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Solving Edges Deletion Problem of Complete Graphs

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Abstract

As the network size increases, there is an increased probability that some components will fail. It is therefore necessary for the network to be able to maintain the interconnection even in the presence of faulty components. The likelihood of a failure in the connections between stations is higher than that of a failure in the stations themselves because of the nature of many networks. Numerous graph-theoretic methods can be used to examine the reliability and efficacy of a network, and the network's dependability is determined by its connectivity. For a parallel and distributed system, the maximum connection time between any two nodes in the network can be determined using the diameter of the graph. Diameter is often used to measure the efficiency of a network with the maximum delay or signal degradation. The diameter of a graph can be altered by adding or removing edges. In this paper, it has been considered how the diameter will increase after deleting the number of edges from the complete graph and calculating the maximum diameter. The maximum diameter $\mathrm{f_n(t', d')}$ of a connected graph with n vertices is obtained after removing t' edges from a connected graph G. The maximum diameter $\mathrm{f(t', d')}$ of a connected graph is derived from $\mathrm{f_n(t', d')}$.

Keywords: Altered graph, Complete graph, Diameter, Edges deletion problem, Graph theory.

Introduction

Graph theory has gained popularity as a result of its many diverse applications in several fields, such as social science, biology, and engineering. A graph can be used to describe data regarding the connections between several items where the things are represented by vertices, relations, and edges[1]. The topology of power grid complex interconnections is regarded as one of the obstacles that might be encountered in terms of understanding and analyzing them. As a result, several nations model their power networks using complicated complicated Network principles[2]. When designing a network, communication is one of the important key concerns. Therefore, a strong network must be challenging to disrupt and should be able to send and receive messages even when nodes or links experience faults[3]. Graphs with the property of being resilient to severe shocks when faults occur on the nodes or links are typically preferred by network designers. The graphs are the perfect tool for modeling interconnection networks due to their reliability[4]. Systems for data transmission and communication must include data security. Its primary responsibility is to safeguard and integrate sensitive data from the source to the recipient[5]. The effect of deleting vertices and edges on a graph is of practical interest because such deletions can have a significant impact on the efficiency of a network modeled by these graphs, particularly when the time delay is proportional to the distance of the shortest path

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between any two network stations[4]. The nodes and links of the network are represented by the vertices and edges of the graph, the highest number of links between two nodes is represented by the diameter of the graph[6]. The diameter of the underlying graph can be used to calculate the maximum connection time between any two nodes in the network. The removal or addition of edges can change a graph's diameter[7]. Many authors have studied and investigated the impact of the edge's deletion on the diameter d of a graph. In the following, some related results concerning the edges deletion problem. Yannakakis discussed the minimum number of edges (nodes) whose removal results in a subgraph fulfilling property π, if π is a graph property [8]. Chen and others are given an overview of possible problem types related to the topic of deleting edges, references to some theorems and open problems, and a study of the class of trivially perfect graphs[9]. Plesnik was the first to observe that the removal of one edge from graph G, which is an undirected graph, can at most multiply the diameter of the graph[4]. Chung discussed the minimum number of edges in a graph with k vertices and the feature that the diameter does not increase when any edge is removed[10]. Revathy, Regitha and Chithra discussed the smallest number of edges can be added to a graph G to reduce the diameter, the least number of edges can be removed from a graph G to grow the diameter, and the greatest number of edges that can be removed from a graph G without increasing its diameter[3]. Kim and others discussed determining how many edges to remove from a graph in order to increase the diameter by a specific amount[11]. Swart examined how the loss of links or nodes affects the effectiveness of communication networks. that also examined graphs that are important in terms of the radius or diameter. The parameter changes depending on whether an edge is eliminated, added, or decreased. It also changes depending on whether a vertex is removed[12]. Alochukwu and Dankelmann presented an upper bound on the t-edge fault diameter of a $(t+1)$-edge connected C_4-free graph and constructed a show that the bound is close to optimal for large values of t. Considering $t = 2$ as a special case[13]. Bouabdallah, Delormea, and Djelloul discussed the problem of removing maximal edges from a hypercube graph Q_n without changing its diameter[14]. Graham and Harary investigated the impact of increasing and decreasing edges on a hypercube graph's diameter[15]. Fundikwa, Mazorodze, and Mukwembi presented an upper bound on the diameter of a $3-$edge-connected C_4-free graph in terms of order[16]. Schoone, Bodlaender, and Leeuwen improved the bounds as follows: they derived an upper bound for $(t+1)d$, and they proved a lower bound $(d-1)t+d$, and d is even, $(d-2)t+2+d$ for d is odd and $d>2$. Also, they considered a complete graph k_n with diameter $d=1$ to be a special case of graph [6]. Najim and Jun-Ming discussed the upper bound to the maximum diameter, which is $f(t,d)$ of a connected graph G, $f(t,d)$ obtained after deleting t edges from G. They indeed showed that $f(t,d) \leq (d-1)t+d+1$ for $t>3$ and $t+7 \geq d \geq t+4$, and $d=(2h-1)t+2h$ and $h \geq 1$,also for $t=4$ and $d=10h+1$ [17].

