^T (y_k^* - B_k s_k) s_k^T z}{s_k^T s_k} \quad \ldots \quad (11)$$

By substituting (6) in (11) we get

$$z^T B_{k+1} z = z^T B_k z + \frac{z^T s_k s_k^T z}{s_k^T s_k}$$

$$z^T B_{k+1} z = z^T B_k z + \frac{(s_k^T z)^T (s_k^T z)}{s_k^T s_k}$$

$$= z^T B_k z + \frac{\|s_k^T z\|^2}{\|s_k\|^2}$$

Since B_k is positive definite then $z_k^T B_k z_k > 0$, $\|s_k^T z\|$ and $\|s_k\|$ is always positive .Therefore $z^T B_{k+1} z > 0$.

3. Local Linear Convergence of SPBR1

In this section, the local linear convergence of **SPBR1** update will be proved.
Consider the formula (10), we need the following assumptions:

Assumption 1, [6]
(A) $f : R^n \to R$ is twice continuously differentiable on convex set $D \subset R^n$.
(B) $f(x)$ is uniformly convex , i.e. , ∃ a positive constants c and C ∋ ∀ $x \in L(x) = \{x \mid f(x) \leq f(x_0)\}$, we have:

$$c\|v\|^2 \leq v^T \nabla^2 f(x) v \leq C\|v\|^2 \quad \ldots (12)$$

for all $v \in R^n$, $c, C \in R^+$, x_0 is starting point.
The assumption 1.B implies that $\nabla^2 f(x)$ is positively definite on L(x), and that f has a unique minimizer x^* in L(x).
By definition of positive definite and since equation (1), satisfy Zhang Xu condition.
then $V\lambda_k y_k = s_k$ and $\lambda_k y_k = V^{-1} s_k$ then

$$\lambda_k y_k = \widetilde{G_k} s_k \quad \ldots (13)$$

where $\widetilde{G_k} = V^{-1}$.

Now by equations (11), (7) and from definition of positive definite we get:

$$c \leq \frac{(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}) y_k^T s_k}{\|s_k\|^2} = \frac{s_k^T \widetilde{G_k} s_k}{\|s_k\|^2} \leq C \quad \ldots (14)$$

Where $\widetilde{G_k}$ is the average Hessian matrix, which is defined as

$$\widetilde{G_k} = \int_0^1 \nabla^2 f(x_k + \mu s_k)\, d\mu$$

and $\frac{1}{C} \leq \frac{\|s_k\|^2}{((\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}) y_k)^T s_k} \leq \frac{1}{c}$

and also

$$\frac{\left\|\left(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}\right) y_k\right\|^2}{s_k^T \left(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}\right) y_k} = \frac{\|\widetilde{G_k} s_k\|^2}{s_k^T \left(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}\right) y_k}$$

$$= \frac{(\widetilde{G_k} s_k)^T (\widetilde{G_k} s_k)}{s_k^T \widetilde{G_k} s_k} = \frac{s_k^T \widetilde{G_k}^2 s_k}{s_k^T \widetilde{G_k} s_k}$$

Assumption 1.B, means that $\widetilde{G_k}$ is positively definite, thus its square root is well defined. This is a symmetric square root $\widetilde{G_k}^{\frac{1}{2}}$ which satisfies

$$\widetilde{G_k} = \widetilde{G_k}^{\frac{1}{2}} \cdot \widetilde{G_k}^{\frac{1}{2}}$$

If we let $v_k = \widetilde{G_k}^{\frac{1}{2}} s_k \quad \ldots \quad (15)$

then,

$$\frac{\left\|\left(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}\right) y_k\right\|^2}{s_k^T \left(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}\right) y_k} = \frac{s_k^T \widetilde{G_k}^{\frac{1}{2}} \widetilde{G_k}^{\frac{1}{2}} \widetilde{G_k}^{\frac{1}{2}} \widetilde{G_k}^{\frac{1}{2}} s_k}{s_k^T \widetilde{G_k}^{\frac{1}{2}} \widetilde{G_k}^{\frac{1}{2}} s_k}$$

$$= \frac{\left(\widetilde{G_k}^{\frac{1}{2}} s_k\right)^T \widetilde{G_k} \left(\widetilde{G_k}^{\frac{1}{2}} s_k\right)}{\left(s_k \widetilde{G_k}^{\frac{1}{2}}\right)^T \left(\widetilde{G_k}^{\frac{1}{2}} s_k\right)} \quad \ldots \quad (16)$$

