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Some Outcomes Involving a Specific Class of Functions over Differential Subordination and Superordination

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Abstract

This work investigates several aspects of differential subordination and superordination, leading to the inclusion of a specific class within the domain of univalent meromorphic functions in a perforated open unit disc and deriving a few sandwich theorems. The purpose of this article is to look into a few of the characteristics of variation subordination for analytic univalent functions over a perforated unit disc. It additionally aims to shed insight into geometric characteristics like coefficient inequality, Hadamard product characteristics, and the Komatu integral operator. A few interesting findings have been discovered for variations in subordination as well as superordination in analytic univalent functions. The outcomes about variations in subordination, including linear algebra operators, were presented employing convolutions involving two linear operators. Everyone evaluates and investigates subordinations as well as superordinations about convolutions using includes from the Komatu integral operator. The convolution operator as a tool was used for obtaining multiple findings over differential subordination within the perforated unit disk employing a generalized hypergeometric function. Appropriate classes of acceptable functions are examined, and the two-dimensional real estate of the differential subordinations is explained by utilizing the linear operator, a technique that Srivastava introduced as well as examined. This leads to the establishment of several sandwich-type theorems for a class of univalent analytical functions. The current work examines several subclasses of star-like functions that are defined by subordination. Additionally, our team provides some relevant links between the results reported here and those acquired previously.

Keywords: Differential Operator, Differential Subordination, Generalized Hypergeometric Functions, Meromorphic Functions, Sandwich Theory, Starlike Functions.

Introduction

The study of analytic univalent and multivalent functions has a rich history in mathematics, particularly in complex analysis. It has attracted many researchers due to the sophistication of its geometric features and the numerous possibilities for study. One of the most significant fields of intricate analysis over only a few factors is the investigation of univalent functions. Since possibly 1907, scientists have become intrigued by the conventional investigation of this topic. Numerous scholars in the discipline of complicated analysis, including Euler, Gauss, Riemann, and Cauchy, as well as a number of others, have emerged throughout this period of time. The geometric function concept is a synthesis of geometry as well as analysis. The primary impetus behind the aforementioned school of thought is the renowned speculation referred to as the Bieberbach speculation or coefficients issue, and this provided an enormous opportunity for expansion to 1916 up until its favorable agreement in 1985 through De Branges, in which numerous leads derived from the aforementioned issue have been achieved. Since that time, geometric function theory has been further studied separately. Geometric Function Theory has become a popular subject matter. Regardless of this, it is still finding novel uses in an array of fields, such as contemporary mathematical physics, medical science, engineering, and many more, in addition to traditional physics topics such as fluid mechanics, not linear compatible system theory, and the basic partial differential equation hypothesis. Rogosinski [1] created the fundamental leads to subordination that are able to be tracked to the work of Littlewood [2]. Srivastava as well as Owa [3] recently employed subordination to investigate the fascinating characteristics that characterize the broader hypergeometric function. Miller as well as Mocanu [4] wrote about differential subordinations, which can be considered a generalization about differential disparities in society. The subject matter is primarily concerned with the differential subordination as well as the superordination of univalent operations on a transparent unit disk.

Take $D = \{r \in \mathbb{C} : |r| < 1\}$ indicate an open unit disc within $\mathbb{C}$. Allow $G(D)$ to become the class of analytic functions found in D as well as $D[i,n]$ to become the subclass about $G(D)$ in the shape of:

$$\varphi(r) = i + i_n r^n + i_{n+1} r^{n+1} + \cdots ,$$

so that $i \in \mathbb{C}, n \in \mathbb{N}, G_0 \equiv G[0,1]$ and $G \equiv G[1,1]$. Assume Σ that is the class that includes all analytic functions in the shape of

$$\varphi(r) = r^{-1} + \sum_{n=0}^{\infty} i_n r^n , \qquad 1$$

inside the pierced unit disk

$$D^* = \{r : r \in \mathbb{C}\, ;\ 0 < |r| < 1\} = D \backslash \{0\}.$$

Every function S has the same indication inside D. Allow

$$S^* = \left\{\varphi \text{ is univalent}, \Re e \frac{r\varphi'(r)}{\varphi(r)} > 0, \right\},$$

indicate the class of starlike functions inside D as well as

$$K = \left\{\varphi \text{ is univalent}, \Re e \frac{r\varphi''(r)}{\varphi'(r)} + 1 > 0, \right\},$$

indicate a convex function class inside D. Assume that φ as well as F are members of $G(D)$. When it's possible to find a Schwarz function u analytic inside D, alongside $u(0) = 0$ and $|u(r)| < 1$, so that $\varphi(r) = F(u(r))$. The function φ is deemed to have been subordinate to a function F or F is deemed to have been superordinate to φ. This position of subordination is denoted through

$$\varphi(r) \prec F(r) \text{ or } \varphi \prec F.$$

likewise, when the function F is univalent in D, then a subsequent equivalence exists [5, 6]

$$\varphi(r) \prec F(r) \Leftrightarrow \varphi(0) = F(0) \text{ and } \varphi(D) \subset F(D).$$