This paper is mainly about investigating the edges deletion problem in the complete graph and calculating the diameter after deleting a certain number of edges, calculating the exact values of $f_n(t',d')$ for given t', d' and n, $(n \geq 3)$. $f_n(t',d')$ represents the maximum diameter of connected graphs G_i, $i \geq 1$ (altered graphs G_i) with n vertices, G_i obtained upon removing t edges from a graph k_n. $f_n(t',d')$ obtained upon removing t' edges from G_i, and $d' \geq 2$ such that the resulting graph remains a connected graph. This is also followed by finding $f(t',d')$ for given $t'$and d', where $f(t',d')$ represents the maximum diameter of connected graphs G_j ,$j \geq 1$ (altered graphs G_j), G_j obtained upon removing t edges from a graph k_n such that the resulting graph remains a connected graph. Our results are obtained with the help of the MATLAB program. As a result, it can be concluded that any connected graph with n vertices can be derived from a graph k_n after deleting a certain number of edges. This makes a graph k_n the foundation for all connected graphs with n vertices.

Preliminaries

Consider $G=(V,E)$ is a simple and undirected graph where $V(G)$ represents the set of vertices of order n and $E(G)$ denotes the set of edges $E(G)$ of size m. Let u and v be two vertices in G, $u,v \in V(G)$. Suppose that $d_G(u,v)$ denotes the distance

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between u and v, which is the length of the shortest path connecting u and v in G. Moreover, $d_G(u,v) = \infty$ if there is no path connecting u and v [18]. The diameter denoted by the symbol $d = diam(G)$, such that $d = \max_{u,v \in V(G)} d_G(u,v)$. If and only if a graph is connected, it is known that it has a finite diameter[19].

Edge Deletion in Graph

Suppose that G is a graph, any edge $e \in G$, $G' = G - e$ is a subgraph with $E(G') = E(G - e)$ and $V(G') = V(G)$. Note that G' is the graph that results from removing edge e from G[11].

Given G is a complete graph K_n with the diameter $d = 1$, then the diameter of the graph that gets after deletion t edges is denoted by d' such that $d' \geq d$ [14]. For later use, note that the following results (Lemma 1 and Lemma 2 are due to [6], and Lemma 3 is due to [20]):

Auxiliary Results

Lemma 1:

Let G' be a graph with a diameter d, $1 < d < n$, that is gotten by removing t edges from a graph k_n , then $t \geq \frac{1}{2}d(d-1) + (d-2)\,(n-d-1)$.