By substituting equation (15) in equation (16), we get

$$\frac{\left\|(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}) y_k\right\|^2}{s_k^T \left(\frac{(B_k s_k + s_k) y_k^T}{y_k^T y_k}\right) y_k} = \frac{v_k^T \widetilde{G_k} v_k}{v_k^T v_k} \quad \ldots \quad (17)$$

and from (12) $u^T \nabla^2 f(x^*) u \leq C\|u\|^2$, we know $\|u\|^2 > 0$. Then by dividing both sides of (12) , we get:

$\frac{v_k^T \widetilde{G_k} v_k}{\|u_k\|^2} \leq C$ that mean $\frac{v_k^T \widetilde{G_k} v_k}{v_k^T v_k} \leq C$

Then from equation (17) , we get :

$$\frac{\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|^2}{s_k^T\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k} \le C$$

From equation (13), we get :

$$\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\| = \|\widetilde{G_k} s_k\|$$

$$\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\| \le \|\widetilde{G_k}\|\|s_k\|, \quad \|s_k\|$$

$$\le \left\|\widetilde{G_k}^{-1}\right\| \left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|$$

Which gives :

$$\frac{\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|}{\|s_k\|} \le C \quad \ldots(18)$$

and $$\frac{\|s_k\|}{\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|} \le \frac{1}{C} \quad \ldots (19)$$

Remark

We use $F(x)$ and $F'(x)$ to mean $\nabla f(x)$ and $\nabla^2 f(x)$ respectively.

Assumption 2, [6]

(A): $F: R^n \to R^n$ is continuosly differentiable in open convex set $D \subset R^n$.

(B): There is x^* in D, such that $F(x^*) = 0$ and $F'(x^*)$ is non singular.

(C): $F'(x)$ satisfies the Lipschitz condition at x^*, that is , there exists a constant $\beta > 0$ such that

$$\|F'(x) - F'(x^*)\| \le \beta\|x - x^*\|, \qquad x^*, x \in D$$

We begin with some general converged results.

Lemma (1), [6]

Let $f: R^n \to R$ satisfy Assumption (1), then

$$\frac{\|s_k\|}{\|y_k\|}, \frac{\|y_k\|}{\|s_k\|}, \frac{s_k^T y_k}{\|s_k\|^2}, \frac{s_k^T y_k}{\|y_k\|^2}, \frac{\|y_k\|^2}{s_k^T y_k}, \text{are}$$

bounded . "

Lemma (2)

Let $f: R^n \to R$ satisfy Assumption (1) then, $\frac{\|s_k\|}{\|y^*\|}, \frac{\|y^*\|}{\|s_k\|}, \frac{s_k^T y^*}{\|s_k\|^2}, \frac{s_k^T y^*}{\|y^*\|^2}, \frac{\|y^*\|^2}{s_k^T y^*}, \frac{s_k^T B_k s_k}{\|s_k\|^2}, \frac{s_k^T B_k H_k B_k s_k}{\|s_k\|^2}, \frac{s_k^T B_k H_k y^*}{\|s_k\|^2}, \frac{s_k^T H_k s_k}{\|s_k\|^2}$ are bounded.

Lemma (3), [6]

Under exact line search, $\sum\|s_k\|^2$ and $\sum\|y_k\|^2$ are convergent.

Lemma (4)

Under exact or inexact line search, $\sum\|s_k\|^2$ and $\sum\|y^*\|^2$ are convergent.

Proof: $\sum\|s_k\|^2$ is convergent. where $f(x^*)$ is the minimum of $f(x)$.

Now, we will prove $\sum\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|^2$ is convergent, from lemma (1), we have $\frac{\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|^2}{\|s_k\|^2}$ is bounded , then $\exists C$ is a positive number which is independent of k such that it satisfy (18). And from (18), we obtain

$$\left\|\left(\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|^2 \le C^2\|s_k\|^2$$

Since $\sum\|s_k\|^2$ is convergent, we get $\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|^2$ is bounded , then

$$\sum_{k=0}^{\infty}\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|^2 \le C^2\sum_{k=0}^{\infty}\|s_k\|^2$$

Which implies $\sum\left\|\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k\right\|^2$ is convergent, where $f(x^*)$ is the minmum value of $f(x)$.

