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Orthogonal Symmetric Higher bi-Derivations *on* Semiprime Γ-Rings

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Abstract:

Let M is a Γ-ring. In this paper the concept of orthogonal symmetric higher bi-derivations on semiprime Γ-ring is presented and studied and the relations of two symmetric higher bi-derivations on Γ-ring are introduced.

Key words: Higher bi-derivations on a Γ-ring, Orthogonal symmetric higher bi-derivations on a Γ-ring, Symmetric bi-derivations on a Γ-ring.

Introduction:

Let M and Γ be two additive abelian groups, M is called a Γ-ring if the following conditions are satisfied for any x,y,zϵM and $\alpha, \beta \epsilon \Gamma$:

i) xαy ϵM

ii) xα(y + z) = xαy + xαz

x(α + β)y = xαy + xβy

(x + y)αz = xαz + yαz

iii) (xαy)βz = xα(yβz)

The nottion of a Γ-riing was first introduced by **Nobusawa** and generalized by **Barnes 1966 (1)** as above definition. It is well known that every ring is Γ-ring . M is called prime if xΓMΓy=0 implies that x=0 or y=0 and it said to be semiprime if xΓMΓx=0 implies that x=0 for all x,y ϵM ,**(2)** .M is said to be n-torsion free if nx=0 , xϵM implies that x=0 where n is positive integer, **(3)**. **Khan A.R, Chaudhry M.A and Javaid I. in (4)** defined a derivation on Γ-ring as follows :An additive mapping d:M→M is said to be derivation on M if d(xαy)=d(x) αy +xα d(y) for all x,y ϵM and αϵΓ. **Rahman M.M. and Paul A.C. in (5)** are defined a Jordan derivation on Γ-ring as follows: An additive mapping d:M→M is said to be Jordan derivation on Γ-ring if d(xαx) = d(x) αx +xα d(x) for all xϵM and αϵΓ . It is clear that every derivation of a Γ-ring M is Jordan derivation of M. **Suliman N.N. and Majeed A.H. in (6)** are introduced the definition of orthogonal derivation on Γ-ring as follows :Let d and g be two derivations on M are said to be orthogonal if d(x) ΓMΓ g(y) =(0)=g(y) ΓMΓd(x) for all x,yϵM. **Ozturk M.A, Sapanci M, Soyturk M, Kim K.H. ,in (7)** presented the definition of symmetric bi-derivation on Γ-ring M as follows:A mapping d:MxM→M is said to be symmetric if d(x,y)=d(y,x) for all x,yϵM.

symmetric bi-derivation on M if d(xαy,z)=d(x,z)αy +xαd(y,z) for all x,y,z ϵM , αϵΓ and d is said to be Jordan bi-derivation on M if d(xαx,y)=d(x,y)αx+xαd(x,y) for all x,y,z ϵM and αϵΓ. It is clear that every bi-derivation of a Γ-ring M is Jordan bi-derivation of M. **Marir A.M in (8)** introduced the concept of higher bi- derivation on Γ-ring M as follows : Let D=$(d_i)_{i\in N}$ be a family of bi-additive mapping on M×M into M is said to be higher bi-derivation if d_n(xαy,zαw)=$\sum_{i+j=n} d_i(x,z)\alpha d_j(y,w)$ for all x,y,z,w ϵM , αϵΓ. **Salah M.Salih and Ahmed M.M. in (9)** introduced the concept of Jordan higher bi-derivations of prime Γ-rings as follows: let D=$(d_i)_{i\in N}$ be a family of bi-additive mapping on M×M into M is said to be Jordan bi-derivation if d_n(xαx,yαy)= $\sum_{i+j=n} d_i(x,y)\alpha d_j(x,y)$ for all $x,y\epsilon$M , α ϵΓ .

In this paper the results of the concept of orthogonal symmetric higher bi-derivations on semiprime Γ-ring are introduced, and some of lemmas and theorem are proved .

Orthogonal Symmetric Higher bi-Derivations on Semiprime Γ-Rings.