Lemma 2:

$f_n(t,d) = \lfloor n + \frac{1}{2} - \sqrt{(n - \frac{3}{2})^2 + 2 - 2t}\, \rfloor$ for $d = 1$ and $t \leq \frac{1}{2}(n-1)\,(n-2)$.

Lemma 3:

$f(t,d) = \lfloor \lceil \sqrt{2t + \frac{1}{4}} + \frac{3}{2} \rceil + \frac{1}{2} - \sqrt{(\lceil \sqrt{2t + \frac{1}{4}} + \frac{3}{2} \rceil - \frac{3}{2})^2 + 2 - 2t}\, \rfloor$ for $t \geq 1$ and $d = 1$.

As applications to the latter lemmas, the following figure was obtained with the help of the MATLAB program. By applying Lemma 2 and Lemma 3 with diameter $d = 1$, the results in Fig 1, represent d' that obtained after deleting t edges from a graph k_n, $n \geq 3$.

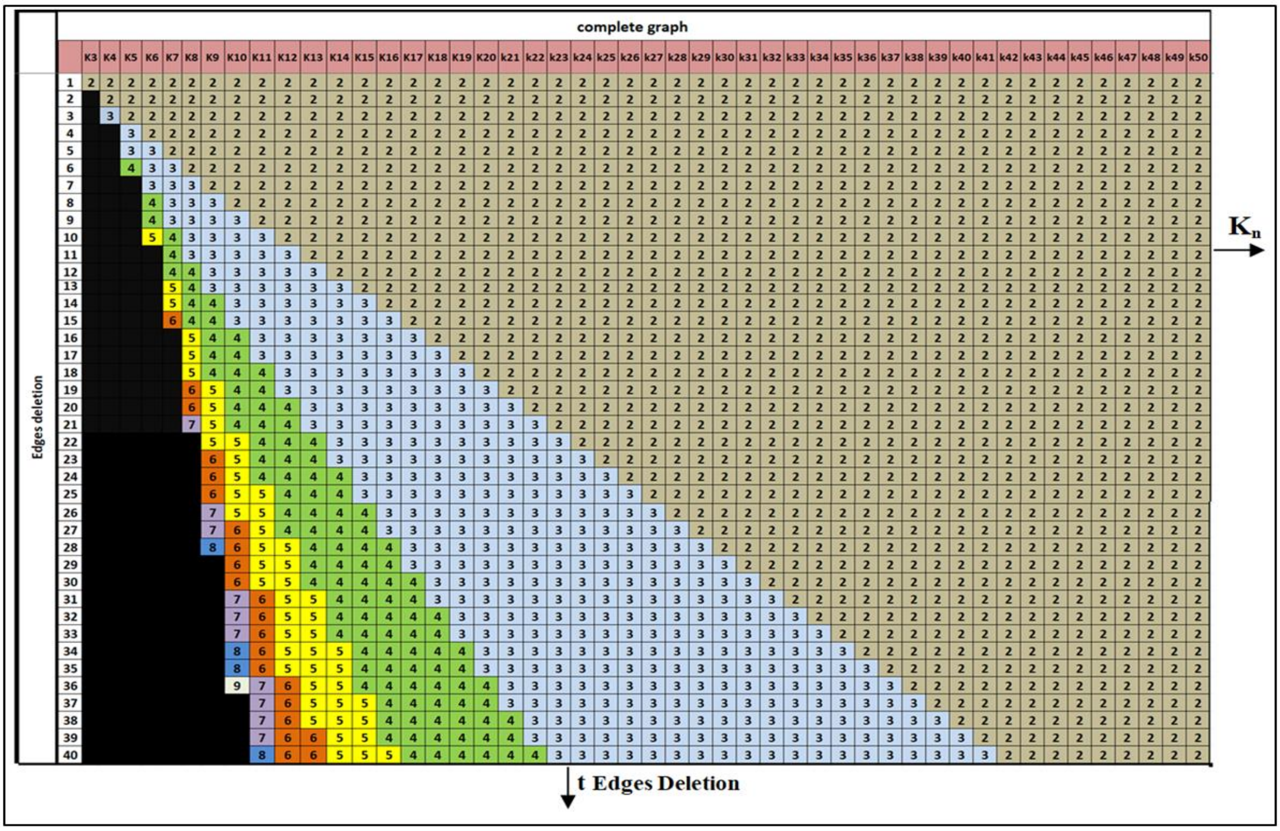

Figure 1. The presentation of $f_n(t,1)$ and $f(t,1)$

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Main Results

Theorem 1:

