

A note on an R -module with (m, n) -pure intersection property

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Abstract

Let R be a ring. Given two positive integers m and n , an R module V is said to be (m, n) -presented, if there is an exact sequence of R -modules $0 \rightarrow K \rightarrow R^m \rightarrow V \rightarrow 0$ with K is n -generated. A submodule N of a right R -module M is said to be (m, n) -pure in M , if for every (m, n) -Presented left R -module V the canonical map $N \otimes_R V \rightarrow M \otimes_R V$ is a monomorphism. An R -module M has the (m, n) -pure intersection property if the intersection of any two (m, n) -pure submodules is again (m, n) -pure. In this paper we give some characterizations, theorems and properties of modules with the (m, n) -pure intersection property.

Key words:- (m, n) -pure submodule, (m, n) -flat module, module with (m, n) -pure intersection property.

Introduction

Throughout, this paper, R is an associative ring with non-zero identity, and all modules are unitary right R -modules. A submodule N of an R -module M is called pure submodule, if for every finitely generated ideal I of R $MI \cap N = NI$ [1]. Following [2], an R -module M has the PIP, if the intersection of any two pure submodules is again pure. For an abelian group G , we write $G^{m \times n}$ for the set of all formal $m \times n$ matrices with entries in G and write G^n (resp. G_n) for $G^{1 \times n}$ (resp. $G^{n \times 1}$). For two position integer m, n . A submodule N of an R -module M is (m, n) -pure in M if and only if $MI \cap N^m = NI$, for all n -generated submodule I of R^m [3]. An R -module M is (m, n) -flat, if $1_M \otimes L_I : M \otimes_R I \rightarrow M \otimes_R R^m$ is monomorphism for all n -generated

submodule I of R^m [4]. In this paper, for two fixed positive integers m and n , we introduce the concept of an R -module M has (m, n) -PIP. We prove that if M is an R -module such that for any two (m, n) -pure submodules A and B of M , $A + B$ is (m, n) - flat R -module, then M has the (m, n) -PIP.

Propeties of module which has (m, n) -PIP

Definition 2.1:- An R -module M has the (m, n) -pure intersection property (briefly (m, n) -PIP) if the intersection of any two (m, n) -pure submodules is again (m, n) -pure. An R -module M has the $(m, *)$ -PIP (resp. $(*, n)$) if for all positive integer n (resp. m) M has the (m, n) -PIP.

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It is clear that if M has the (1,1)-PIP, then M has the PIP. The converse is not true.

Proposition 2.2:-

(1) If an R -module M has the (m, n) -PIP, then every (m, n) -pure submodule of M has the (m, n) -PIP.

(2) Let N be (m, n) -pure submodule of an R -module M . M has the (m, n) -PIP, if and only if $\frac{M}{N}$ has the (m, n) -PIP.

Proof:- (1) trivial

(2) \Rightarrow Let $\frac{A}{N}$ and $\frac{B}{N}$ be (m, n) -pure submodules of $\frac{M}{N}$. We want to

show that $\frac{A}{N} \cap \frac{B}{N} = \frac{A \cap B}{N}$ is (m, n) -pure. Now, A and B are

(m, n) -pure in M [3, Proposition (1.9-4)]. And since M has the (m, n) -PIP,

then $A \cap B$ is (m, n) -pure. Then

$\frac{A \cap B}{N} = \frac{A}{N} \cap \frac{B}{N}$ is (m, n) -pure in $\frac{M}{N}$ [4, Proposition (1.9-3)].

\Leftarrow Let E and F be (m, n) -pure submodules in M . Then $\frac{E}{N}$ and $\frac{F}{N}$

is (m, n) -pure submodules of $\frac{M}{N}$. [3, Proposition (1.9-3)]. Since $\frac{M}{N}$ has

the (m, n) -PIP, then

$\frac{E}{N} \cap \frac{F}{N} = \frac{E \cap F}{N}$ is (m, n) -pure in

$\frac{M}{N}$. hence $A \cap B$ is (m, n) -pure

in M [4, Proposition (1.9-4)]. Thus M has the (m, n) -PIP.

Remarks 2.3:-

(1) Let A and B be R -modules, it is clear that $(A \cap B)^m = A^m \cap B^m$

(2) Every summand submodule N of an R -module M is (m, n) -pure submodule.