In this section the concept of orthogonal symmetric higher bi-derivations are presented and studied on Γ-rings and introduced an example and some lemmas.

Definition (2-1):

Let M be a Γ-ring, let D=$(d_i)_{i\in N}$ and G=$(g_i)_{i\in N}$ be two symmetric higher bi-derivations on a Γ-ring M .Then d_n and g_n are said to be orthogonal if for ali x,y,zϵM and nϵN

d_n(x,y) ΓMΓg_n(y,z)=(0)= g_n(y,z) ΓMΓd_n(x,y)

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Where d_n(x,y) ΓMΓg_n(y,z)=$\sum_{i=1}^{n} d_i(\mathrm{x,y})\alpha \mathrm{m}\beta\ g_i(\mathrm{y,z}) = 0$, for all mϵM and $\alpha, \beta\epsilon\Gamma$

Now, the following example is about orthogonal symmetric higher bi-derivations:

Example (2-2) :

Let M be a Γ-ring , d_n and g_n be two symmetric higher bi-derivations on M. Put M′=M×M and Γ′=Γ×Γ ,we define d'_n and g'_n on M′ by d'_n:M′→M′ and g'_n:M′→M′ such that d'_n((x,y))= (d_n(x),0) and g'_n((x,y))=(0, $g_n(y)$) for all x,yϵM′ and nϵN. Then d'_n and g'_n are symmetric higher bi-derivations such that d'_n and g'_n are orthogonal .

Now, the following lemmas of orthogonal symmetric higher bi-derivations on Γ- ring M.

Lemma (2-3):(2) Let M be a 2-torsion free semiprime Γ-ring an x,y be elements of M. If for all $\alpha, \beta\epsilon\Gamma$, then the following conditions are equivalent:

- **i)** $x\alpha M\beta y$ =0
- **ii)** $y\alpha M\beta x$ =0
- **iii)** $x\alpha M\beta y + y\alpha M\beta x$ =0

If one of these conditions is fulfilled , then $x\alpha y = y\alpha x = 0$.

Lemma (2-4) :(2) Let M be a 2-torsion free semiprime Γ-ring and x,y be elements of M such that $x\alpha M\beta y + y\alpha M\beta x = 0$ for all $\alpha, \beta\epsilon\Gamma$,then $x\alpha M\beta y = y\alpha M\beta x$=0.

Lemma (3-5) :

Let M be a semiprime Γ- ring . Suppose that the bi-additive mappings d_n and g_n on M×M into M satisfies d_n(x,y)ΓMΓg_n(x,y)=(0) for all $x, y\epsilon$M and nϵN. Then d_n(x,y)ΓMΓg_n(y,z)=(0) for all zϵM and nϵN.

Proof:

Suppose that $d_n(x,y)$ΓMΓg_n(x,y)=(0)

d_n(x,y)ΓMΓg_n(x,y)=$\sum_{i=1}^{n} d_i(\mathrm{x,y})\alpha\mathrm{m}\beta g_i(\mathrm{x,y}) = 0$ …(1) for all mϵM and α, βϵΓ

Replace x by $x + z$ in (1)

$\sum_{i=1}^{n} d_i(\mathrm{x+z,y})\alpha\mathrm{m}\beta g_i(\mathrm{x+z,y}) = 0$

$\sum_{i=1}^{n}(d_i\,(x,y) + d_i(z,y)\alpha m\beta\big(g_i(x,y) + g_i(z,y)\big) = 0$ $\quad \sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(x,y) + d_i(x,y)\alpha m\beta g_i(z,y) + d_i(z,y)\alpha m\beta g_i(x,y) + d_i(z,y)\alpha m\beta g_i(z,y) = 0$

By equation (1)

$\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(z,y) = -\sum_{i=1}^{n} d_i(z,y)\alpha m\beta g_i(x,y)$ ……(2)

Multiplication(2) by $\gamma t\delta \sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(z,y)$ for all tϵM and $\gamma, \delta\epsilon\Gamma$

