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Estimation of Survival Function for Rayleigh Distribution by Ranking function:-

Iden H. Hussein * *Hadeer A. KHammas*

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Abstract:

In this article, performing and deriving te probability density function for Rayleigh distribution is done by using ordinary least squares estimator method and Rank set estimator method. Then creating interval for scale parameter of Rayleigh distribution. Anew method using ($\bar{x}$ $\pm s^2$) is used for fuzzy scale parameter. After that creating the survival and hazard functions for two ranking functions are conducted to show which one is beast.

Key words: Fuzzy number, Hazard function, Ordinary least squares estimator method, Rank set estimator method, Survival function.

Introduction:

One of the most popular functions in statistic is Rayleigh distribution which used in failure and survival times. Many authors tend to fuzzfiy data in studying some distribution as follows:-

In (2013) Pak and Saraj (1) studied two parameters of weibull distribution. In (2014) Pak and Saraj (2) studied the parameter of exponential distribution. In (2014) Shafiq and Viertl (3) used the maximum likelihood estimator for two parameters of weibull distribution. In (2016) Pak (4) studied inference for one parameter of lognormal distribution. In (2016) Jasim and Hussein (5) studied the two parameters of weibull distribution by using maximum likelihood method. In (2017) Shafiq (6) studied the two parameters of Pareto distribution. In (2017) Shafiq (7) studied statistical inference for the two parameters of Lindley distribution. The aim of this article is to estimate the parameter of Rayleigh distribution by using ordinary least squares method and rank set method then estimating the survival and hazard functions. After that, the researcher uses interval estimation to find the scale parameter of Rayleigh the distribution. The estimation is fuzzfied of scale parameter by using trapezoidal membership depending on ($\bar{x}$ $+s^2$) and ($\bar{x}$ $-s^2$) to fuzzify this parameter, then utilizing the ranking function procedure to transform the fuzzy parameter to crisp parameter.

Department of Applied Science, University of Technology, Baghdad, Iraq.
*Corresponding author: Iden_Alkanani@yahoo.com

Finally, the researcher estimates the fuzzy survival and hazard performance the compare between crisp and fuzzy survival functions by using mean square error to know which one is better.

Rayleigh Distribution:-

The Rayleigh distribution is widely used in Probability, Reliability, and Survival analysis. The Rayleigh distribution is as follows:-

$$f(t;\, B) = \begin{cases} Bt\, e^{-\frac{B}{2}t^2} & 0 \le t < \infty \\ 0 & o.w \end{cases} \quad \ldots.(1)$$

$\Omega = \{B; B > 0\}$, where B is scale parameter.

The cdf function of Rayleigh distribution is:-

$$F(t) = 1 - e^{-\frac{B}{2}t^2} \quad \ldots.(2)$$

The survival function and hazard of Rayleigh distribution is:-

$$S(t) = e^{-\frac{B}{2}t^2} \quad \ldots.(3)$$

$$h(t) = Bt \quad \ldots.(4)$$

Ordinary Least Squares Method :-

The ordinary least squares method is one of the most popular procedures to estimate the parameter B in this distribution. The aim of the ordinary least square method is minimizing the sum squares of error.

In this method, the CDF of one-parameter Rayleigh distribution is used as follows:-

$$F(t_i) = 1 - e^{\frac{-B\,t_i^2}{2}} \quad t \in [0, \infty) \quad \ldots.(5)$$

Taking the Logarithm for the function above, and equaling it to zero we get:-

$\ln[1-F(t_i)]+\frac{Bt_i^2}{2}=0$ …. (6)

$s(B)=\sum_{i=1}^{n}[\ \ln(1-F(t_i))+\frac{Bt_i^2}{2}\ ]^2$ …. (7)

Taking the partial derivatives for the above equation, then:-

$\frac{\partial s(B)}{\partial B}=2\sum_{i=1}^{n}[\ln(1-F(t_i))+\frac{Bt_i^2}{2}].\ \frac{t_i^2}{2}$ …. (8)