In [2, CH 2, theorem 3.1] an R -module M has the PIP, if and only if $(A \cap B)I = AI \cap BI$ for every finitely generated ideal I of R and for every pure submodules A and B of M .

Theorem 2.4:- Let M be an R -module. Then M has the (m, n) -PIP, if and only if $(A \cap B)I = AI \cap BI$ for every n -generated submodule I of ${}_R R^m$ and for every (m, n) -pure submodules A and B of M .

Proof:- Suppose that M has the (m, n) -PIP and let each of A and B is (m, n) -pure. Let I be n -generated submodule of ${}_R R^m$, then $(A \cap B)^m \cap MI = AI \cap BI$

${}_R R^m$, [3, theorem (1.5)]. It is clear that

$(A \cap B)I \subseteq AI \cap BI$. But $AI \cap BI \subseteq (MI \cap A^m) \cap B^m = MI \cap (A^m \cap B^m) = MI \cap (A \cap B)^m = (A \cap B)I$. Conversely,

let A and B be (m, n) -pure submodules in M and let I be n -generated submodule of ${}_R R^m$. Then $MI \cap (A \cap B)^m = (MI \cap A^m) \cap B^m = AI \cap B^m$.

Similarly, $MI \cap (A \cap B)^m = BI \cap A^m$, because A and B are (m, n) -pure in M .

Thus, $MI \cap (A \cap B)^m = AI \cap BI = (A \cap B)I$.

Therefore M has the (m, n) -PIP.

Corollary 2.5:- Let M be an R -module, then M has the $(1, *)$ -PIP, if and only if $(A \cap B)I = AI \cap BI$ for every finitely generated ideal I of R and for every for every $(1, *)$ -pure submodules A and B of M .

Proof:- It follows by [3, corollary 1.6]

In [2, CH 2, theorem 3.3], an R -module M has the PIP, if and only if for every pure submodule A and B of M and for every R -homomorphism $f : (A \cap B) \rightarrow M$ such that $A \cap \text{Im } f = 0$ and $A + \text{Im } f$ is pure in M , $\ker f$ is pure in M .

Theorem 2.6 :- Let M be an R -module, then M has the (m, n) -PIP, if and only if for every (m, n) -pure submodules A and B of M and for every R -homomorphism $f : (A \cap B) \rightarrow M$ such that $A \cap \text{Im } f = 0$ and $A + \text{Im } f$ is (m, n) -pure in M , $\ker f$ is (m, n) -pure in M .

Proof:- Assume that M has the (m, n) -PIP. Let A and B be (m, n) -pure submodules of M and $f : (A \cap B) \rightarrow M$ be an R -homomorphism such that $A \cap \text{Im } f = 0$ and $A + \text{Im } f$ is (m, n) -pure in M . Let $T = \{x + f(x) / x \in A \cap B\}$,

It is clear that T is a submodule of M . To show that T is (m, n) -pure in M . Let I be n -generated submodule of ${}_R R^m, I = Rb_1 + Rb_2 + \dots + Rb_n,$
 $b_j = (\alpha_{1j}, \dots, \alpha_{mj}) \in R^m$ and $y \in MI \cap T^m, m_j \in M, b_j \in R^m, \forall j = 1, \dots, n$. Hence

$$y = \sum_{j=1}^n m_j b_j = (u_1, \dots, u_m), u_1, \dots, u_m \in T,$$

$$u_i = x_i + f(x_i), i = 1, \dots, m. \text{ For some } x_i \in A \cap B. \text{ Since } y = \sum_{j=1}^n m_j b_j = (x_1 + f(x_1), \dots, x_m + f(x_m)) = (x_1, \dots, x_m) + (f(x_1), \dots, f(x_m)) \in (A \cap B)^m + (\text{Im } f)^m \subseteq A + (\text{Im } f)^m = (A + \text{Im } f)^m \text{ and } A + \text{Im } f \text{ is } (m, n)\text{-pure in } M. \text{ Thus}$$

$$y = \sum_{j=1}^n m_j b_j \in MI \cap (A + \text{Im } f)^m = (A + \text{Im } f)I$$