Allow Θ as well as Λ be take hold $\mathbb{C}$, allow $\phi : \mathbb{C}^3 \times D \to \mathbb{C}$ as well as h be univalent inside D. When p is analytic in D with $p(0) = i$ and implications conclusions,

$$\{\phi(p(r), rp'(r), rp''(r); r\,)\} \subset \Theta \Longrightarrow p(D) \subset \Lambda,$$

alongside meets the second-order differential subordination requirement

$$\phi(\,p(r), rp'(r), rp"(r); r\,) \prec g(r), \qquad 2$$

subsequently, p is referred to as a variations subordination of a solution. In addition to p, $\Phi(p(r), rp'(r), rp"(r); r)$ are univalent alongside meets the second-order variations subordination requirement

$$g(r) \prec \phi(\,p(r), rp'(r), rp"(r); r), \qquad 3$$

then p is referred to as a variations superordination: by Eq 3, obtaining

$$\Theta \subset \{\phi(p(r), rp'(r), rp''(r); r)\}.$$

Bulboaca [7] investigated specific types of first-order variations superordinations, including superordination safeguarding integral operators, employing the outcomes of Miller as well as Mocanu [8]. Using the results of Bulboaca [7], Ali [9] obtained adequate circumstances for particular normalized analytic functions in satisfy:

$$w_1(r) \prec \frac{r\varphi'(r)}{\varphi(r)} \prec w_2(r),$$

in which w_1 as well as w_2 are somewhat univalent functions inside D and $w_1(0) = w_2(0) = 1$. Tuneski [10] gathered sandwich leads to feed certain classes about analytic functions in order to satisfy:

$$w_1(r) \prec \frac{\varphi(r)}{z\varphi'(r)} \prec w_2(r),$$

and

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Baghdad Science Journal

$$\frac{\varphi''(r)f(r)}{\left(\varphi'(r)\right)^2}.$$

A few weeks ago Shanmugam et al. [11] gathered sandwich outcomes from their analytic function [12- 15]. Can look at a few variations of subordination as well as superordination leads to include the operator $E^m_{v,\alpha}(r)$.

Basic Concepts

At this point, we'll go over the definitions in addition to the lemmas that will be required in this piece of writing.

Definition 1: [16] Indicating by W a collection of functions w which are analytic as well as injective on $\overline{D}/T(w)$, which

$$T(w) = \left\{x \,\epsilon\, \partial D \,;\, \lim_{r\to x} w(r) = \infty\right\}, \qquad 4$$

so that $w'(x) \neq 0$ for $x \,\epsilon\, \partial D/t(w)$. The subclass for W about which $w(0) = b$ is represented by $W(b)$.

Definition 2: [17] In terms of parameters $\lambda_i \in \mathbb{C}$, $i = 1, 2, \dots, q, \beta_j \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$ and $j = 1, 2, \dots, s$, the general hypergeometric function is characterized as follows [9,10,13]

$$_qF_s(\lambda_1, \dots, \lambda_q; \beta_1, \dots, \beta_s; z) = \sum_{n=0}^{\infty} \frac{(\lambda_1)_n \dots (\lambda_q)_n}{(\beta_1)_n \dots (\beta_s)_n\, n!} r^n, q \le s+1; s \in \mathbb{N} \cup \{0\};\ \mathbb{N} = \{1,2,\dots\}, \qquad 5$$

the Pochhammer representation $(x)_n$ it can be defined as a component of the Gamma function Γ through

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} x(x+1)(x+2)\dots(x+n-1) \;;\; n \in \mathbb{N} \text{ and } x \in \mathbb{C} \\ 1 \quad \text{if } n = 0 \text{ and } x \in \mathbb{C}\setminus\{0\} \end{cases}. \qquad 6$$

Definition 3: [18] If functions $\varphi \in \Sigma$ with v, α to be numbers that are real, ($v \ge \alpha \ge 0$ and $m \in \mathbb{N}$ characterize the operator as comes next:

$$D^m_{v,\alpha}\varphi(r) = r^{-1} + \sum_{n=0}^{\infty} A(v,\alpha,n)^m\, i_n\, r^n, \quad (r \in D^*) \qquad 7$$

so that

$$A(v,\alpha,n) = [(n+2)v\alpha + v - \alpha](n+1) + 1, \qquad 8$$

and

$$D^0_{v,\alpha}\varphi(r) = \varphi(r)$$
$$D^1_{v,\alpha}\varphi(r) = E_{v,\alpha}\varphi(r)$$
$$D^m_{v,\alpha}\varphi(r) = D_{v,\alpha}\left(D^{m-1}_{v,\alpha}\,\varphi(r)\right). (r \in D^*) \qquad 9$$

Let $v = 1$ as well as $\alpha = 0$, receiving the difference operator established in [14]. Using the operator $D^m_{v,\alpha}\varphi(r)$. Currently, characterizing the linear operator $E^m_{v,\alpha}(r)$ on Σ which comes next:

$$E^m_{v,\alpha}(r) = D^m_{v,\alpha}\varphi(r) * \omega(r), \qquad (r \in D^*)$$

in which

$$\omega(r) = r^{-1} + \sum_{n=0}^{\infty} \frac{(i)_{n+1}}{(c)_{n+1}} i_n\, r^n \ , \ i \in C^*, c \in C\setminus\{0,-1,-2,\dots\},$$

then

$$E^m_{v,\alpha}(r) = r^{-1} + \sum_{n=0}^{\infty} A(v,\alpha,n)^m \ \frac{(i)_{n+1}}{(c)_{n+1}} i_n\, r^n. \qquad 10$$

It is easy to see for Eq 10, which

$$r\left(E^m_{v,\alpha}(r)\varphi(r)\right)' = iE^m_{v,\alpha}(i+1;r)\varphi(r) - (i+1)E^m_{v,\alpha}(r)\varphi(r), \qquad 11$$

and

$$r\left(E^{m+1}_{v,\alpha}(r)\varphi(r)\right)' = vE^m_{v,\alpha}(r)\varphi(r) - (v+1)E^{m+1}_{v,\alpha}(r)\varphi(r). \qquad 12$$

Lemma 1: [19] The function

$$w(r) = \frac{1}{(1-r)^{2bc}}$$

is univalent inside D if as well as only if $|2bc-1| \le 1$ or $|2bc+1| \le 1$.