$\frac{\partial s(B)}{\partial B}=\sum_{i=1}^{n}[t_i^2\ln\left(1-F(t_i)\right)+\frac{Bt_i^4}{2}]$ …. (9)

Equaling the partial derivative for log-likelihood with respect to zero, the equation is:-

$\frac{\partial s(B)}{\partial B}=\sum_{i=1}^{n}[t_i^2\ln\left(1-F(t_i)\right)+\frac{B^{\wedge}t_i^4}{2}]=0$ …. (10)

$B^{\wedge}=\frac{-2\sum_{i=1}^{n}\ln(1-F(t_i))t_i^2}{\sum_{i=1}^{n}t_i^4}$ …. (11)

Rank Set Method:-

Rank set sampling estimator method (RSS) was introduced by McIntyre for the first time in (1952) for estimating pasture yields.

The procurer to compute to estimator for Relight distribution is:-

$g(y_i)=\frac{n!}{(i-1)!(n-i)!}[F(y_i)]^{i-1}[1-F(y_i)]^{n-i}f(y_i$ …. (12)

By using the p. d. f 0f one-parameter Rayleigh distribution is:-

$f(t_i;B)=Bt_ie^{-\frac{Bt_i^2}{2}}$ …. (13)

$Put\ \ f(t_i;B)=\ f(y_i;B)$

$where\ f(y_i;B)=By_ie^{-\frac{By_i^2}{2}}$ …. (14)

The c .d .f of one –parameter Rayleigh distribution is:-

$F(t_i;B)=1-e^{-\frac{Bt_i^2}{2}}$ …. (15)

Therefore $F(t_i;B)=\ F(y_i;B)$ …. (16)

$F(y_i;B)=1-e^{-\frac{By_i^2}{2}}$ …. (17)

$Let\ \frac{n!}{(i-1)!(n-i)!}=k$ …. (18)

$g(y_i)=k\,By_i[e^{-\frac{By_i^2}{2}}]^{n-i+1}\,[1-e^{-\frac{By_i^2}{2}}]^{i-1}$ …. (19)

The likelihood function of sample $y_{1,}y_{2,}y_{3,,,,,,,,,,,}y_n$ is: (20)

$L(B;y_i)=$

$k^nB^n\prod_{i=1}^{n}y_i\,e^{-\sum_{i=1}^{n}(n-i+1)\frac{By_i^2}{2}}.\prod_{i=1}^{n}[1-e^{-\frac{By_i^2}{2}}]^{i-1}$

Taking the loqarithm ofabove equation, getting : - (21)

$$ln\,L=n\,ln\,k+n\,ln\,B+\sum_{i=1}^{n}ln\,y_i-\sum_{i=1}^{n}(n-i+1)\frac{By_i^2}{2}+\sum_{i=1}^{n}(i-1)\,ln\left[1-e^{-\frac{By_i^2}{2}}\right]$$

Taking the partial derivatives for above equation, then: - (22)

$\frac{\partial\,ln\,L}{\partial B}=\frac{n}{B}-\sum_{i=1}^{n}(n-i+1)\frac{y_i^2}{2}+\sum_{i=1}^{n}(i-1)\frac{(-e^{-\frac{By_i^2}{2}}\frac{-y_i^2}{2})}{1-e^{-\frac{By_i^2}{2}}}$

equall above equation to zero as follows: –

$\frac{\partial\ln L}{\partial B}=\frac{n}{B^{\wedge}}-\sum_{i=1}^{n}(n-i+1)\frac{y_i^2}{2}+\sum_{i=1}^{n}(i-1)\frac{\left(e^{-\frac{B^{\wedge}y_i^2}{2}}\frac{y_i^2}{2}\right)}{1-e^{-\frac{B^{\wedge}y_i^2}{2}}}=0$ …. (23)