$\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(z,y)$ $\gamma t\delta$ $\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(z,y)$ = 0

Since M is semiprime Γ-ring

$\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(z,y)$ = 0 ……(3)

Replace $g_i(z,y)$ by $g_i(y,z)$ in (3)

$\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(y,z)$ = 0

Thus $d_n(x,y)\Gamma M\Gamma g_n(y,z)$ =(0)

Lemma (2-6) :

Let M be a 2-torsion free semiprime Γ- ring such that $x\alpha y\beta z = x\beta y\alpha z$ for all α, βϵΓ , x, y, zϵM and two symmetric higher bi-derivations d_n and g_n for all nϵN are orthogonal if and only if $d_n(x,y)\alpha g_n(y,z) + g_n(x,y)\alpha d_n(y,z) = 0$ for all nϵN .

Proof :

Suppose that $d_n(x,y)\alpha g_n(y,z) + g_n(x,y)\alpha d_n(y,z) = 0$

$\sum_{i=1}^{n} d_i(x,y)\alpha g_i(y,z) + g_i(x,y)\alpha d_i(y,z) = 0$ ……(1)

Replace x by $x\beta w$ in (1) for all wϵM

$\sum_{i=1}^{n} d_i(x\beta w,y)\alpha g_i(y,z) + g_i(x\beta w,y)\alpha d_i(y,z) = 0$

$\sum_{i=1}^{n} d_i(x,y)\beta d_i(w,y)\alpha g_i(y,z) + g_i(x,y)\beta g_i(w,y)\alpha d_i(y,z)$=0…..(2)

Replace $d_i(w,y)$ by $g_i(w,y)$ in (2)

$\sum_{i=1}^{n} d_i(x,y)\beta g_i(w,y)\alpha g_i(y,z) + g_i(x,y)\beta g_i(w,y)\alpha d_i(y,z)$=0….(3)

By Lemma (2-4)

$\sum_{i=1}^{n} d_i(x,y)\beta g_i(w,y)\alpha g_i(y,z)$=0= $\sum_{i=1}^{n} g_i(x,y)\beta g_i(w,y)\alpha d_i(y,z)$..(4)

Replace $g_i(w,y)$ by m in (4) for all mϵM

$d_n(x,y)\Gamma M\Gamma g_n(y,z) = (0) = g_n(x,y)\Gamma M\Gamma d_n(y,z)$

Thus d_n and g_n are orthogonal

Conversely, suppose that d_n and g_n are orthogonal

$d_n(x,y)\Gamma M\Gamma g_n(y,z) = (0) = g_n(x,y)\Gamma M\Gamma d_n(y,z)$

$\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(y,z) = 0 = \sum_{i=1}^{n} g_i(x,y)\alpha m\beta d_i(y,z)$

$\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(y,z) + \sum_{i=1}^{n} g_i(x,y)\alpha m\beta d_i(y,z) = 0$

By Lemma (2-3) $\sum_{i=1}^{n} d_i(x,y)\alpha g_i(y,z) = 0 = \sum_{i=1}^{n} g_i(x,y)\alpha d_i(y,z)$

$\sum_{i=1}^{n} d_i(x,y)\alpha g_i(y,z) + g_i(x,y)\alpha d_i(y,z) = 0$

Thus $d_n(x,y)\alpha g_n(y,z) + g_n(x,y)\alpha d_n(y,z) = 0$

Main Results:

In this section introduce some theorems of orthogonal symmetric higher bi-derivations on semiprime Γ-ring.

Theorem (3-1): Let M be a 2-torsion free semiprime Γ-ring such that $x\alpha y\beta z = x\beta y\alpha z$ for all $\alpha, \beta\epsilon\Gamma, x, y, z\epsilon M$.Two symmetric higher bi-derivations d_n and g_n for all nϵN are orthogonal if and only if $d_n(x,y)\alpha g_n(y,z)$=0 or $g_n(x,y)\alpha d_n(y,z)$="0 for all x,y,z ϵM and nϵN .