Lemma 2: [10] Allow w to be univalent inside D,

$$w(r) = \frac{1+Ar}{1-Ar}, \text{as well } A \in (-1,0) \cup (0,1), \qquad 13$$

as well as allow $\sigma \in (0,1], \lambda, \beta > 0$, so that

$$\frac{2\lambda}{\sigma}\frac{1+A}{1-A} + \frac{\beta}{\sigma}\frac{1+A}{1-A} > 0.$$

When p univalent in D alongside $p(0) = w(0) = 1$, in addition to

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Baghdad Science Journal

$$\lambda p^2(r)+\beta p(r)+\sigma r p'(r) \prec \lambda\left(\frac{1+Ar}{1-Ar}\right)^2+\beta\left(\frac{1+Ar}{1-Ar}\right)+\sigma\frac{2Ar}{(1-Ar)^2}, \qquad 14$$

then

$$p(r)\prec w(r).$$

Lemma 3: [20] Allow w to be univalent alongside $w(0)=b$ and ϕ,η be analytic in a domain $\wp$ containing $w(D)$. Assume that

(i) $\Re e\left(\frac{\phi'(w(r))}{\eta(w(r))}\right)>0$. $(ii) W(r)=rw'(r)\eta(w(r))$ is starlike univalent in D.

When $p\in G[b,1]\cap W$ alongside $p(D)\subseteq\wp$, $\phi(p(r))+rp'(r)\eta(p(r))$ is univalent inside D, then

$$\phi(w(r))+rw'(r)\eta(w(r)) \prec \phi(p(r))+rp'(r)\eta(p(r)), \qquad 15$$

indicates that $w(r)\prec p(r)$, w is the most subordinate dominant Eq 15.

Lemma 4: [15] Allow w to be univalent within the unit disc D as well as allow ϕ,η to be analytic within a domain $\wp$ containing $w(D)$, alongside $\eta(u)\neq 0$ so that $u\in w(D)$. Set

$W(r)=rw'(r)\eta(w(r)), g(r)=\phi(w(r))+W(r)$

and assume that either

(i) g is convex, or (ii) W is a starlike function within D. Furthermore, suppose (iii) $\Re e\left(\frac{rg'(r)}{W(r)}\right)>0$.

When p is analytic within D, alongside $p(0)=w(0)$, $p(D)\subseteq\wp$ and

$$\phi(p(r))+rp'(r)\eta(p(r)) \prec \phi(w(r))+rw'(r)\eta(w(r)), \qquad 16$$

then $p(r)\prec w(r)$, in addition to w is the most suitable dominant for Eq 16.

Lemma 5: [8] Allow w to be convex inside D alongside $w(0)=b$ as well as allow $\beta\in\mathbb{C}$, and $\Re e(\beta)>0$. If $p\in G[b,1]\cap W$ as well as $p(r)+\beta rp'(r)$ univalent within D, then

$$w(r)+\beta rw'(r) \prec p(r)+\beta rp'(r), (r\in D) \qquad 17$$

indicates that $w(r)\prec p(r)$, w is the most suitable dominant for Eq 17.

Lemma 6: [11] Allow w to be convex univalent inside D alongside $w(0)=b$ and assume that $\Re e\left\{\frac{\alpha_1}{\alpha_2}\right\}>0, \alpha_I\in\mathbb{C}(i=1,2)$ and $\alpha_2\neq 0$. If $p\in G[b,1]\cap W$ as well as $\alpha_1 p(r)+\alpha p'(r)$ is univalent within D, then

$$\alpha_1 w(r)+\alpha_2 rw'(r) \prec \alpha_1 p(r)+\alpha_2 p'(r), \qquad 18$$

which implies $w\prec p$ in addition to w is the best subordinant.

Lemma 7: [15] Allow $\lambda,\beta\in\mathbb{C}$ alongside $\beta\neq 0$ as well as allow w be a convex function within D, and

$$\Re e\left\{1+\frac{rw''(r)}{w'(r)}\right\} > max\left\{0; -\Re e\left(\frac{\lambda}{\beta}\right)\right\}. (r\in D) \qquad 19$$

When p is analytic within D and

$$\lambda p(r)+\beta rp'(r)\prec \lambda w(r)+\beta rw'(r), \qquad 20$$

then $p(r)\prec w(r)$, w is the most suitable dominant for Eq 20.

Results and Discussion

Theorem 1: If $\varphi\in\Sigma$ with p is univalent inside the unit disc D alongside $p(0)=1$, as well as what follows:

$$\Re e\left\{1+\frac{rp''(r)}{p'(r)}\right\} > max\left\{0; -\upsilon\mu\Re e\left(\frac{1}{\lambda}\right)\right\}, \qquad 21$$

for $rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\neq 0$, $0<\mu<1, \upsilon\geq 1, \lambda\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$. In addition to the subsequent variations subordination

$$\left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu} \prec p(r)+\lambda\frac{rp'(r)}{\mu\upsilon}, \qquad 22$$

then

$$\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}\prec p(r),$$

along with the function $p(r)$ is the most dominant of Eq 22.