[4, theorem(1.5)]. Therefore $\sum_{j=1}^n m_j b_j =$

$$\sum_{j=1}^n (a_j + c_j) b_j, a_j \in A, c_j \in \text{Im } f.$$

$$\text{Thus } y = \sum_{j=1}^n m_j b_j = \sum_{j=1}^n a_j b_j + \sum_{j=1}^n c_j b_j,$$

$$\text{hence } (x_1, \dots, x_m) - \sum_{j=1}^n a_j b_j = \sum_{j=1}^n c_j b_j -$$

$$(f(x_1), \dots, f(x_m)) \in (A \cap \text{Im } f)^m. \text{ Since}$$

$$A \cap \text{Im } f = 0, \text{ then } (x_1, \dots, x_m) = \sum_{j=1}^n a_j b_j$$

$$\in AI \cap (A \cap B)^m. \text{ But } A \cap B \text{ is}$$

$$(m, n)\text{-pure in } M, \text{ hence } A \cap B \text{ is}$$

$$(m, n)\text{-pure in } A, \text{ thus}$$

$$AI \cap (A \cap B)^m = (A \cap B)I.$$

$$\text{Then } (x_1, \dots, x_m) \in (A \cap B)I.$$

$$\text{Let } (x_1, \dots, x_m) = \sum_{j=1}^n w_j b_j, w_j \in A \cap B,$$

$$\text{then } (f(x_1), \dots, f(x_m)) = \sum_{j=1}^n f(w_j) b_j.$$

$$\text{Now, } y = (x_1, \dots, x_m) + (f(x_1), \dots, f(x_m)) =$$

$$\sum_{j=1}^n w_j b_j + \sum_{j=1}^n f(w_j) b_j =$$

$$\sum_{j=1}^n (w_j + f(w_j)) b_j \in TI. \text{ Therefore } T$$

$$\text{is } (m, n)\text{-pure in } M. \text{ Next we show}$$

$$\text{that } \ker f = (A \cap B) \cap T. \text{ Let}$$

$$x \in \ker f, \text{ then } x \in A \cap B \text{ and}$$

$$f(x) = 0. \text{ Hence } x \in T. \text{ Now,}$$

$$\text{let } x \in (A \cap B) \cap T,$$

$$\text{then } x = y + f(y), y \in A \cap B. x -$$

$$y = f(y) \in A \cap \text{Im } f = 0. \text{ Therefore}$$

$$f(x) = f(y) = 0 \text{ and } x \in \ker f. \text{ Since}$$

$$M \text{ has the } (m, n)\text{-PIP,}$$

$$(A \cap B) \cap T = \ker f \text{ is } (m, n)\text{-pure}$$

in M . Conversely, let A and B be (m, n) -pure submodules of M . Define $f : (A \cap B) \rightarrow M$ by $f(x) = 0 \forall x \in A \cap B$. It is clear that $A \cap \text{Im} f = 0$ and $A + \text{Im} f = A$ is (m, n) -pure in M . Then $\ker f = A \cap B$ is (m, n) -pure in M . Then M has the (m, n) -PIP.

By the same argument one can prove the following:

Theorem 2.7:- Let M be an R -module. Then M has the (m, n) -PIP if and only if for every (m, n) -pure submodules A and B of M and for every R -homomorphism $f : (A \cap B) \rightarrow C$ where C is a submodule of M such that $A \cap C = 0$ and $A + C$ is (m, n) -pure, in M , $\ker f$ is (m, n) -pure in M .

Proof:- it is clear

Corollary 2.8:- Let M be an R -module with the (m, n) -PIP. Then for every decomposition $M = A \oplus B$ and for every R -homomorphism $f : A \rightarrow B$, $\ker f$ is (m, n) -pure in M .

Proof:- Since $(A \cap B) = 0$ and $A + B = M$ is (m, n) -pure, in M and $A = A \cap M$. Then by (theorem 2.9), $\ker f$ is (m, n) -pure in M .

Corollary 2.9:- Let M be an R -module with the (m, n) -PIP. let A and B be (m, n) -pure submodules in M such that $(A \cap B) = 0$ and $A + B$ is (m, n) -pure, in M . Then for each R -homomorphism $f : A \rightarrow B$, $\ker f$ is (m, n) -pure in M .

Remark 2.10:- Let A be (m, n) -pure submodule of an R -

module, M then there exists (m, n) -pure \bar{A} in M such that \bar{A} is maximal with respect property $A + \bar{A}$ is (m, n) -pure in M and $A \cap \bar{A} = 0$.