$f_n(t', d') = \lfloor n + \frac{1}{2} - \sqrt{(n - \frac{3}{2})^2 - 2t'} \rfloor$ for $d' \geq 2$ and $t' \leq \frac{1}{2}(n-1)(n-2) - 1$.

Proof: Let G_i, with $i \geq 1$, be connected graphs obtained by removing t edges from a graph k_n, $n \geq 4$. The diameter of G_i is denoted by d , and $d \geq 2$.

Suppose that G_i', with $i \geq 1$ is connected graphs obtained by removing t' edges from connected graphs G_i. The diameter of G_i' is denoted by d', $d' \geq 2$. Also, $f_n(t', d')$ denotes the maximum diameter of a connected graph obtained upon removing edges from connected graphs G_i of order n , $n \geq 4$.

From Lemma 1, get the following:

$t \geq \frac{1}{2}d(d-1) + (d-2)(n-d-1)$ and $t \leq \frac{1}{2}(n-1)(n-2)$.

Therefore, assuming $t' \geq t - 1$ implies that $t' \geq \frac{1}{2}d(d-1) + (d-2)(n-d-1) - 1$.

On the other hand, the largest number of edges that can be removed from G_i' without necessarily disconnecting it is by $L = \frac{1}{2}(n-1)(n-2) - 1$. Thus, $t' \leq L$.

Hence $t' \leq \frac{1}{2}(n-1)(n-2) - 1$

Hence:

$$f_n(t', d') = max\{d' | 2 \leq d' \leq n-1, n \geq 4, \frac{1}{2}d'(d'-1) + (d'-2)(n-d'-1) - 1 \leq t' \leq L\}. \quad 1$$

Now, suppose that the function $h: R \to R$ is defined by

$$h(d') = \frac{1}{2}d'(d'-1) + (d'-2)(n-1-d') - 1. \quad 2$$

Next, differentiating both sides of Eq 2, get the following: $h'(d') = -d' + n + \frac{1}{2}$.

By letting $h'(d') = 0$, then $d' = n + \frac{1}{2}$.

Thus, h is increasing for all $d' \leq n - 1$. By substituting the obtained upper bound of d' in Eq 2, get the following:

$$h(n-1) = \frac{1}{2}(n-1)(n-2) - 1. \quad 3$$

Therefore, from Eq 1

$$f_n(t', d') = max\{d' | 2 \leq d' \leq n-1, n \geq 4, h(d') \leq t' \leq L\} \quad 4$$

Hence

$$f_n(t', d') = max\{d' | 2 \leq d' \leq n-1, n \geq 4, h(d') \leq t'\}. \quad 5$$

which give that $h(d') - t' \leq 0$.

By substituting Eq 2 in the inequality Eq 5, get the following:

$$d' \leq n + \frac{1}{2} - \sqrt{\left(n + \frac{1}{2}\right)^2 + 2 - 4n - 2t'}. \quad 6$$

Since the value of $f_n(t', d')$ is an integer,

$$f_n(t', d') = \left\lfloor n + \frac{1}{2} - \sqrt{\left(n + \frac{1}{2}\right)^2 + 2 - 4n - 2t'} \right\rfloor.$$

which can be written in the following form (for later use)

$$f_n(t', d') = \left\lfloor n + \frac{1}{2} - \sqrt{\left(n - \frac{3}{2}\right)^2 - 2t'} \right\rfloor.$$

Theorem 2:

$f(t', d') = \left\lfloor \lceil \sqrt{2t' + \frac{9}{4}} + \frac{3}{2} \rceil + \frac{1}{2} - \sqrt{(\lceil \sqrt{2t' + \frac{9}{4}} + \frac{3}{2} \rceil - \frac{3}{2})^2 - 2t'} \right\rfloor$ for $t' \geq 1$ and $d' \geq 2$.