Proof: Allow

$$\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}=g(r),$$

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Baghdad Science Journal

subsequently $g(r)$ is analytic in D alongside $p(0)=1$, by Eq 11, getting

$$\left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu} = g(r)+\lambda\frac{rg'(r)}{\mu\upsilon},$$

As a result, the Eq 22 provides

$$g(r)+\lambda\frac{rg'(r)}{\mu\upsilon}\prec p(r)+\lambda\frac{rp'(r)}{\mu\upsilon}.$$

Employing Lemma 7, in which $x=1$ and $\alpha=\frac{\lambda}{\mu\upsilon}$, getting the outcome.

When $p(r)=\frac{1\_Ar}{1+Br}$ is substituted in Theorem 1, in which $-1\le B<A\le 1$, the circumstance Eq 21 turns into

$$\Re e\left\{\frac{1-Br}{1+Br}\right\}>max\left\{0;\ -\upsilon\mu\Re e\left(\frac{1}{\lambda}\right)\right\}, \qquad 23$$

It is simple to demonstrate which function $f(\vartheta)=\frac{(1-\vartheta)}{(1+\vartheta)}, |\vartheta|<|B|$, is convex inside D since $\overline{f(\vartheta)}=\overline{f(\vartheta)}$ every time $|\vartheta|<|B|$, it comes that $f(r)$ is a convex domain symmetrical in relation to its real axis, hence

$$inf\left\{\Re e\left\{\frac{1-Br}{1+Br}\right\}\right\}=\frac{1-|B|}{1+|B|}>0. \qquad 24$$

In that case, Eq 23 is the same as

$$\upsilon\mu\Re e\left(\frac{1}{\lambda}\right)\ge\frac{1-|B|}{1+|B|}.$$

Corollary 1: Allow $rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\neq 0, 0<\mu<1, \mu\ge 1$, $\lambda\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$ as well as $\Re e\left(\frac{1}{\lambda}\right)\ge 0$.
If $\varphi\in\Sigma$ alongside

$$\left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu} \prec \frac{1+r}{1-r}\frac{\lambda}{\upsilon\mu}\frac{2r}{(1-r)^2}. \qquad 25$$

Then

$$\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}\prec\frac{1+r}{1-r},$$

in addition to $\frac{1+r}{1-r}$ is the most dominant for Eq.25.

Corollary 2: Allow $rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\neq 0, \lambda\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$, $0<\mu<1, \mu\ge 1$ as well as $-1\le B<A\le 1$ and the one that follows:

$$max\left\{0;\ -\upsilon\mu\Re e\left(\frac{1}{\lambda}\right)\right\}\le\frac{1-|B|}{1+|B|}.$$

If $\varphi\in\Sigma$ with

$$\left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu} \prec \frac{1+Ar}{1+Br}\frac{\lambda}{\upsilon\mu}\frac{(A-B)r}{(1+Br)^2}. \qquad 26$$

Then

$$\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}\prec\frac{1+Ar}{1+Br},$$

in addition to $\frac{1+Ar}{1+Br}$ is the most dominant about Eq 25. If $A=1$ as well as $B=-1$.

Theorem 2: Let p become convex inside the unit disc D as well as $p(0)=1$, the values are $\varphi\in\Sigma, rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\neq 0, 0<\mu<1, \upsilon\ge 1$ alongside $\lambda\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$, as well as $\Re e(\lambda)>0$. Assume the following $\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}\in G[p(0),1]\cap W$.
If

$$\left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu}$$

is univalent within the unit disc D, in addition to

$$p(r)+\lambda\frac{rp'(r)}{\mu\upsilon} \prec \left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu}, \qquad 27$$

then

$$p(r)\prec\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1},$$

The function $p(r)$ is the most dominant for Eq.27.

Proof: Allow

$$\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}=g(r).$$

in which $g(r)$ is analytic in D via $p(0)=1$. Eq 12, gives us

$$g(r)+\lambda\frac{rg'(r)}{\mu\upsilon} = \left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu}.$$

Satisfied with this result by Lemma 5. Given $p(r)=\frac{1+Ar}{1+Br}$ within Theorem 2, as well as $-1\le B<A\le 1$, receiving the following outcome.

Corollary 3: Let p become convex inside the unit disc D alongside $p(0)=1$ as well as allow $-1\le B<A\le 1, \upsilon\ge 1, \lambda\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$ such so $\varphi\in\Sigma$, $rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\neq 0, 0<\mu<1$, and $\Re e(\lambda)>0$.

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Assume the following $\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} \in G[p(0),1] \cap W$. For the function

$$\left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu}$$

is univalent within the unit disc D, as well as

$$\frac{1+Ar}{1+Br}\frac{\lambda}{\upsilon\mu}\frac{(A-B)r}{(1+Br)^2} \prec \left(\frac{\lambda\upsilon+1}{\upsilon}\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} - r\lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu}, \quad 28$$

then

$$\frac{1+Ar}{1+Br} \prec \left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1},$$

as well as $\frac{1+Ar}{1+Br}$ is the most dominant for Eq.28.