$g\left(y_i\ ,\ B^{\wedge}\right)=\frac{n}{B^{\wedge}}-\sum_{i=1}^{n}(n-i+1)\frac{y_i^2}{2}+\sum_{i=1}^{n}(i-1)\frac{\left(e^{-\frac{B^{\wedge}y_i^2}{2}}\frac{y_i^2}{2}\right)}{1-e^{-\frac{B^{\wedge}y_i^2}{2}}}$ …. (24)

This likelihood functions are difficult to be solved. It is impossible to find the estimate B. We use the numerical procedure to estimate B, that means using the following formula

$\tilde{B}_{k+1}=\tilde{B}_k-\frac{g(y_i\ ,B)}{g'(y_i\ ,B)}$ …. (25)

$g'\left(y_i\ ,\ B^{\wedge}\right)=-\frac{n}{B^{\wedge 2}}+\sum_{i=1}^{n}(i-1)\frac{\frac{-y_i^4}{4}e^{-\frac{B^{\wedge}y_i^2}{2}^2}}{\left(1-e^{-\frac{B^{\wedge}y_i^2}{2}}\right)^2}$ …. (26)

- The interval estimation is as follows:-

$[B^{\wedge}-t\sqrt{var(B^{\wedge})}\ ,B^{\wedge}+t\sqrt{var(B^{\wedge})}\ ]$ …. (27)

Fuzzy Sets (8):-

Definition (1) (9): A crisp set is a special case of a fuzzy set, in which the membership function has only two values, 0 and 1.

Definition (2) (9): Let$x$ be a nonempty set (universal set).A fuzzy set$\tilde{A}$ in x is characterized by its membership function $\mu_{\tilde{A}}:x\to[0,1]$ $\mu_{\tilde{A}}(x)$ is the interpreted as a degree of membership of element x in fuzzy set A for each $x\in X$ and denoted for its set by $\tilde{A}$. $\tilde{A}=\{(x,\mu_{\tilde{A}}(x):x\in X\}$

Definition (3) (9): The fuzzy set $\tilde{A}$ is normal if its core is nonempty equivalently; we can find at least one element $x\in X$ s.t $\mu_{\tilde{A}}(x)$ =1

Ranking Function (10):-

The method of ranking function was first introduced by Yager in (1981) proposed four indices that may be employed for the purpose of ordering fuzzy quantities in [0,1].

A ranking function is defined $R: F(R) \rightarrow R$, which maps each fuzzy number into the real line. Now, suppose that $\tilde{\alpha}$ and $\tilde{b}$ are two trapezoidal fuzzy numbers. Therefore, the orders on $F(R)$are defined as following:-

(1) $\tilde{\alpha} \geq \tilde{b}$ if and only if $R(\tilde{\alpha}) \geq R(\tilde{b})$
(2)$\tilde{\alpha} > \tilde{b}$ if and only $R(\tilde{\alpha}) > R(\tilde{b})$
(3)$\tilde{\alpha} = \tilde{b}$ If and only if $R(\tilde{\alpha}) = R(\tilde{b})$ where $\tilde{\alpha}$ and $\tilde{b}$ are in $F(R)$. Also
$\tilde{\alpha} \leq \tilde{b}$ If and only if $\tilde{\alpha} \geq \tilde{b}$

Lemma:-(10) let R be any linear ranking function then:-

i- $\tilde{\alpha} \geq \tilde{b}$ iff $-\tilde{b} \geq 0$ iff $-\tilde{b} \geq \tilde{\alpha}$
ii- if $\tilde{\alpha} \geq \tilde{b}$ and $\tilde{c} \geq \tilde{d}$, then $\tilde{\alpha} + \tilde{c} \geq \tilde{d} + \tilde{b}$

Algorithms of the Ranking Function:-

The First Algorithm:-

Yager (1981) (11) studied the ranking function, $R: F(R) \rightarrow R$

Let $\tilde{A} = (a, b, c, d)$ be trapezoidal fuzzy number, and then the following formula is applied to find the ranking function of $\tilde{A}$