Proof :

Suppose that $d_n(x,y)\alpha g_n(y,z)$=0

$d_n(x,y)\alpha g_n(y,z) = \sum_{i=1}^{n} d_i(x,y)\alpha g_i(y,z) = 0$ ……(1)

Replace x by xβt in (1) for all tϵM and $\beta\epsilon\Gamma$

$\sum_{i=1}^{n} d_i(x\beta t,y)\alpha g_i(y,z) = 0$

$\sum_{i=1}^{n} d_i(x,y)\beta d_i(t,y)\alpha g_i(y,z) = 0$ ……(2)

Replace $d_i(t,y)$ by m in (2) for all mϵM

$\sum_{i=1}^{n} d_i(x,y)\beta m\alpha g_i(y,z) = 0$

$d_n(x,y)\Gamma M\Gamma g_n(y,z) = (0)$ by Lemma (2-3) , d_n and g_n are orthogonal

Similarly way if $g_n(x,y)\alpha d_n(y,z)$=0 , d_n and g_n are orthogonal

Conversely, suppose that d_n and g_n are orthogonal

$d_n(x,y)\Gamma M\Gamma g_n(y,z) = (0)$

$\sum_{i=1}^{n} d_i(x,y)\alpha m\beta g_i(y,z) = 0$

By Lemma (2-3)

$\sum_{i=1}^{n} d_i(x,y)\alpha g_i(y,z) = 0$

Thus $d_n(x,y)\alpha g_n(y,z) = 0$

And by $g_n(x,y)\Gamma M\Gamma d_n(y,z) = (0)$

$\sum_{i=1}^{n} g_i(x,y)\alpha m\beta d_i(y,z) = 0$

By Lemma (2-3)

$\sum_{i=1}^{n} g_i(x,y)\alpha d_i(y,z) = 0$

Thus $g_n(x,y)\alpha d_n(y,z) = 0$

Theorem (3-2) : Let d_n and g_n be two symmetric higher bi-derivations of a 2-torsion free semiprime Γ-ring M and where d_n is commuting and g_n is commuting for all nϵN . Then the following conditions are equivalent for every nϵN :

i) d_n and g_n are orthogonal.
ii) $d_n g_n = 0$.
iii) $g_n d_n = 0$.
iv) $d_n g_n + g_n d_n = 0$.
v) $d_n g_n$ is symmetric higher bi-derivation .
vi) $g_n d_n$ is symmetric higher bi-derivation .

Proof : $(\boldsymbol{i}) \Leftrightarrow (\boldsymbol{ii})$

Suppose that d_n and g_n are orthogonal

$g_n(x,y)\Gamma M\Gamma d_n(y,z) = (0)$

$d_n(g_n(x,y)\Gamma M\Gamma d_n(y,z),m_1) = (0)$ for all $m_1\epsilon$M

$\sum_{i=1}^{n} d_i(g_i(x,y)\alpha m\beta d_i(y,z),m_1) = 0$ for all m ϵM

$\sum_{i=1}^{n} d_i(g_i(x,y),m_1)\alpha d_i(m,m_1)\beta d_i(d_i(y,z),m_1) = 0$ ……(1)

Replace $d_i(d_i(y,z),m_1)$by $d_i(g_i(x,y),m_1)$ in (1)

$\sum_{i=1}^{n} d_i(g_i(x,y),m_1)\alpha d_i(m,m_1)\beta d_i(g_i(x,y),m_1) = 0$

Since M is semiprime Γ-ring

$\sum_{i=1}^{n} d_i(g_i(x,y),m_1) = 0$

$d_n(g_n(x,y),m_1) = 0$

$d_n g_n = 0$

Conversely , suppose that $d_n g_n = 0$

$d_n(g_n(x,y),m) = 0$ for all $m\epsilon$M

$\sum_{i=1}^{n} d_i(g_i(x,y),m) = 0$ ……(2)

Replace x by xαz in (2)