Theorem 3: Let p become univalent inside the unit disc D as well as $p(0)=1$, allow $\varphi \in \Sigma$ everybody have $r \in D$ as well as $\delta, \Lambda, \rho \in \mathbb{C}$, alongside $\Lambda+\rho \neq 0$, $\alpha, \upsilon \in \mathbb{C}^* = \mathbb{C}\backslash\{0\}$. Assume that φ as well as p meet the subsequent circumstances:

$$\left\{\frac{r\{\Lambda E_{\upsilon,\alpha}^{m}(r)\varphi(r)+\rho E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right\} \neq 0, \quad 29$$

as well as

$$\Re e\left\{1+\frac{rp''(r)}{p'(r)}\right\} > max\left\{0; -\Re e\left(\frac{\delta}{\alpha}\right)\right\}. \quad 30$$

If

$$\phi(r) \equiv \left(\frac{r\{\Lambda E_{\upsilon,\alpha}^{m}(r)\varphi(r)+\rho E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right)^{-\mu+1} \left[\delta + \alpha\mu\left(-1 - \frac{\Lambda r\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)' + \rho r\left(E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)'}{\Lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)+\rho\left(E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)}\right)\right], \quad 31$$

as well as

$$\phi(r) \prec \delta p(r) + \alpha r p'(r), \quad 32$$

then

$$\left(\frac{r\{\Lambda E_{\upsilon,\alpha}^{m}(r)\varphi(r)+\rho E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right)^{-\mu+1} \prec p(r),$$

the value of the function $p(r)$ is the most dominant about Eq.32.

Proof: Let

$$\left(\frac{r\{\Lambda E_{\upsilon,\alpha}^{m}(r)\varphi(r)+\rho E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right)^{-\mu+1} = g(r). \quad 33$$

in which $g(r)$ is analytic in D and $p(0)=1$. By dividing Eq 33, receiving

$$\mu\left(-1 - \frac{\Lambda r\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)' + \rho r\left(E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)'}{\Lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)+\rho\left(E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)}\right) = \frac{rg'(r)}{g(r)}.$$

Hence,

$$\mu g(r)\left(-1 - \frac{\Lambda r\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)' + \rho r\left(E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)'}{\Lambda\left(E_{\upsilon,\alpha}^{m}(r)\varphi(r)\right)+\rho\left(E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)}\right) = rg'(r).$$

Furthermore, allowing the functions:

$$\phi(u) = \delta u, \quad \eta(u) = \alpha, \ u \in \mathbb{C},$$
$$W(r) = rp'(r)\eta(p(r)) = \alpha r p'(r),$$

and

$$h(r) = \phi(p(r)) + W(r) = \delta p(r) + \alpha r p'(r).$$

Based on Eq 30, can also see which W is a starlike function within D. Such as

$$\Re e\frac{rh'(r)}{W(r)} = \Re e\left\{\frac{\delta}{\alpha}+1+\frac{rp''(r)}{p'(r)}\right\} > 0.$$

The proof has become complete thanks to Lemma 4.

Allowing, $p(r) = \frac{1+Ar}{1+Br}$ within Corollary 3, as well as $-1 \le A < B \le 1$ alongside Eq 24, a value of Eq 30 is obtained.

$$max\left\{0; -\Re e\left(\frac{\delta}{\alpha}\right)\right\} \le \frac{1-|B|}{1+|B|}.$$

As a result of the particular scenario $\Lambda = 0, \rho = \alpha = 1$, getting what follows:

Corollary 4: If $rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r) \neq 0$ for every $r \in D, \varphi \in \Sigma$ alongside $-1 \le A < B \le 1, \mu \in \mathbb{C}^*$ and $\delta \in \mathbb{C}$ suppose as

$$max\{0; -\Re e(\delta)\} \le \frac{1-|B|}{1+|B|}.$$

If

$$\left(rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}\left[\delta + \upsilon\left(-1 - \frac{r\left(E_{\upsilon,\alpha}^{m+1}(r)\varphi(r)\right)'}{rE_{\upsilon,\alpha}^{m+1}(r)\varphi(r)}\right)\right] \prec \delta\frac{1+Ar}{1+Br} + \frac{r(A-B)}{(1+Br)^2}, \quad 34$$

subsequently,

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$$\left(rE_{v,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1} \prec \frac{1+Ar}{1+Br},$$

as well as $\frac{1+Ar}{1+Br}$ is the most dominant of Eq 34.

Allowing, $p(r) = \frac{1+r}{1-r}, \Lambda = 0, \alpha = \rho = 1$ as well as $\lambda_i = \beta_i (i = 1,2, \dots, s)$ within Corollary 4, obtain what follows:

Corollary 5: If $\varphi \in \Sigma$ as well as $r\varphi(r) \neq 0$ for every $r \in D$ such so $\mu \in \mathbb{C}^*$. And

$$\left(r\varphi(r)\right)^{-\mu+1}\left[\delta + v\left(-1 - \frac{r\left(\varphi(r)\right)'}{\varphi(r)}\right)\right] \prec \delta\frac{1+r}{1-r} + \frac{2r}{(1-r)^2}, \qquad 35$$

then

$$\left(r\varphi(r)\right)^{-\mu} \prec \frac{1+r}{1-r},$$

as well as $\frac{1+r}{1-r}$ is the most advantageous dominant of Eq 35.