$R(\tilde{A}) = \frac{1}{2} \int_0^1 (inf\tilde{A}_\mu + sup\tilde{A}_\mu)\, d\mu$

Let $\mu^4 = \frac{(x-a)}{b-a}$ by using inverse transformation:-

$\mu^4(b-a) = (x-a)$

$x = \mu^4(b-a) + a = inf\tilde{A}_\mu$

$\mu^2 = \frac{(d-x)}{(d-c)}$ by using inverse transformation

$\mu^2(c-d) = (x-d)$

$x = \mu^2(c-d) + d = sup\tilde{A}_\mu$

$R(\tilde{A}) = \frac{1}{2} \int_0^1 (\mu^4 b - \mu^4 a + a)\, d\mu + \int_0^1 (\mu^2 c - \mu^2 d + d)\, d\mu$

$R(\tilde{A}) = \frac{1}{30}\, [3b + 12a + 5c + 10d]$ …. (28)

The Second Algorithm:-

Maleki (2002) studied the ranking function, $R: F(R) \rightarrow R$

Let $\tilde{A} = (a, b, c, d)$ be trapezoidal fuzzy number, and then the following formula is applied to find the ranking function of $\tilde{A}$

$R(\tilde{A}) = \frac{1}{2} \int_0^1 (inf\tilde{A}_\mu + sup\tilde{A}_\mu)\, d\mu$

$R(\tilde{A}) = \frac{1}{2} \int_0^1 (inf\tilde{A}_\mu + sup\tilde{A}_\mu)\, d\mu$

$\mu = \frac{(x-a)}{b-a}$ by using inverse transformation:-

$\mu\,(b-a) = (x-a)$

$x = \mu\,(b-a) + a = inf\tilde{A}_\mu$

$\mu = \frac{(d-x)}{(d-c)}$ by using inverse transformation

$\mu\,(c-d) = (x-d)$

$x = \mu\,(c-d) + d = sup\tilde{A}_\mu$

$R(\tilde{A}) = \frac{1}{2} \int_0^1 (\mu\,(b-a) + a + \mu\,(c-d) + d)\, d\mu$

$R(\tilde{A}) = \frac{1}{4}[\, b + a + c + d]$ …. (29)

Definition (4) (11): The support of a fuzzy set $\tilde{A}$, $S(\tilde{A})$ is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}} > 0$ i.e. supp($\tilde{A}$) = $\{x \in X: \mu_{\tilde{A}}(x) > 0\}$

Definition (5) (11):-The height h (A) of a fuzzy set A is the largest membership grade obtained by any element in that set, formally, $h(A) = sup_{x \in X} A(x)$

Definition (6) (11): The elements of x, such that $\mu_{\tilde{A}}(x) = \frac{1}{2}$ are called crossover points of $\tilde{A}$.

Definition (7) (11): The crisp set of element that belongs to the fuzzy set $\tilde{A}$ at least to the degree $\propto$ is called the $\propto$ -level set that is:- $A_\propto = \{x \in X: \mu_{\tilde{A}} \geq \propto\}$

$A'_\propto = \{x \in X: \mu_{\tilde{A}} > \propto\}$ Is called strong $\propto$ - level set or strong $\propto$-cut.

Trapezoidal Function:-(11)

A fuzzy number $\tilde{A}(a, b, c, d; 1)$ is said to be a trapezoidal fuzzy number if its membership function is given by:-

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{b-a} & , a \leq x < b \\ 1 & , b < x \leq c \\ \frac{(d-x)}{(d-c)} & , c < x \leq d \\ 0 & otherwise \end{cases}$$

Mean Time To failure:-

MTTF=$\int_0^\infty s(t)\, dt$, MTTF=$\int_0^\infty e^{-\frac{B}{2}t^2}\, dt$

Let u=$\frac{B}{2}t^2$, $du = \frac{1}{\sqrt{B}\sqrt{2u}}$

MTTF=$\int_0^\infty e^{-u} \frac{1}{\sqrt{B}\sqrt{2}}\, u^{-\frac{1}{2}} du$

MTTF=$\frac{\sqrt{\pi}}{\sqrt{B}\sqrt{2}}$ …. (30)