Proof :- Let $F = \{ B : B \text{ is } (m, n)\text{-pure in } M \text{ such that } (A \cap B) = 0 \text{ and } A + B \text{ is } (m, n)\text{-pure in } M \}$. It is clear that $(0) \in F$ and hence $F \neq \emptyset$.

Let $\{C_\alpha\}_{\alpha \in I}$ be a chain in F . It is clear that $\bigcup_{\alpha \in I} C_\alpha$ is a submodule of M and since $C_\alpha \cap A = 0 \forall \alpha \in I$. Then $(\bigcup_{\alpha \in I} C_\alpha) \cap A = 0$. To show that $\bigcup_{\alpha \in I} C_\alpha$ is (m, n) -pure in M . Let $I = Rb_1 + Rb_2 + \dots + Rb_n$ be n -generated submodule of ${}_R R^m$ and,

$$\sum_{i=1}^n m_i b_i \in MI \cap (\bigcup_{\alpha \in I} C_\alpha)^m.$$

$$\sum_{i=1}^n m_i b_i \in MI \cap (C_{\alpha_0})^m \text{ for}$$

some $\alpha_0 \in I$. Thus,

$$\sum_{i=1}^n m_i b_i \in C_{\alpha_0} I \subseteq (\bigcup_{\alpha \in I} C_\alpha) I.$$

Therefore, $MI \cap (\bigcup_{\alpha \in I} C_\alpha)^m = (\bigcup_{\alpha \in I} C_\alpha) I$. To

show that $A + \bigcup_{\alpha \in I} C_\alpha$ is (m, n) -pure in M . Let

$$\sum_{i=1}^n m_i b_i \in MI \cap (A + \bigcup_{\alpha \in I} C_\alpha)^m, \text{ then}$$

$$\sum_{i=1}^n m_i b_i \in MI \cap (A + C_{\alpha_0})^m \text{ for some}$$

$$\alpha_0 \in I, \text{ and hence } \sum_{i=1}^n m_i b_i \in (A + C_{\alpha_0}) I.$$

Thus $\sum_{i=1}^n m_i b_i \in (A + \bigcup_{\alpha \in I} C_\alpha) I$. By

zeron's lemma F has a maximal element say $\bar{A} = \bigcup_{\alpha \in I} C_\alpha$.

In [2, theorem 3.8], let M be an R -module such that for every pure submodules A and B of M either $A \subseteq B \oplus \bar{B}$ or $B \subseteq A \oplus \bar{A}$, then M has

the PIP if and only if for every R -homomorphism

$$f : A \cap (B \oplus \bar{B}) \rightarrow \bar{A}, \ker f \text{ is pure in } M.$$

Theorem 2.11:- Let M be an R -module such that for every (m, n) -pure submodules A and B of M either $A \subseteq B \oplus \bar{B}$ or $B \subseteq A \oplus \bar{A}$, then M has the (m, n) -PIP if and only if for every R -homomorphism

$$f : A \cap (B \oplus \bar{B}) \rightarrow \bar{A}, \ker f \text{ is } (m, n)\text{-pure in } M.$$

Proof:- Suppose that M has the (m, n) -PIP and A and B are (m, n) -pure submodules of M . Let $f : A \cap (B \oplus \bar{B}) \rightarrow \bar{A}$, be an R -homomorphism, then by (theorem 2.9), $\ker f$ is (m, n) -pure in M . The converse, let A and B be (m, n) -pure submodules of M . Assume that $A \subseteq B \oplus \bar{B}$. $\pi_1 : A \oplus \bar{A} \rightarrow A$ and $\pi_2 : B \oplus \bar{B} \rightarrow \bar{B}$ be the natural projections. Put $h = \pi_2 \pi_1 |_{B \cap (A \oplus \bar{A})}$. We show that $\ker h = (B \cap A) \oplus (B \cap \bar{A})$. Let $x \in \ker h, x \in B \cap (A \oplus \bar{A})$ and then $x = a + \bar{a}, a \in A, \bar{a} \in \bar{A}$. Now, $\pi_2 \circ \pi_1(a + \bar{a}) = \pi_2(a) = 0$. So $a \in B$, then $\bar{a} \in B, x \in (B \cap A) \oplus (B \cap \bar{A})$. Now, let $x \in (B \cap A) \oplus (B \cap \bar{A})$, then $x = a + \bar{a}, a \in B \cap A, \bar{a} \in B \cap \bar{A}$. Thus, $\pi_2 \circ \pi_1(a + \bar{a}) = \pi_2(a) = 0$. Therefore $\ker h = (B \cap A) \oplus (B \cap \bar{A})$ is (m, n) -pure in M . $B \cap A$ is (m, n) -pure in $\ker h$ (Remark 2.12), then $B \cap A$ is (m, n) -pure in M [3, prop.1.9]. That is M has the (m, n) -PIP.