Theorem 4: If the function p is univalent in D and $r \in D$, $p(0) = 1$, $p(r) \neq 0, \Lambda, \rho \in \mathbb{C}, \alpha, v \in \mathbb{C}^* = \mathbb{C}\backslash\{0\}$ as well as $\Lambda + \rho \neq 0$. If $\varphi \in \Sigma$ Assume that φ as well as p meet the following circumstances:

$$\left\{\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r) + \rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda + \rho}\right\} \neq 0, \qquad 36$$

and

$$\Re e\left\{1 + \frac{rp''(r)}{p'(r)} - \frac{rp'(r)}{p(r)}\right\} > 0. \qquad 37$$

If

$$\frac{1}{} + \alpha v\left[-1 - \frac{\Lambda r\left(E_{v,\alpha}^{m}(r)\varphi(r)\right)' + \rho r\left(E_{v,\alpha}^{m+1}(r)\varphi(r)\right)'}{\Lambda\left(E_{v,\alpha}^{m}(r)\varphi(r)\right) + \rho\left(E_{v,\alpha}^{m+1}(r)\varphi(r)\right)}\right] \prec 1 + \alpha\frac{rp'(r)}{p(r)}, \qquad 38$$

then

$$\left(\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r) + \rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda + \rho}\right)^{-\mu+1} \prec p(r),$$

As well as the function $p(r)$ is the most advantageous dominant of Eq 38.

Proof: Let

$$\left(\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r) + \rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda + \rho}\right)^{-\mu+1} = g(r), \qquad 39$$

therefore the function $g(r)$ is analytic within D as well as $p(0) = 1$. Eq 30 is differentiated to give us

$$\mu\left[-1 - \frac{\Lambda r\left(E_{v,\alpha}^{m}(r)\varphi(r)\right)' + \rho r\left(E_{v,\alpha}^{m+1}(r)\varphi(r)\right)'}{\Lambda\left(E_{v,\alpha}^{m}(r)\varphi(r)\right) + \rho\left(E_{v,\alpha}^{m+1}(r)\varphi(r)\right)}\right] = \frac{rg'(r)}{g(r)}.$$

By Lemma 4, letting

$$\phi(u) = 1 \text{ and } \eta(u) = \frac{\alpha}{u},$$

therefore w is analytic within $\mathbb{C}$ alongside $\eta(u) \neq 0$ is analytic in $\mathbb{C}^*$. Believing that

$$W(r) = rp'(r)\eta\left(p(r)\right) = \alpha\frac{rp'(r)}{p(r)},$$

and

$$h(r) = \phi\left(p(r)\right) + W(r) = 1 + \alpha\frac{rp'(r)}{p(r)},$$

since $W(0) = 0$ as well as $W'(0) \neq 0$, hence Eq 28 would result in W as a starlike function inside D. Eq 28, obtaining

$$\Re e\frac{rh'(r)}{W(r)} = \Re e\left\{1 + \frac{rp''(r)}{p'(r)} - \frac{rp'(r)}{p(r)}\right\} > 0,$$

using Lemma 4, getting $g(r) \prec p(r)$, $p(r)$ is the most advantageous dominant of the Eq 38. Next result, supposing that $\Lambda = 0, \rho = \alpha = 1$, $-1 \leq A < B \leq 1$, alongside $p(r) = \frac{1+Ar}{1+Br}$.

Corollary 6: Let $\varphi \in \Sigma$, and $b, c \in \mathbb{C}^*$ such so $r\varphi(r) \neq 0$ for every $r \in D$, $|2bc - 1| \leq 1$ or $|2bc + 1| \leq 1$. Alongside Eq 27. If

$$1 + \frac{1}{b}\left[-1 - \frac{r\varphi'(r)}{\varphi(r)}\right] \prec \frac{1+r}{1-r}, \qquad 40$$

therefore,

$$\left(rf(r)\right)^{-b} \prec \frac{1}{(1-r)^{2bc}},$$

as well as $\frac{1}{(1-r)^{2bc}}$ is the most advantageous dominant of Eq.40, consider the following, $b, c \in \mathbb{C}^*$, $\alpha = \frac{e^{i\lambda}}{cbcos\lambda}$, $\Lambda = 0, \mu = b$, $\rho = 1$ and $\lambda_i = \beta_i (i = 1,2, \dots, s)$ alongside $|\lambda| < \Lambda/2$, $q(z) = \frac{1}{(1-r)^{2cbcos\lambda e^{-i\lambda}}}$, the following is the outcome.

Corollary 7: If $\varphi \in \Sigma$, and $r\varphi(r) \neq 0$ for every $r \in D$, allow $|\lambda| < \Lambda/2$ and $b, c \in \mathbb{C}^*$. consider the follows $\left|2cbcos\lambda e^{-i\lambda} - 1\right| \leq 1$ $or\left|2cbcos\lambda e^{-i\lambda} + 1\right| \leq 1$ by Eq 29. If

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$$1+\frac{e^{i\lambda}}{bcos\lambda}\left[-1-\frac{r\varphi'(r)}{\varphi(r)}\right]\prec\frac{1+r}{1-r}, \qquad 41$$

then

$$\left(r\varphi(r)\right)^{-b}\prec\frac{1}{(1-r)^{2cbcos\lambda e^{-i\lambda}}},$$

as well as $(1-r)^{2cbcos\lambda e^{-i\lambda}}$ is the most advantageous dominant of Eq 41.