- The mean squared error by following equation is:-

MSE $[S^{\wedge}(t_i)] = \sum_{i=1}^{n} \frac{[S^{\wedge}(t_i) - S(t_i)]^2}{n}$

…. (31) Where: - $S^{\wedge}(t_i)$ Is estimated survival function, $S(t_i)$ is empirical survival which:-

$S(t_i) = \frac{i-0.5}{n}$

Application:-

Choosing real data for lung cancer disease because it is widespread and deadly in Iraq. Depending data for the lung cancer disease from Radiation and Nuclear Medicine Hospital. For period 1-1-2017 until 31-12-2017 .The number of patients in this time is (68): twenty patients are dead

and forty eight patients remained alive, this means the data became complete data are (20) patients where:-
T=[15,22,26,30,35,42,44,58,60,65,66,71,73,75,80,86,91,104,121,190]

(a) – Ordinary Least Squares Method:-
***** The value of$B^{\wedge}$ from equation (11) is:- $B^{\wedge} = 0.00026$
***** f(t), s(t), h(t) from equations (1), (3), (4) then tabulating in following table:-

Table 1. Estimate value for functions $\boldsymbol{f(t), S(t), h(t)}$ functions

T	f(t)	S(t)	h(t)
15	0.0038	0.9711	0.0039
22	0.0054	0.9389	0.0057
26	0.0062	0.9157	0.0068
30	0.0070	0.8894	0.0078
35	0.0078	0.8525	0.0091
42	0.0087	0.7947	0.0109
44	0.0089	0.7771	0.0115
58	0.0097	0.6452	0.0151
60	0.0098	0.6257	0.0156
65	0.0098	0.5768	0.0169
66	0.0097	0.5670	0.0172
71	0.0096	0.5186	0.0185
73	0.0095	0.4995	0.0190
75	0.0094	0.4806	0.0195
80	0.0091	0.4345	0.0208
86	0.0085	0.3816	0.0224
91	0.0081	0.3401	0.0237
104	0.0066	0.2445	0.0271
121	0.0047	0.1485	0.0315
190	0.0004	0.0091	0.0495

-By applying the equation (30) is: - MTTF=77.7075
- By applying the equation (31) is: - MSE $[S^{\wedge}(t_i)]$ =0.3117

***** To find the interval estimation applying the equation (27) as follows:-

$B^{\wedge} = [0.00013,0.00038]$ = [a, d]

***** Then applying $(\bar{x} - s^2) = b$ and $(\bar{x} + s^2) = c$, therefore the trapezoidal becomes as follow:-

$B^{\wedge} = [0.00013,0.00024,0.00025,0.00038]$

(1)- applying the first ranking function by using equation (28) as follow:-

$B^{\wedge} = 0.00024$

Finding the f(t), s(t), h(t) and tabulating in following table:-

Table 2. Estimate value for functions $\boldsymbol{f(t), S(t), h(t)}$ functions

T	f(t)	S(t)	h(t)
15	0.0035	0.9734	0.0036
22	0.0050	0.9436	0.0053
26	0.0058	0.9221	0.0062
30	0.0065	0.8976	0.0072
35	0.0073	0.8633	0.0084
42	0.0082	0.8092	0.0101
44	0.0084	0.7927	0.0106
58	0.0093	0.6679	0.0139
60	0.0093	0.6492	0.0144
65	0.0094	0.6023	0.0156
66	0.0094	0.5929	0.0158
71	0.0093	0.5461	0.0170
73	0.0092	0.5276	0.0175
75	0.0092	0.5092	0.0180
80	0.0089	0.4639	0.0192
86	0.0085	0.4117	0.0206
91	0.0081	0.3702	0.0218
104	0.0068	0.2731	0.0250
121	0.0050	0.1726	0.0290
190	0.0006	0.0131	0.0456