$\sum_{i=1}^{n} d_i(g_i(x\alpha z,y),m) = 0$

$\sum_{i=1}^{n} d_i(g_i(x,y)\alpha g_i(z,y),m) = 0$

$\sum_{i=1}^{n} d_i(g_i(x,y),m)\alpha d_i(g_i(z,y),m) = 0$

$\sum_{i=1}^{n} d_i(g_i(x,y),m)\alpha g_i(d_i(z,y),m) = 0$……(3)

Replacing $d_i(g_i(x,y),m)$ by d_i(x,y) and $g_i(d_i(z,y),m)$ by g_i(y,z) in (3)

$\sum_{i=1}^{n} d_i(x,y)\alpha g_i(y,z) = 0$

$d_n(x,y)\alpha g_n(y,z) = 0$

By Theorem (3-1) d_n and g_n are orthogonal

Proof : $(\boldsymbol{i}) \Leftrightarrow (\boldsymbol{iii})$

Suppose that d_n and g_n are orthogonal

$d_n(x,y)\Gamma M\Gamma g_n(y,z) = (0)$ we get $g_n(d_n(x,y)\Gamma M\Gamma g_n(y,z),m_1) = (0)$ for all $m_1\epsilon$M

$\sum_{i=1}^{n} g_i(d_i(x,y)\alpha m\beta g_i(y,z),m_1) = 0$ for all m ϵM

$\sum_{i=1}^{n} g_i(d_i(x,y),m_1)\alpha g_i(m,m_1)\beta g_i(g_i(y,z),m_1) = 0$ ……(2)

Replace $g_i(g_i(y,z),m_1)$ by $g_i(d_i(x,y),m_1)$ in (2)

$\sum_{i=1}^{n} g_i(d_i(x,y),m_1)\alpha g_i(m,m_1)\beta g_i(d_i(x,y),m_1) = 0$

Since M is semiprime Γ-ring

$\sum_{i=1}^{n} g_i(d_i(x,y),m_1) = 0$

$g_n(d_n(x,y),m_1) = 0$

$g_n d_n = 0$

Conversely , suppose that $g_n d_n = 0$

$g_n(d_n(x,y),m) = 0$ for all $m\epsilon$M

$\sum_{i=1}^{n} g_i(d_i(x,y),m) = 0$ ……(3)

Replace x by xαz in (3)

$\sum_{i=1}^{n} g_i(d_i(x\alpha z,y),m) = 0$

$\sum_{i=1}^{n} g_i(d_i(x,y)\alpha d_i(z,y),m) = 0$

$\sum_{i=1}^{n} g_i(d_i(x,y),m)\alpha g_i(d_i(z,y),m) = 0$

$\sum_{i=1}^{n} g_i(d_i(x,y),m)\alpha d_i(g_i(z,y),m) = 0$……(4)

Replacing $g_i(d_i(x,y),m)$ by g_i(x,y) and $d_i(g_i(z,y),m)$ by g_i(y,z) in (4)

$\sum_{i=1}^{n} g_i(x,y)\alpha d_i(y,z) = 0$ we get $g_n(x,y)\alpha d_n(y,z) = 0$

By Theorem (3-1) d_n and g_n are orthogonal

Proof : $(\boldsymbol{i}) \Leftrightarrow (\boldsymbol{iv})$

Suppose that d_n and g_n are orthogonal

By Theorem (3-2) (ii) $d_n g_n = 0$ ……(5)

And by Theorem (3-2) (iii) $g_n d_n = 0$ ……(6)

By (5) and (6) $d_n g_n + g_n d_n = 0$

Conversely, suppose that $d_n g_n + g_n d_n = 0$

$(d_n g_n + g_n d_n)$(x,y)=0

$(d_n g_n)(x,y) + (g_n d_n)(x,y) = 0$

$d_n(g_n(x,y),m) + g_n(d_n(x,y),m) = 0$ for all $m\epsilon M$

$\sum_{i=1}^{n} d_i(g_i(x,y),m) + g_i(d_i(x,y),m) = 0$ ……(7)