Theorem 5: If the function p is univalent in D as well as $p(0)=1$ such so $\varphi\in\Sigma,\mu,\alpha\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$, consider following $\delta,\Lambda,\rho\in\mathbb{C},\ \Lambda+\rho\neq0$ alongside $\Re e\left(\frac{\delta}{\alpha}\right)>0$. Allow φ meet the following requirements:

$$\left\{\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r)+\rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right\}\neq0,$$

and

$$\left\{\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r)+\rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right\}^{-\mu+1}\in G[p(0),1]\cap W.$$

When $\phi(r)$ from Eq 29 is univalent within D alongside

$$\delta w(r)+\alpha rw'(r)\prec\phi(r), \qquad 42$$

then

$$p(r)\prec\left\{\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r)+\rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right\}^{-\mu+1},$$

the function $p(r)$ is the most advantageous dominant of Eq 42. Now by Theorem 1, Theorem 2 as well as Theorem 5, the subsequent two sandwich theorems accordingly:

Theorem 6: If p_1 and p_2 become two convex functions within the unit disc D alongside $p_1(0)=p_2(0)=1$, for $\varphi\in\Sigma$ such so $rE_{v,\alpha}^{m+1}(r)\varphi(r)\neq0, 0<\mu<1$, $\lambda\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$ and $\Re e(\lambda)>0$. Assume the following scenario $\left(rE_{v,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}\in G[p(0),1]\cap W$. If

$$\left(\frac{\lambda v+1}{v}\right)\left(rE_{v,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}-r\lambda\left(E_{v,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{v,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu}$$

is univalent inside the unit disc D, so that

$$p_1(r)+\lambda\frac{rp_1'(r)}{\mu v}\prec\left(\frac{\lambda v+1}{v}\right)\left(rE_{v,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}-r\lambda\left(E_{v,\alpha}^{m}(r)\varphi(r)\right)\left(rE_{v,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu}\prec p_2(r)+\lambda\frac{rp_2'(r)}{\mu v}, \qquad 43$$

then

$$p_1(r)\prec\left(rE_{v,\alpha}^{m+1}(r)\varphi(r)\right)^{-\mu+1}\prec p_2(r),$$

the functions p_1 as well as p_2 are the most subordinate as well as the most dominant for Eq 43.

Theorem 7: Allow p_1 as well p_2 become two convex within the unit disc D and $p_1(0)=p_2(0)=1$, such so $\delta,\Lambda,\rho\in\mathbb{C},\mu,\alpha\in\mathbb{C}^*=\mathbb{C}\backslash\{0\},\Lambda+\rho\neq0$ and $\Re e\left(\frac{\delta}{\alpha}\right)>0$. Allow $\varphi\in\Sigma$ Assume the following scenario φ meet the following requirements:

$$\left\{\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r)+\rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right\}\neq0,$$

so that

$$\left\{\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r)+\rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right\}^{-\mu+1}\in G[p(0),1]\cap W.$$

When the function ϕ provided by Eq 31 is univalent and

$$\delta p_1(r)+\alpha rp_1'(r)\prec\phi(r)\prec\delta p_2(r)+\alpha rp_2'(r), \qquad 44$$

then

$$p_1(r)\prec\left\{\frac{r\{\Lambda E_{v,\alpha}^{m}(r)\varphi(r)+\rho E_{v,\alpha}^{m+1}(r)\varphi(r)\}}{\Lambda+\rho}\right\}^{-\mu}\prec p_2(r),$$

then p_1 as well as p_2 are the most subordinate as well as the most dominant for Eq.44.

Conclusion

The paper investigates a subclass of analytical univalent functions associated with the concept of power source variations in subordination. Looking at the variations in subordination as well as super ordination that leads to including a particular class about univalent functions stated upon the open unit disc time for univalent functions gathering multiple real estate subordinations as well as super ordinations linked alongside the Hadamard product, employing the characteristics of the operator. Investigating the different types of subordinations as well as super ordinations employing the real estate of the broader derived operator and demonstrating sure theorems. Additional inferences were gathered on variation subordination employing a linear operator.

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Authors' Declaration

- Conflicts of Interest: None.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Anbar.

Authors' Contribution Statement

M.I. and I.A. contributed to the research creation and execution. K. and S.J. helped with the analysis of the findings. H. and A.F. assisted in the editing and proofreading of the results. Both authors collaborated to improve the final research by discussing the findings.

References

1. Rogosinski W. On Subordination Functions. Math Proc Camb Philos Soc. 1939; 35(1): 1 – 26. https://doi.org/10.1017/S0305004100020703
2. Littlewood JE. Lectures on the Theory of Functions. UK: Oxford University Press; 1944. 244 p.
3. Srivastava HM, Owa S. Some Applications of the Generalized Hypergeometric Function Involving Certain Subclasses of Analytic Functions. Publ Math Debr. 1987; 34(3-4): 299 – 306.
4. Miller SS, Mocanu PT. Second-order Differential Inequalities in the Complex Plane. J Math Anal Appl. 1978; 65(2): 298–305. https://doi.org/10.1016/0022-247X(78)90181-6
5. Sambo B, Lasode AO. Differential Subordination and Superordination for a Family of Analytic Functions Defined by a New Multiplier Differential Operator. Ann Math Comput Sci. 2023; 17: 40-49.
6. Hameed MI, Shihab BN, Jassim KA. An Application of Subclasses of Goodman-Salagean-Type Harmonic Univalent Functions Involving Hypergeometric Function. 1st International Conference on Advanced Research in Pure and Applied Science (ICARPAS 2021): Third Annual Conference of Al-Muthanna University/College of Science 24–25 March 2021, Al-Samawah, Iraq. AIP Conf. Proc. 2022; 2398(1): 060012. https://doi.org/10.1063/5.0093392 .
7. Bulboacă T. Classes of First Order Differential Superordinations. Demonstr Math. 2002; 35(2): 287-292. https://doi.org/10.1515/dema-2002-0209 .
8. Miller SS, Mocanu PT. Subordinates of Differential Superordinations. Complex Var. 2003; 48(10): 815-826. https://doi.org/10.1080/02781070310001599322 .
9. Cotîrlă L-I, Cătaş A. Differential Sandwich Theorem for Certain Class of Analytic Functions Associated with an Integral Operator. Stud Univ Babes-Bolyai Math. 2020; 65(4): 487-494. http://dx.doi.org/10.24193/subbmath.2020.4.01 .
10. Avdiji S, Tuneski N. Sufficient Conditions for Starlikeness Using Subordination Method. Adv Math Sci J. 2020; 9(12): 10707-10716. https://doi.org/10.37418/amsj.9.12.55 .
11. Lupaş AA. Applications of a Multiplier Transformation and Ruscheweyh Derivative for Obtaining New Strong Differential Subordinations. Symmetry. 2021; 13(8): 1312. https://doi.org/10.3390/sym13081312 .
12. Ali M, Saeed M. On A Differential Subordination and Superordination of New Class of Meromorphic Functions. Matematiche. 2014; 69(1): 259-274. https://doi.org/10.4418/2014.69.1.20 .
13. Hameed MI, Ali MH, Shihab BN. A Certain Subclass of Meromorphically Multivalent Q-Starlike Functions Involving Higher-Order Q-Derivatives. Iraqi J Sci. 2022; 63(1): 251-258. https://doi.org/10.24996/ijs.2022.63.1.26 .
14. Shehab NH, Juma ARS. Third Order Differential Subordination for Analytic Functions Involving Convolution Operator. Baghdad Sci J. 2022; 19(3): 0581-0581. http://dx.doi.org/10.21123/bsj.2022.19.3.0581 .
15. Alb Lupaş A, Oros GI. On Special Differential Subordinations Using Fractional Integral of Sălăgean and Ruscheweyh Operators. Symmetry. 2021; 13(9): 1553. https://doi.org/10.3390/sym13091553 .