Lemma (5)

For **SPBR1** update, the determinant of the next Hessian matrix is given by

$$|B_{k+1}| = |B_k| \left[\frac{s_k w_k}{s_k^T s_k}\right] \quad \text{where}$$

$$s_k = \left(\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k - B_k s_k\right),$$

$w_k = s_k - H_k s_k$, and $H_k = B_k^{-1}$.

Proof: we can prove from the **SPBR1** update (10)

$$|B_{k+1}| = \left|B_k + \frac{\left(\left(\frac{(B_k s_k+s_k)y_k^T}{y_k^T y_k}\right)y_k - B_k s_k\right)s_k^T}{s_k^T s_k}\right|$$

$$|B_{k+1}| = \left|B_k - \frac{s_k s_k^T}{s_k^T s_k}\right|$$

$$|B_{k+1}| = |B_k| \left|I - \frac{L_k^{-1} s_k s_k^T L_k^{T^{-1}}}{s_k^T s_k}\right|$$

Where $B_k = L_k L_k^T$

$$|B_{k+1}| = |B_k| \left|I - \frac{(L_k^{-1}s_k)(L_k^{-1}s_k)^T}{s_k^T s_k}\right|$$

Let $u_k = \frac{L_k^{-1}s_k}{s_k^T s_k}$, $v_k = L_k^{-1}s_k$, … (20)

By formula ($|I + u_k v_k^T| = 1 + u_k^T v_k$) and equation (20)

$$|B_{k+1}| = |B_k| \left[1 - \frac{(L_k^{-1}s_k)^T(L_k^{-1}s_k)}{s_k^T s_k}\right] \quad \ldots (21)$$

$$|B_{k+1}| = |B_k| \left[1 - \frac{(L_k^{-1}s_k)^T(L_k^{-1}s_k)}{s_k^T s_k}\right]$$

$$|B_{k+1}| = |B_k| \left[1 - \frac{s_k^T H_k s_k}{s_k^T s_k}\right]$$

Where $H_k = L_k^{-1^T} L_k^{-1}$

$$|B_{k+1}| = |B_k| \left[\frac{s_k^T s_k - s_k^T H_k s_k}{s_k^T s_k}\right]$$

$$\left|B_{k+1}\right| = \left|B_k\right|\left[\frac{s_k^T(s_k - H_k s_k)}{s_k^T s_k}\right]$$

Let $w_k = s_k - H_k s_k$, we get:

$$\left|B_{k+1}\right| = \left|B_k\right|\left[\frac{s_k^T w_k}{s_k^T s_k}\right] \qquad \dots (22)$$

Theorem (4)

Suppose that f(x) satisfies Assumption (1). Then under exact or inexact line search the sequence $\{x_k\}$ generated by **SPBR1** update converges to the minimizer x^* of f .

Proof :

Consider **SPBR1** update (10) of Hessian matrix

$$B_{k+1} = B_k + \frac{\left(\left(\frac{(B_k s_k + s_k)y_k^T}{y_k^T y_k}\right)y_k - B_k s_k\right)s_k^T}{s_k^T s_k}$$

Subtracting equation (6), in (10), we get :

$$B_{k+1} = B_k + \frac{s_k s_k^T}{s_k^T s_k} \qquad ...(23)$$

By taking the trace operator for both sides of equation (23), we get:"

$$tr(B_{k+1}) = tr(B_k) + \frac{s_k^T s_k}{s_k^T s_k} \qquad ... (24)$$

By taking the ln operator for equation (22), we get:

$$\ln\left(\left|B_{k+1}\right|\right) = \ln\left(\left|B_k\right|\frac{s_k^T w_k}{s_k^T s_k}\right) \qquad ... \quad (25)$$

Define $\psi(B_k) = tr(B_k) - \ln\left(\left|B_k\right|\right) \geq 0$ (26)

By replacing B_k by B_{k+1} in equation …(26)

$$\psi(B_{k+1}) = tr(B_{k+1}) - \ln\left(\left|B_{k+1}\right|\right) \qquad \dots (27)$$