In [3, proposition 3.10], for any two pure submodule A and B of an R -module M , if $A+B$ is flat, then M has the PIP.

Proposition 2.12:- Let M be an R -module such that for any two (m, n) -pure submodules A and B of $M, A+B$ is (m, n) - flat R -module, then M has the (m, n) -PIP.

Proof:- Let A and B be (m, n) -pure submodules of M . Consider the following short exact sequence

$$0 \rightarrow A \cap B \xrightarrow{i_1} A \xrightarrow{f_1} \frac{A}{A \cap B} \rightarrow 0$$

$$0 \rightarrow B \xrightarrow{i_2} A + B \xrightarrow{f_2} \frac{A+B}{B} \rightarrow 0$$

Where i_1, i_2 are the inclusion maps and f_1, f_2 are the natural epimorphism by the second isomorphism theorem,

$$\frac{A}{A \cap B} \cong \frac{A+B}{B}.$$

Since $A+B$ is (m, n) - flat R -module, and B is (m, n) -pure submodule of M , then [4, theorem 3.6] $\frac{A+B}{B}$ is (m, n) -flat and B is (m, n) -pure in $A+B$.

Thus $\frac{A}{A \cap B}$ is (m, n) -flat and hence $A \cap B$ is (m, n) -pure in A . But A is (m, n) -pure in M , so $A \cap B$ is (m, n) -pure in M , thus M has the (m, n) -PIP.

Lemma 2.13 :- Let $M = \bigoplus_{i \in I} M_i$ where M_i is a submodule of $M \forall i \in I$ and let W_i be a submodule of M_i , for each $i, j \in I$. Then $\bigoplus W_i$ is (m, n) -pure in M if and only if W_i is (m, n) -pure in $M_i \forall i$.

Proof :- Assume $\bigoplus_{i \in I} W_i$ is (m, n) -pure in M . since W_i is a summand of $\bigoplus W_i$, then W_i is (m, n) -pure in $\bigoplus_{i \in I} W_i$. So W_i is (m, n) -pure

in M [3,proposition1.9]. Since W_i is a submodule of M_i , then W_i is (m, n) -pure in $M_i \forall i$, [3,proposition 1.9]. The converse ,let J be n -generated submodule of ${}_R R^m$ and $x \in MJ \cap (\bigoplus_{j=1}^n W_j)^m . x = \sum_{j=1}^n m_j b_j , m_j \in \bigoplus_{i \in I} M_i .$ Then $m_j = \sum_{i \in I} m_{ij} , m_{ij} \in M_i$ for $i \in I .$ Thus $x \in \sum_{j=1}^n \sum_{i \in I} m_{ij} b_j = \sum_{i \in I} \sum_{j=1}^n m_{ij} b_j .$ Since $\sum_{j=1}^n m_{ij} b_j \in M_i , M = \bigoplus_{i \in I} M_i .$ The element x can be written uniquely as $\sum_{i \in I} \sum_{j=1}^n m_{ij} b_j .$ But $x \in \bigoplus_{i \in I} W_i .$ Thus $\sum_{j=1}^n m_{ij} b_j \in W_i \forall i$ and hence $\sum_{j=1}^n m_{ij} b_j \in M_i J \cap (W_i)^m = W_i J$ (W_i is (m, n) -pure in M_i), $\sum_{j=1}^n m_{ij} b_j = \sum_{j=1}^n w_{ij} b_j$ $w_{ij} \in W_i$ for each $j .$ Thus $x = \sum_{i \in I} \sum_{j=1}^n w_{ij} b_j \in (\bigoplus_{i \in I} W_i) J .$

Proposition 2.14 :- Let $M = \bigoplus_{i \in I} M_i$ be an R -module where each M_i is a submodule of M , If M has the (m, n) -PIP, then each M_i has the (m, n) -PIP. The converse is true if each (m, n) -pure submodule of M is fully invariant.