Replace x by xαz in (7) we get $\sum_{i=1}^{n} d_i(g_i(x\alpha z,y),m) + g_i(d_i(x\alpha z,y),m) = 0$

$\sum_{i=1}^{n} d_i(g_i(x,y)\alpha g_i(z,y),m) + g_i(d_i(x,y)\alpha d_i(z,y),m) = 0$

$\sum_{i=1}^{n} d_i(g_i(x,y),m)\alpha d_i(g_i(z,y),m)+$
$g_i(d_i(x,y),m)\alpha g_i(d_i(z,y),m)=0$

$\sum_{i=1}^{n} d_i(g_i(x,y),m)\alpha g_i(d_i(z,y),m)+$
$g_i(d_i(x,y),m)\alpha d_i(g_i(z,y),m)=0$ ……(8)

Replacing $d_i(g_i(x,y),m)$ *by* $d_i(x,y)$ and $g_i(d_i(z,y),m)$ *by* $g_i(y,z)$ and $g_i(d_i(x,y),m)$ *by* $g_i(x,y)$ and $d_i(g_i(z,y),m)$ *by* $d_i(y,z)$ in (8)

$\sum_{i=1}^{n} d_i(x,y)\alpha g_i(y,z) + g_i(x,y)\alpha d_i(y,z) = 0$

$d_n(x,y)\alpha g_n(y,z) + g_n(x,y)\alpha d_n(y,z) = 0$

By Lemma (2-5) d_n and g_n are orthogonal

Proof : $(\boldsymbol{i}) \Leftrightarrow (\boldsymbol{v})$

To prove $(d_n g_n)(x\alpha z,y) = (d_n g_n)(x,y)\alpha(d_n g_n)(z,y)$

$(d_n g_n)(x,y)\alpha(d_n g_n)(z,y)$
$= d_n(g_n(x,y),m) d_n(g_n(z,y),m)$ for all $m\epsilon$M
$= \sum_{i=1}^{n} d_i(g_i(x,y),m)\alpha d_i(g_i(z,y),m)$ ……(9)

And $(d_n g_n)(x\alpha z,y) = d_n(g_n(x\alpha z,y),m) = \sum_{i=1}^{n} d_i(g_i(x\alpha z,y),m)$
$= \sum_{i=1}^{n} d_i(g_i(x,y)\alpha g_i(z,y),m)$
$= \sum_{i=1}^{n} d_i(g_i(x,y),m)\alpha d_i(g_i(z,y),m)$ ……(10)

By (9) and (10) $d_n g_n$ is higher bi-derivation

Conversely , to prove d_n and g_n are orthogonal

$d_n(x,y)\Gamma M\Gamma g_n(y,z) =$
$d_n(d_n(x,y)\Gamma M\Gamma g_n(y,z),t)$ for all $t\epsilon$M
$= \sum_{i=1}^{n} d_i(d_i(x,y)\alpha m\beta g_i(y,z),t)$ for all $\alpha,\beta\epsilon\Gamma$ and mϵM
$= \sum_{i=1}^{n} d_i(g_i(x,y),t)\alpha d_i(m,t)\beta d_i(g_i(y,z),t)$ ……(11)

Replace $g_i(y,z)$ by $g_i(x,y)$ in (11)
$= \sum_{i=1}^{n} d_i(g_i(x,y),t)\alpha d_i(m,t)\beta d_i(g_i(x,y),t)$

Since R is semiprime Γ-ring and by Theorem (3-2) (ii)

$0 = \sum_{i=1}^{n} d_i(g_i(x,y),t)$ we get $d_n g_n = 0$

Thus $d_n(x,y)\Gamma M\Gamma g_n(y,z) = (0)$ by lemma (2-3) d_n and g_n are orthogonal

Proof : $(\boldsymbol{i}) \Leftrightarrow (\boldsymbol{vi})$

Similar way used in the proof of (v)