By substitution (25), and (26), in (27), we get:

$$\psi(B_{k+1}) = tr(B_k) + 1 - \ln\left(\left|B_k\right|\frac{s_k^T w_k}{s_k^T s_k}\right)$$

$$= tr(B_k) + 1 - \ln\left(\left|B_k\right|\frac{s_k^T(s_k - H_k s_k)}{s_k^T s_k}\right)$$

$$= tr(B_k) + 1 - \ln\left(\left|B_k\right|\right) - \ln\left(s_k^T s_k - s_k^T H_k s_k\right) + \ln\left(s_k^T s_k\right) \qquad \dots (28)$$

Substituting (26) in (28), we get:

$$\psi(B_{k+1}) = \psi(B_k) + 1 - \ln\left(s_k^T s_k - s_k^T H_k s_k\right) + \ln\left(s_k^T s_k\right) \qquad \dots (29)$$

Without loss the generality of the prove we can assume that $\ln\left(s_k^T s_k - s_k^T H_k s_k\right)$ is positive.

$$\psi(B_{k+1}) \leq \psi(B_k) + 1 + \ln\left(s_k^T s_k\right) \quad ... (30)$$

Define $\cos\vartheta_k = \frac{s_k^T B_k s_k}{\|s_k\|\| B_k s_k\|}$ and $q_k = \frac{(s_k^T B_k s_k)^2}{\|s_k\|^4\| B_k s_k\|^2}$

$$s_k^T s_k = \frac{\cos^2\vartheta_k}{q_k} \qquad ... \quad (31)$$

Substituting (31), in (30) we get:

$$\psi(B_{k+1}) = \psi(B_k) + 1 + \ln\left(\frac{\cos^2\vartheta_k}{q_k}\right)$$

$$\psi(B_{k+1}) \leq \psi(B_k) + 1 + \ln(\cos^2\vartheta_k) - \ln(q_k) \qquad \dots (32)$$

Summing from j = 1 up to k

$$\sum_{j=1}^{k}\psi(B_{j+1}) \leq \sum_{j=1}^{k}\psi(B_j) + \sum_{j=1}^{k}\left(1 - \ln(q_k)\right) + \sum_{j=1}^{k}\ln\left(\cos^2\vartheta_j\right)$$

$$0 < \psi(B_2) + \dots + \psi(B_{k+1}) \leq \psi(B_1) + \dots + \psi(B_k) + C.k + \sum_{j=1}^{k}\ln\left(\cos^2\vartheta_j\right)$$

$$0 < \psi(B_{k+1}) \leq \psi(B_1) + C.k + \sum_{j=1}^{k}\ln\left(\cos^2\vartheta_j\right) \quad \dots (33)$$

Where the constant $C = 1 - \ln(q_k)$ is assumed to be positive without loss of generality and from Zoutendijk condition [4], we have:

$$\lim_{k\to\infty} F(x_k)\cos\vartheta_k = 0$$

If ϑ_k is bounded away from 90°, $\exists\mu \in R^+ \ni \cos\vartheta_k > \mu > 0$, for sufficient large k and hence $F(x_k) \to 0$ and the proof is complete.

Now, assume by contradiction that $\cos\vartheta_k \to 0$, then $\exists k_1 > 0 \ni \forall j > k_1$ we have

$$\ln\left(\cos^2\vartheta_j\right) < -2C \quad \dots (34)$$

By (33), we get:

$$0 < \psi(B_1) + C.k + \sum_{j=1}^{k_1}\ln\left(\cos^2\vartheta_j\right) + \sum_{j=k_1+1}^{k}\ln\left(\cos^2\vartheta_j\right) \quad , k_1 + 1 < k \quad \dots (35)$$

By use the equation (34), in (35), we have :

$$0 < \psi(B_1) + C.k + \sum_{j=1}^{k_1}\ln\left(\cos^2\vartheta_j\right) + \sum_{j=k_1+1}^{k}(-2C)$$

$$0 < \psi(B_1) + \sum_{j=1}^{k_1}\ln\left(\cos^2\vartheta_j\right) + 2C.k_1 - Ck.$$

Hence $\psi(B_{k+1}) < 0$ for sufficiently large k which is a contradiction then, $\cos^2\vartheta_j \to 0$ is not true. Then $\lim_{k\to\infty}\inf F(x_k) \to 0$ or $x_k \to x^*$.