- By applying the equation (30) is: - MTTF=80.8806
- By applying the equation (31) is: - MSE $[S^{\wedge}(t_i)]$ =0.3089

*** (2)** applying the second ranking function method by using equation (29) as follow:- $B^{\wedge} = 0.00025$
Finding the f (t), s (t), h (t) and tabulating in following table:-

Table 3. Estimate value for functions $\boldsymbol{f(t), S(t), h(t)}$ functions

t	f(t)	S(t)	h(t)
15	0.0036	0.9723	0.0037
22	0.0052	0.9413	0.0055
26	0.0060	0.9190	0.0065
30	0.0067	0.8936	0.0075
35	0.0075	0.8580	0.0088
42	0.0084	0.8021	0.0105
44	0.0086	0.7851	0.0110
58	0.0095	0.6567	0.0145
60	0.0096	0.6376	0.0150
65	0.0096	0.5897	0.0163
66	0.0096	0.5801	0.0165
71	0.0095	0.5325	0.0178
73	0.0094	0.5137	0.0182
75	0.0093	0.4950	0.0187
80	0.0090	0.4493	0.0200
86	0.0085	0.3967	0.0215
91	0.0081	0.3552	0.0227
104	0.0067	0.2587	0.0260
121	0.0049	0.1604	0.0302
190	0.0005	0.0110	0.0475

-By applying the equation (30) is: - MTTF=79.2465
- By applying the equation (31) is: - MSE $[S^{\wedge}(t_i)]$ =0.3103

(b)-Rank Set Method:-

***** The value of$B^{\wedge}$ from equation (25)

$$B^{\wedge}=0.00036$$

***** f(t), s(t), h(t) from equations (1), (3), (4) then tabulating in following table:-

Table 4. Estimate value for functions $\boldsymbol{f(t), S(t), h(t)}$ functions

T	f(t)	S(t)	h(t)
15	0.0052	0.9599	0.0055
22	0.0073	0.9158	0.0080
26	0.0084	0.8844	0.0095
30	0.0093	0.8491	0.0109
35	0.0102	0.8004	0.0127
42	0.0111	0.7257	0.0153
44	0.0113	0.7034	0.0160
58	0.0114	0.5426	0.0211
60	0.0113	0.5198	0.0218
65	0.0110	0.4640	0.0236
66	0.0109	0.4531	0.0240
71	0.0103	0.4000	0.0258
73	0.0101	0.3796	0.0265
75	0.0098	0.3597	0.0273
80	0.0091	0.3125	0.0291
86	0.0082	0.2607	0.0313
91	0.0073	0.2220	0.0331
104	0.0053	0.1400	0.0378
121	0.0031	0.0699	0.0440
190	0.0001	0.0014	0.0691

- By applying the equation (30) is: - MTTF=66.0387

- By applying the equation (31) is: - MSE $[S^{\wedge}(t_i)]$ =0.3265

***** To find the interval estimation applying the equation (27) as follows:-

$$B^{\wedge}=[0.00019,0.00052]=[a,d\ ]$$

***** Then applying $(\ \bar{x}\ -s^2)=b$ and $(\bar{x}\ +s^2)=c$, therefore the trapezoidal becomes as follow:-

$$B^{\wedge}=[0.00019,0.00034,0.00035,0.00052]$$

(1)- applying the first ranking function by using equation (28) as follow:-

$$B^{\wedge}=0.00034$$

Finding the f(t), s(t), h(t) and tabulating in following table:-

Table 5. Estimate value for functions $\boldsymbol{f(t), S(t), h(t)}$ functions

T	f(t)	S(t)	h(t)
15	0.0049	0.9625	0.0051
22	0.0069	0.9210	0.0075
26	0.0079	0.8914	0.0088
30	0.0088	0.8581	0.0102
35	0.0097	0.8120	0.0119
42	0.0106	0.7409	0.0143
44	0.0108	0.7196	0.0150
58	0.0111	0.5645	0.0197
60	0.0111	0.5423	0.0204
65	0.0108	0.4876	0.0221
66	0.0107	0.4769	0.0224
71	0.0102	0.4244	0.0241
73	0.0100	0.4042	0.0248
75	0.0098	0.3843	0.0255
80	0.0092	0.3369	0.0272
86	0.0083	0.2844	0.0292
91	0.0076	0.2447	0.0309
104	0.0056	0.1590	0.0354
121	0.0034	0.0830	0.0411
190	0.0001	0.0022	0.0646