Proof. Let G_i and G_i', with $i \geq 1$, be connected graphs obtained by removing t and t' edges from a graph k_n and G_i , respectively.

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Suppose that $f(t',d')$ denotes the maximum diameter of the connected graphs $G_i{}'$ obtained upon removing t' edges from connected graphs G_i with $d' \geq 2$.

Hence, from Theorem 1, get the following:

$$f_n(t',d') = \left\lfloor n + \frac{1}{2} - \sqrt{\left(n - \frac{3}{2}\right)^2 - 2t'} \right\rfloor \qquad 7$$

Suppose that L= $\frac{1}{2}(n-1)(n-2)-1$

L is the largest number of edges that can be removed from G_i without necessarily disconnecting, $t' \leq$ L

Then, $t' \leq \frac{1}{2}(n-1)(n-2)-1 \qquad 8$

From the inequality 8, then

$$n \geq \sqrt{2t' + \frac{9}{4}} + \frac{3}{2} \qquad 9$$

Now, suppose that the function $g: R \to R$ is defined by

$$g(n) = n + \frac{1}{2} - \sqrt{\left(n - \frac{3}{2}\right)^2 - 2t'}. \qquad 10$$

Next, differentiating both sides of Eq 10 gives that:

$$g'(n) = 1 - \frac{2n-3}{2\sqrt{\left(n-\frac{3}{2}\right)^2 - 2t'}} \Longrightarrow g'(n) \leq 0 \text{ for}$$

$$2n - 3 \geq 2\sqrt{(n - \tfrac{3}{2})^2 - 2t'} \qquad 11$$

that inequality 11 is true for $t' \geq 1$.

hence $g(n)$ is decreasing when $n \geq \sqrt{2t' + \frac{9}{4}} + \frac{3}{2}$.

It follows that

$$f(t',d') = max\left\{ g(n): n \geq \sqrt{2t' + \frac{9}{4}} + \frac{3}{2}\right\} . \qquad 12$$

By substituting inequality 9 in the 10, it achieves that

$$f(t',d') = \left\lfloor \left\lceil \sqrt{2t' + \frac{9}{4}} + \frac{3}{2} \right\rceil + \frac{1}{2} - \sqrt{\left(\left\lceil \sqrt{2t' + \frac{9}{4}} + \frac{3}{2} \right\rceil - \frac{3}{2}\right)^2 - 2t'} \right\rfloor.$$

Therefore, Theorem 2 is proved.

As done in the previous Fig 1 represents applications to our results given in Theorems 1 and 2 with $d' \geq 2$. Indeed, Fig 2 gives all results $f(t',d')$ and $f_n(t',d')$ by removing t' edges from the altered connected graph of n vertices with $n \geq 4$, and again these results are obtained with the help of MATLAB.