Numerical Results:

This section is devoted to numerical experiments. Our purpose is to check whether the **SPBR1** algorithm provides improvements on the corresponding standard Broyden algorithm. The programs are written in MATLAB with single precision. The test functions commonly use unconstrained test problems with the same starting point and a summary of which is given in Table (1). The test functions are chosen as follows:

1 − Least square equation for two dimention [2]

$f(x) = (1 - x_1)^2 + (1 - x_2)^2$.

2 − A quartic function . [2]

$f(x) = \sum_{i=1}^{4}\left(10^{i-1}x_i^4 + x_i^3 + 10^{1-i}x_i^2\right)$.

3 − Rosenbroc'k cliff function [3]

$f(x) = 10^{-4}(x_1 - 3)^2 - (x_1 - x_2) + e^{20(x_1 - x_2)}$.

4 − Generalized Edger function [2]

$f(x) = \sum_{i=1}^{n/2}\left[(x_{2i-1} - 2)^4 + (x_{2i-1} - 2)^2 x_{2i}^2 + (x_{2i} + 1)^2\right]$.

5 − Rosen rock's function [3]

$$f(x)=\sum_{i=1}^{n/2}[100(x_i-x_i^3)^2+(1-x_i)^2].$$

6 − Trigonometric function [3]

$$f(x)=\sum_{i=1}^{n}\left[n-\sum_{j=1}^{n}\cos x_j+i(1-\cos x_i)-\sin x_i+e^{xi}-1\right]^2.$$

7 − Watson function [2]

$$F(x)=\sum_{i=1}^{j}f_i^2(x)$$

$$f_i(x)=\sum_{j=2}^{3}(j-1)x_jt_j^{j-2}-\left(\sum_{j=1}^{3}x_jt_j^{j-1}\right)^2-1.$$

and $t_j=\frac{i}{29}$.

8-Trigonometric function [2]

$$f(x)=100(x_2-\sin x_1)^2+0.25x_1^2$$

9- Rosen function [1]

$$f(x)=100(x_2-x_1^2)^2+(1-x_1)^2$$

10- Cubic function [1]

$$f(x)=100(x_2-x_1^3)^2+(1-x_1)^2$$

11- Trigonometric function [6]

$$f(x)=\sum_{i=1}^{n}\left[n+i(1-\cos x_i)-\sin x_i-\sum_{j=1}^{n}\cos x_j\right]^2$$

12-Extended Rosenbrock function [6]

$$f(x)=\sum_{i=1}^{n-1}[100(x_{i+1}-x_i^2)^2+(1-x_i)^2]$$

13 − Least sequare function [6]