Proof:- Suppose that M has the (m, n) -PIP. Since M_i is a summand of M , then M_i is (m, n) -pure in M [Remark 2.3], and hence M_i has the (m, n) - PIP .To prove the

converse, let S be (m, n) - pure submodule of M and $\pi_i : M \rightarrow M_i$ be the natural projection on M_i , for each $i \in I .$ Let $x \in S$, then $x = \sum_{i \in I} m_i , m_i \in M_i , \pi_i(x) = m_i .$ Since S is (m, n) -pure in M , then S is fully invariant (By assumption),and hence $\pi_i(S) \subseteq S \cap M_i .$ Thus $\pi_i(x) = m_i \in S \cap M_i .$ i.e $x \in \bigoplus_{i \in I} (S \cap M_i) .$ Therefore $S \subseteq \bigoplus_{i \in I} (S \cap M_i) .$ But $\bigoplus_{i \in I} (S \cap M_i) \subseteq S$, so $S = \bigoplus_{i \in I} (S \cap M_i) .$ Now suppose S and T are (m, n) -pure submodules of M , then $S \cap T = (\bigoplus_{i \in I} (S \cap M_i)) \cap (\bigoplus_{i \in I} (T \cap M_i)) = \bigoplus ((S \cap M_i) \cap (T \cap M_i)) .$ Since $S = \bigoplus_{i \in I} (S \cap M_i)$, then $S \cap M_i$ is (m, n) -pure in S [Remark 2.3]. But S is (m, n) -pure in M , so $S \cap M_i$ is (m, n) -pure in M . Then $S \cap M_i$ is (m, n) -pure in M_i . By lemma 2.13, $\bigoplus ((S \cap M_i) \cap (T \cap M_i))$ is (m, n) -pure in $\bigoplus_{i \in I} M_i = M .$

Proposition 2.15 :- Let M and N be R -module with the (m, n) - PIP, such that $r_R(M) + r_R(N) = R$, then $M \oplus N$ has the (m, n) -PIP .

Proof:-Let C and D be (m, n) -pure submodules of $M \oplus N$. Since $r_R(M) + r_R(N) = R$, then by the same way of the proof of ([5],proposition 4.2,CH.1), $C = A \oplus B$ and $D = A_1 \oplus B_1$ where A and A_1 are submodules of M , B and B_1 are submodules of N .Since M and N has the (m, n) -PIP ,then $A \cap A_1$ is (m, n) -pure in M and $B \cap B_1$ is (m, n) -pure in N .Thus by lemma 2.13 $(A \cap A_1) \oplus (B \cap B_1)$,

is (m, n) -pure in $M \oplus N$. But $(A \cap A_1) \oplus (B \cap B_1) = (A \cap A_1) \cap (B \cap B_1) = C \cap D$. So $C \cap D$ is (m, n) -pure in $M \oplus N$ and that is $M \oplus N$ has (m, n) -PIP.

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ملاحظات حول المقاسات التي تمتلك خاصية التقاطع النقي من النمط (m, n)

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الكلمات المفتاحية: مقياس جزئي نقي من النمط (m, n) ، مقياس مسطح من النمط (m, n) ، مقياس يمتلك خاصية التقاطع النقي من النمط (m, n) .

الخلاصة:

لتكن R حلقة فإن المقياس يسمى رئيسي من النمط (m, n) لكل عددين صحيحين موجبين m, n إذا وجدت متتابعة مضبوطة بالشكل $0 \rightarrow K \rightarrow R^m \rightarrow V \rightarrow 0$ بحيث K هو مقياس متولد ب n من العناصر. المقياس الجزئي N من المقياس الأيمن M يسمى نقي من النمط (m, n) في M . إذا كان كل مقياس رئيسي من النمط (m, n) وليكن V بحيث إن التشاكل $N \otimes_R V \rightarrow M \otimes_R V$ متباين. أما المقياس M فإنه يسمى مقياس يمتلك خاصية التقاطع النقي من النمط (m, n) إذا كان تقاطع كل مقاسين نقيين من النمط (m, n) يكون مقياس نقي من النمط (m, n) . الهدف من هذا البحث هو اعطاء بعض الخواص والنظريات والخصائص للمقاسات بأستخدام خاصية التقاطع النقي من النمط (m, n) للمقاسات.