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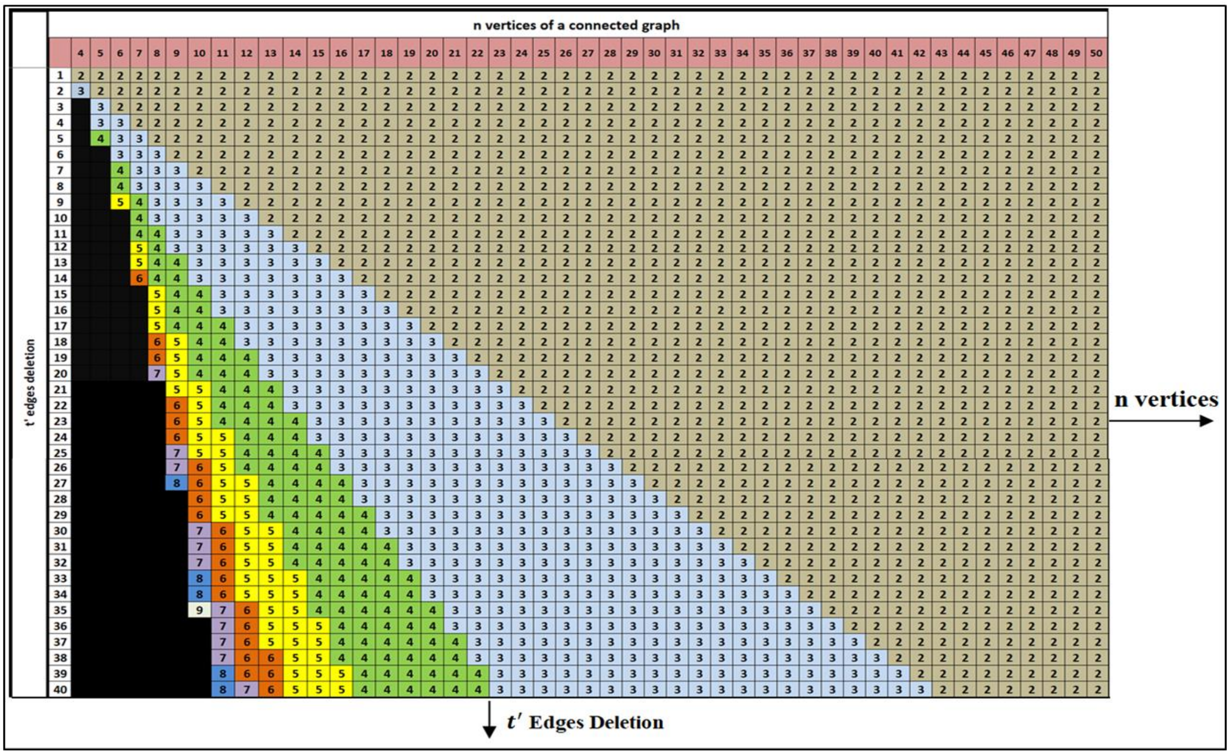


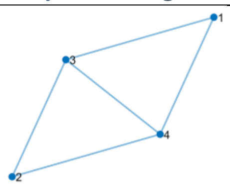
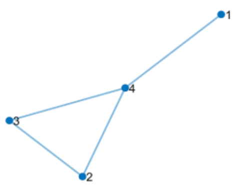
Figure 2. The presentation of $f_n(t',d')$ and $f(t',d')$

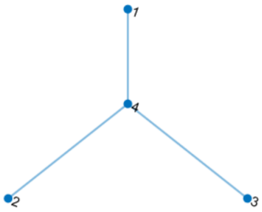
In the following examples, sample cases of deleting edges and calculating $f_n(t',d')$ and $f(t',d')$ for n=4 and 5.

Example 1:

Let's consider the complete graph K_4 with the diameter $d = 1$. Suppose that G is a connected graph obtained by removing $t = 1$ edge from a graph K_4. Table 1 gives all the results of $f_4(t',d')$ by removing t' edges from graph G. The results were obtained with the help of the MATLAB program and using the isomorphism concept in graphs to reduce the number of results.