$$f(x)=(x_1-2)^2+(x_2-1)^2$$

Table 1.Numerical results for Broyden and SPBR1 update

Fun.	Starting point	Dim.	Broyden		SPBR1	
			Feval	Iter.	Feval	Iter.
1	$(-15;5)^T$	2	2.8692e-015	2	2.8692e-015	2
1	$(0.5;-0.8)^T$	2	5.0919e-017	2	5.0919e-017	2
1	$(1/5;1/5)^T$	2	3.7024e-017	2	3.7024e-017	2
2	$(-1;0;0;0)^T$	4	-0.1055	3	-0.1055	3
2	$(-2;0;0;0)^T$	4	-0.1055	3	-0.1055	3
2	$(0;5;0;5)^T$	4	11.6337	67	-2.3963	12
3	$(12;4.8)^T$	2	Nan	2	1.9485e+016	2
3	$(12;5)^T$	2	Nan	2	1.0458e+059	2
3	$(12;4.9)^T$	2	Nan	2	5.7100e+060	2
4	$(2;-2)^T$	2	1.5103e-0.16	2	1.5103e-016	2
4	$(1/4;1/4)^T$	2	4.4951e-008	9	2.4152e-010	6
4	$(0;1)^T$	2	5.7967e-010	13	5.004e-010	7
5	$(5;2)^T$	2	0.9901.	7	0.9901	5
5	$(1.2;1.2)^T$	2	5.2958e-015	3	1.8357e-015	3
5	$(1.2;0)^T$	2	5.2958e-015	3	5.2958e-015	3
6	$(0.5;0.5)^T$	2	3.9206e-005	31	1.8340e-006	4
6	$(2;-2)^T$	2	0.0091	39	6.7395e-007	5
6	$(-5;5)^T$	2	0.0153	34	1.4279e-006	18
7	$(\frac{1}{8};\frac{1}{8};\frac{1}{8})^T$	3	6.9016e-017	3	2.8104e-018	3
7	$(-1;0;0)^T$	3	1.6335e-016	4	3.2750e-019	4
7	$(0.5;0.5;0.5)^T$	3	4.0131e-013	3	2.6114e-018	3
8	$(\frac{2}{3};\frac{1}{3})^T$	2	2.4564e-005	19	1.7914e-005	10
8	$(\frac{1}{2};\frac{1}{4})^T$	2	Nan	62	4.8289e-005	32
8	$(3.3;6.6)^T$	2	2.0472e-004	5	7.9348e-008	10
9	$(1.1;1.1)^T$	2	0.0011	32	4.7181e-007	6
9	$(1;1)^T$	2	0	1	0	1
9	$(1.2;1)^T$	2	3.2764e-006	21	3.2764e-006	21
10	$(1.5;1)^T$	2	0.0496	37	7.4441e-006	9
10	$(1;1.2)^T$	2	0.0026	8	1.2313e-005	26
10	$(-15;5)^T$	2	Nan	44	2.9365e-006	11
11	$(0.5;0.5)^T$	2	0.0043	30	1.1238e-006	10
11	$(-1;0)^T$	2	7.0570e-008	7	2.0684e-012	4
11	$(\frac{1}{4};\frac{1}{4})^T$	2	4.0107e-008	13	1.70420e-009	5
12	$(0;11)^T$	2	9.8078e-006	19	0.3805	58
12	$(1.1;1.1)^T$	2	0.0011	32	4.7181e-007	6
12	(1;1;1;1;1;1)	6	0	1	0	1
13	$(\frac{1}{4};\frac{1}{4})^T$	2	2.0118e-018	2	2.0118e-018	2
13	$(10;3)^T$	2	2.3306e-015	2	2.3306e-015	2
13	$(4;5)^T$	2	5.4879e-018	2	5.4879e-018	2

Remark: the mean of Nan is $\frac{\infty}{\infty}$ or $0.\infty$ or $\frac{0}{0}$.

From Table (1), it's clear that SPBR1 has a performance better than Broyden update, and we can note that the Broyden update cannot solve problem 3 at all starting points because the singularity of the Hessian matrix but SPBR1 can get the minimum of this problem at all starting points. Moreover, the Broyden update in a several test problems cannot get the minimizer and the program break the loop before getting the minimizer but SPBR1 can continue to the minimizer and that's clear in problems 2, 4, 6,8, and 10.

Conclusion:

In this paper, the Broyden update is modified to guarantee the symmetric and positively definite properties and the so called symmetric positive definite Broyden update. The convergence of the proposed method is established and the numerical results are reported in table (1). It can be seen that the proposed method in most tests is better than the Broyden method.

Conflicts of Interest: None.

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تحديث برويدن متناظر وموجب التعريف للمثلية غير المقيدة

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الخلاصه:

تحديث برويدن هو احد التحديثات من الرتبة الاولى الذي يعطي حل لمسائل الامثلية غير المقيدة ولكن هذا التحديث لا يضمن التعريف الموجب وخاصية التناظر للمصفوفة الهيسيه. في هذا البحث قمنا باقتراح تطوير على تحديث برويدن يضمن التناظر والتعريف الموجب للمصفوفة الهيسيه وذلك بتحديث المتجه y_k والذي يمثل الفرق بين الانحدار التالي والانحدار الحالي لدالة الهدف التي فرضناها مستمرة وقابله للاشتقاق مرتين.

قمنا بتقديم تقرير حول النتائج العددية حيث اجرينا مقارنة بين الطريقة المقترحة و طريقة برويدن لمسائل قياسية.

الكلمات المفتاحية: تحديث برويدن، مصفوفة هيسين، شرط شبه نيوتن