- By applying the equation (30) is: - MTTF=67.9533

- By applying the equation (31) is: - MSE $[S^{\wedge}(t_i)]$ =0.3235

*** (2)** applying the second ranking function method by using equation (29) as follow:- $B^{\wedge}=0.00035$

Finding the f(t), s(t), h(t) and tabulating in following table:-

Table 6. Estimate value for functions $\boldsymbol{f(t), S(t), h(t)}$ functions

T	f(t)	S(t)	h(t)
15	0.0050	0.9614	0.0053
22	0.0071	0.9188	0.0077
26	0.0081	0.8884	0.0091
30	0.0090	0.8543	0.0105
35	0.0099	0.8070	0.0123
42	0.0108	0.7344	0.0147
44	0.0110	0.7126	0.0154
58	0.0113	0.5550	0.0203
60	0.0112	0.5326	0.0210
65	0.0109	0.4774	0.0227
66	0.0108	0.4666	0.0231
71	0.0103	0.4139	0.0249
73	0.0101	0.3935	0.0256
75	0.0098	0.3737	0.0262
80	0.0091	0.3263	0.0280
86	0.0083	0.2741	0.0301
91	0.0075	0.2348	0.0318
104	0.0055	0.1506	0.0364
121	0.0033	0.0771	0.0423
190	0.0001	0.0018	0.0665

- By applying the equation (30) is: - MTTF=66.9755

-By applying the equation (31) is: - MSE $[S^{\wedge}(t_i)]$ =0.3250

- Algorithms Comparison:-

Algorithm	MTTF	MSE
Crisp $\begin{cases} OLS \\ R.S \end{cases}$	$\begin{cases} 77.7075 \\ 66.0387 \end{cases}$	$\begin{cases} 0.3117 \\ 0.3265 \end{cases}$
First Algorithm $\begin{cases} OLS \\ R.S \end{cases}$	$\begin{cases} 80.8806 \\ 67.9533 \end{cases}$	$\begin{cases} 0.3089 \\ 0.3235 \end{cases}$
second Algorithm $\begin{cases} OLS \\ R.S \end{cases}$	$\begin{cases} 79.2465 \\ 66.9755 \end{cases}$	$\begin{cases} 0.3103 \\ 0.3250 \end{cases}$

-Noting that from above table, that minimum mean squares error is of first algorithm of ordinary least squares method but the high mean squares error is crisp of rank set method. Therefor the mean time of failure the first algorithm of ordinary least squares method, but the minimum mean time to failure is crisp of rank set method.

Conflicts of Interest: None.

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تقدير دالة البقاء لتوزيع رالي باستخدام الدالة الرتبية

ايدن حسن حسين **هدير أحمد خماس**

قسم الرياضيات، كلية العلوم للبنات، جامعة بغداد، بغداد، العراق.

الخلاصة:

في هذا البحث تم تقدير دالة البقاء لتوزيع رالي من خلال تقدير معلمة هذا التوزيع باستخدام (طريقة متوسط المربعات الصغرى ، طريقة الرتبية) وتم استخدام دالة العضوية شبه المنحرف لتحويل هذا التوزيع الى التوزيع المضبب وتم تقدير دالة البقاء باستخدام بعض الدوال الرتبية وتم استخدام متوسط مربعات الخطأ لدوال البقاء لمعرفة من الافضل من البقية.

الكلمات المفتاحية: الاعداد الضبابية، دالة المخاطرة، طريقة متوسط المربعات الصغرى، دالة البقاء.