Table 1. The presentation of (G and G_i , $i \geq 1$) and ($f_4(1,\mathrm{d})$ and $f_4(t',d')$ with $n = 4$)

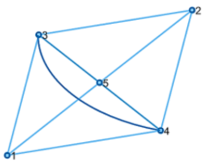
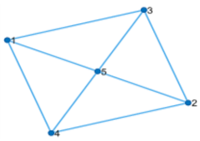
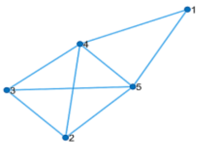
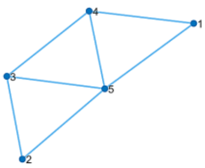
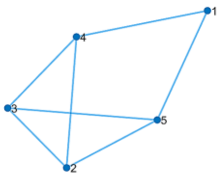
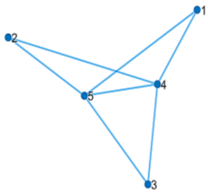
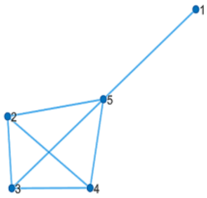
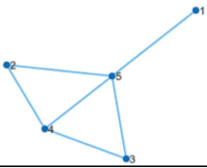
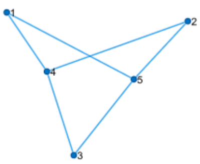
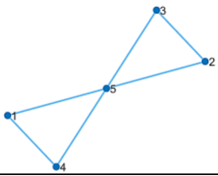
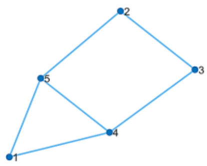
Deletion	Graph G obtained by removing t from the graph K_4	d	$f_4(1,d)$
$t = 1$		2	2
Deletion t'	Graphs $G_i{}'$ obtained by removing t' edges from graph G with $i \geq 1, n = 4$	d'	$f_4(t',d')$
$t' = 1$	 	2	2

$t' = 2$		2	3
		3	

Example 2:

Let's consider the complete graph K_5 with the diameter $d = 1$. Suppose that G is a connected graph obtained by removing $t = 1$ edge from a graph K_5. Table 2 gives all the results of $f_5(t', d')$ by removing t' edges from graph G. Our results were obtained with the help of the MATLAB program and using the isomorphism concept in graphs to reduce the number of results.

Table 2. The presentation of (G and G_i , $i \geq 1$) and ($f_5(1, \mathrm{d})$ and $f_5(t', d')$ with $n = 5$)

Deletion	Graph G obtained by removing t from the graph K_5	d	$f_5(1, d)$
$t = 1$		2	2
Deletion t'	Graphs $G_i{}'$ obtained by removing t' edges from graph G with $i \geq 1, n = 5$	d'	$f_5(t', d')$
$t' = 1$	 	2	2
$t' = 2$	   	2	2
$t' = 3$	   	2	3

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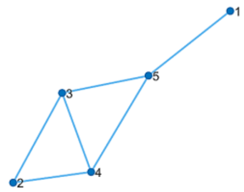
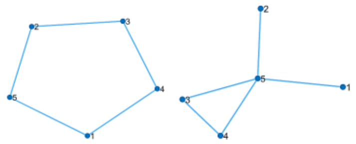
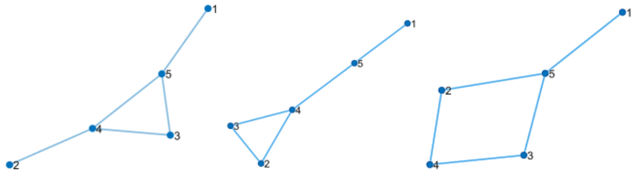
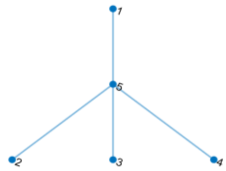
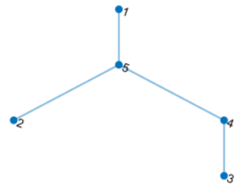
		3	
$t' = 4$		2	3
		3	
$t' = 5$		2	4
		3	
		4	

Table 3, can summarize the results of the presentation of $f_n(t', d')$ and $f(t', d')$ in Examples 1 and 2.