16. Sokol J. Convolution and Subordination in the Convex Hull of Convex Mappings. Appl Math Lett. 2006; 19(4): 303-306. https://doi.org/10.1016/j.aml.2005.06.005 .
17. Patel J, Cho NE, Srivastava HM. Certain Subclasses of Multivalent Functions Associated with a Family of Linear Operators. Math Comput Model. 2006; 43(3-4): 320-338. https://doi.org/10.1016/j.mcm.2005.06.014 .
18. Frasin B. A New Differential Operator of Analytic Functions Involving Binomial Series. Bol Soc Paran Mat. 2020; 38(5): 205-213. https://doi.org/10.5269/bspm.v38i5.40188 .
19. Royster WC. On the Univalence of a Certain Integral. Mich Math J. 1965; 12(4): 385–387. http://dx.doi.org/10.1307/mmj/1028999421 .
20. Rashid AM, Juma ARS. Some Subclasses of Univalent and Bi-Univalent Functions Related to K-Fibonacci Numbers and Modified Sigmoid Function. Baghdad Sci J. 2023; 20(3): 0843-0843. http://dx.doi.org/10.21123/bsj.2022.6888 .

بعض النتائج التي تنطوي على فئة معينة من الدوال على التبعية التفاضلية والتبعية العليا

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الخلاصة

في هذا العمل، يؤدي التحقيق في عدد قليل من التبعية التفاضلية بالإضافة إلى التنسيق الفائق إلى تضمين فئة محددة مذكورة في مجال الدوال الميرومورفية أحادية التكافؤ داخل قرص وحدة مفتوحة مثقوبة. واستخلاص بعض نظريات الساندويتش. الغرض من هذه المقالة هو النظر في عدد قليل من خصائص التبعية المتغيرة للدوال التحليلية أحادية التكافؤ على قرص وحدة مثقوب. ويهدف بالإضافة إلى ذلك إلى إلقاء نظرة ثاقبة على الخصائص الهندسية مثل عدم مساواة المعامل، وخصائص منتج هادامارد، وعامل كوماتو التكاملي. تم اكتشاف بعض النتائج المثيرة للاهتمام فيما يتعلق بالاختلافات في التبعية وكذلك الفوقية في الدوال التحليلية أحادية التكافؤ. تم عرض النتائج المتعلقة بالاختلافات في التبعية، بما في ذلك عوامل الجبر الخطي، باستخدام التلافيف التي تتضمن عاملين خطيين. حيث نقوم بتقييم التبعيات والتحقيق فيها وكذلك التبعية العليا فيما يتعلق بالتلافيف باستخدام التضمين من عامل التكامل كوماتو. وتم استخدام عامل الالتواء كأداة للحصول على نتائج متعددة حول التبعية التفاضلية داخل قرص الوحدة المثقب باستخدام وظيفة هندسية مفرطة معممة. يتم فحص الفئات المناسبة من الدوال المقبولة، ويتم شرح مصوغات ثنائية الأبعاد للتبعيات التفاضلية من خلال استخدام العامل الخطي، وهي تقنية قدمها سريفاستافا وفحصها. وهذا يؤدي إلى إنشاء العديد من النظريات من نوع الساندويتش لفئة من الدوال التحليلية أحادية التكافؤ. يفحص العمل الحالي عددًا من الفئات الفرعية للدوال الشبيهة بالنجوم والتي يتم تحديدها من خلال التبعية. بالإضافة إلى ذلك، يوفر فريقنا بعض الروابط ذات الصلة بين النتائج الواردة هنا وتلك التي تم الحصول عليها سابقًا.

الكلمات المفتاحية: العامل التفاضلي، التبعية التفاضلية، الدوال الهندسية الفوقية المعممة، الدوال الميرومورفية، نظرية الساندويتش، الدوال النجمية.