Theorem (3-3): Let M be a 2-torsion free semiprime Γ-ring, $\mathrm{d_n}$ and be two symmetric higher bi-derivations for all nϵN . Suppose that $\mathrm{d_n}^2 = \mathrm{g_n}^2$, then $\mathrm{d_n} - \mathrm{g_n}$and $\mathrm{d_n} + \mathrm{g_n}$ are orthogonal **.**

Proof :

Suppose that $\mathrm{d_n}^2 = \mathrm{g_n}^2$, then for all x,yϵM :

$= [(\mathrm{d_n} - \mathrm{g_n})(\mathrm{d_n} + \mathrm{g_n}) + (\mathrm{d_n} + \mathrm{g_n})(\mathrm{d_n} - \mathrm{g_n})](x,y)$
$= [\,(\mathrm{d_n} - \mathrm{g_n})(\mathrm{d_n} + \mathrm{g_n})](x,y) + [(\mathrm{d_n} + \mathrm{g_n})(\mathrm{d_n} - \mathrm{g_n})](x,y)$
$= \sum_{i="1"}^{n} [\,(\mathrm{d_i} - \mathrm{g_i})(\mathrm{d_i} + \mathrm{g_i})](x,y) + [(\mathrm{d_i} + \mathrm{g_i})(\mathrm{d_i} - \mathrm{g_i})]\,(x,y)$
$= \sum_{i=1}^{n} (\mathrm{d_i}^2 + \mathrm{d_i g_i} - \mathrm{g_i d_i} - \mathrm{g_i}^2)(x,y) + (\mathrm{d_i^2} - \mathrm{d_i g_i} + \mathrm{g_i d_i} - \mathrm{g_i^2})(x,y)$
$=$
$\sum_{i=1}^{n} \mathrm{d_i}^2(x,y) + \mathrm{d_i}(\mathrm{g_i}(x,y),m) - \mathrm{g_i}(\mathrm{d_i}(x,y),m) - \mathrm{g_i}^2(x,y) + \mathrm{d_i}^2(x,y) - \mathrm{d_i}(\mathrm{g_i}(x,y),m) + \mathrm{g_i}(\mathrm{d_i}(x,y),m) - \mathrm{g_i}^2(x,y)$ for all mϵM

$0 = \mathrm{d_n}^2(x,y) + \mathrm{d_n}(\mathrm{g_n}(x,y),m) - \mathrm{g_n}(\mathrm{d_n}(x,y),m) - \mathrm{g_n}^2(x,y) + \mathrm{d_n}^2(x,y) - \mathrm{d_n}(\mathrm{g_n}(x,y),m) + \mathrm{g_n}(\mathrm{d_n}(x,y),m) - \mathrm{g_n}^2(x,y)$
$(\mathrm{d_n} - \mathrm{g_n})(\mathrm{d_n} + \mathrm{g_n}) + (\mathrm{d_n} + \mathrm{g_n})(\mathrm{d_n} - \mathrm{g_n}) = 0$

By Theorem (3-1-6) (iv) $\mathrm{d_n} - \mathrm{g_n}$and $\mathrm{d_n} + \mathrm{g_n}$are orthogonal

Conflicts of Interest: None.

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تعامد المشتقات الثنائية العليا المتناظرة على الحلقات شبه الأولية من النمط –Γ

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الخلاصة:

لتكن M حلقة من النمط - Γ . في هذا البحث قدمنا مفهوم تعامد المشتقات الثنائية العليا المتناظرة على الحلقات شبه أولية النمط - Γ ودرسنا العلاقات بين المشتقات الثنائية العليا المتناظرة على الحلقات شبه اولية من النمط - Γ وايجاد خواص مكافئة للتعامد .

الكلمات المفتاحية:

المشتقات الثنائية العليا المتناظرة على الحلقة شبه أولية النمط –Γ ،تعامد المشتقات الثنائية العليا المتناظرة على الحلقة شبه أولية النمط –Γ ، المشتقات الثنائية المتناظرة على الحلقة شبه أولية النمط –Γ .