Table 3. The presentation of $f_n(t', d')$ and $f(t', d')$ to Examples 1 and 2

	$t' = 1$	$t' = 2$	$t' = 3$	$t' = 4$	$t' = 5$
$f_4(t', d')$	2	3	------------	----------	----------
$f_5(t', d')$	2	2	3	3	4
$f(t', d')$	2	3	3	3	4

Note that the Fig 2 summarizes the results of $f_n(t', d')$ and $f(t', d')$ for any t' and n that are obtained by Theorem 1 and 2.

Results and Discussion

A graph is a visual representation of a collection of things in a network where some object pairs are linked together, with the vertices denoting the switching components or processors and the edges denoting the communication lines. An altered graph is a new graph that can be generated from an existing graph, which is done by graph operations such as the addition or deletion of a vertex(s) or an edge(s).

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A network's effectiveness can be significantly impacted by the deletion of vertices or edges. Since the diameter is a measure of network efficiency used to study the effects of link failures in networks, it is crucial to understand how deleting a certain number of edges can affect the diameter of the graph.

A study was done to solve the edges deletion problem for a complete graph and calculate the diameter after deleting t edges. Calculating the $f_n(t', d')$ and $f(t', d')$ of altered graph G that is obtained by deleting t'edges from K_n with n vertices (for $n \geq 4$ and $d' \geq 2$). Our results are obtained with the help of the MATLAB program, and the results are represented in Figs 1 and 2. As a result, it can be concluded that any connected graph G with n vertices, $n \geq 3$, can be obtained by deleting some edges from the complete graph K_n, $n \geq 3$.

Conclusion

As the edges deletion problem in graph theory has been of interest to many authors, it is crucial to understand how deleting edges can affect the graph. In this paper, it has been considered how deleting a certain number of edges can affect the diameter of the complete graph and calculated the diameter and the maximum diameter after deleting edges. As a result, it's obtained general formulas representing the diameter and the maximum diameter after deletion. Our results are obtained from Theorems 1 and 2, as given in Fig. 1 and 2.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Basrah.

Authors' Contribution Statement

This work was carried out in collaboration between all authors. A.N.J contributed to the design, implementation, analysis of the results, and writing of the manuscript through continuous discussions with the A.A.N. A.A.N contributed to the revision and proofreading of the manuscript. A.N.J. and A.A.N. read and approved the final manuscript.

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حل مشكلة حذف الحافات في الرسم البياني الكامل

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الخلاصة

مع زيادة حجم الشبكة فان احتمالية الفشل في بعض مكوناتها (العقد او الحافات) يزداد، لذلك من الضروري ان تكون الشبكة قادرة على التواصل بين مكوناتها في حالة وجود خلل في بعض مكوناتها. ان احتمالية حدوث عطل في التوصيلات بين محطات الشبكة أكبر من احتمالية حدوث عطل في المحطات نفسها بسبب طبيعة عمل العديد من الشبكات. يمكن دراسة فعالية وموثوقية الشبكة باستخدام مجموعة من تقنيات نظرية البيان. ان التواصل في الشبكة هو ما يحدد مدى صلابتها (موثوقيتها) حيث ان فعالية الشبكة تتأثر عند حدوث تلف في أحد مكوناتها. يستخدم قطر الرسم البياني لقياس كفاءة عمل الشبكة من خلال قياس اقصى تأخير في استلام إشارة (رسالة) معينة او حدوث خلل في الاستلام. في هذا البحث ندرس مدى زيادة القطر d للرسم البياني G الناتج بعد حذف t من الحافات من الرسم البياني k_n . وايجاد أكبر قطر للرسم بياني $f_n(t', d')$ الذي يحتوي n من الرؤوس ونحصل عليه بعد حذف t' من الحافات من G وتحديد $f_n(t', d')$ بعد حساب $f(t', d')$

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