Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Minimizing the Total Waiting Time for Fuzzy Two-Machine Flow Shop Scheduling Problem with Uncertain Processing Time

K. Ranjith*, *K. Karthikeyan*

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, India.
*Corresponding Author.

Received 23/01/2024, Revised 07/06/2024, Accepted 09/06/2024, Published Online First 20/10/2024

Abstract

Scheduling involves assigning resources to jobs within specified constraints over time. Traditionally, the processing time for each job was considered a constant value. However, in practical scenarios, job processing times can dynamically fluctuate based on prevailing circumstances. In this article, a novel approach is presented for the two-machine fuzzy Flow-Shop Scheduling Problem (FSSP) in a fuzzy environment, where job processing times are represented by trapezoidal fuzzy numbers. Additionally, a novel algorithm based on Goyal's approach for the two-machine FSSP with fuzzy processing times has been proposed and developed. Simultaneously, an existing algorithm has been enhanced to improve its performance in this specific context. The study aims to minimize the total waiting time for jobs. A defuzzification function is employed to rank fuzzy numbers, with the ultimate goal of minimizing the total waiting time. Furthermore, the article evaluates the performance of these methods in terms of solution quality, using test problems with 10, 20, 30, 40, 50, 60, 80, 90, 100, 120, 200, 250, 300, and 500 jobs, along with 2 machines. The mean total waiting time is compared to existing algorithms, including the NEH algorithm, Palmer’s method, Johnson’s method, B. Goyal and B. Kaur, and the proposed algorithm. Additionally, the obtained results, along with a well-structured ANOVA test, highlight the effectiveness of the proposed method in addressing the scheduling problem under investigation. The experimental results showcase that the proposed algorithm can effectively minimize the waiting time in fuzzy two-machine FSSP and achieve superior results when compared to various existing algorithms.

Keywords: Fuzzy flow shop scheduling problem, Fuzzy logic, Heuristic algorithm, Optimal sequence, Trapezoidal Fuzzy Number (TrFN), Total waiting time.

Introduction

Flow shop scheduling problems consist of n identical jobs processed in the same order across m machines. Real scheduling problems often encounter challenges where data cannot be accurately recorded or collected, particularly in unexpected circumstances. An example is the imprecise measurement of job processing times, making it challenging to gather precise data. In 1954, Johnson introduced algorithm [1] known as Johnson’s rule, aimed at minimizing the makespan in the two-machine flow shop problem. This concept, renowned for its ability to yield precise results in the two-machine scenario, has catalyzed the development of heuristics for more complex m-machine flow shop problems, including notable algorithms like Palmer’s [2], Gupta’s [3], and the CDS algorithm [4]. While Johnson’s algorithm assumed deterministic processing times, setting precise values for job

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

processing times is often challenging in real-world scenarios. Variability is inherent, with processing times often fluctuating within intervals rather than fixed values. Consequently, representing such uncertainties is both natural and realistic [5]. As a result, there arises a necessity to extend the classical Johnson's algorithm to accommodate fuzzy processing times. Therefore, there has been a growing interest among researchers in recent years in using fuzzy processing time to tackle job scheduling problems, with particular emphasis on the flow-shop scheduling problem. Additionally, researchers propose employing fuzzy set theory [6], to tackle uncertainty in scheduling problems, a crucial approach for domains such as healthcare and production [7]. In flow shop scheduling problem-solving, NP-complete problems [8] are quick to verify but slow to solve. This limits the effectiveness of exact optimization algorithms for large-scale problems. Instead, Industries [9] relies on scheduling to allocate resources to tasks over time, aiming to minimize metrics such as makespan [10, 11], total flow time, tardiness [12], and waiting time [13].

In flow shop scheduling, the focus is on minimizing waste, including idle time of machines and waiting time of jobs. These factors significantly impact production efficiency and are crucial considerations in scheduling objectives [14]. The significant impact of waiting time, as seen in industries like steel production, involves the wastage of time, raw materials, and resources. In the context of the problem, Asif et al. [15] studied an effective algorithm aimed at reducing total elapsed time and idle time for solving FSSPs. McCahon and Lee [16], discussed a method for predicting the job sequence with an optimal value for a two-machine FSSP employing a triangular fuzzy number.

Previous studies in flow-shop scheduling primarily focused on deterministic environments. However, real-world manufacturing processes inherently involve uncertainty [17]. Recently, significant attention has been devoted to addressing uncertain shop scheduling problems, with fuzzy shop scheduling emerging as a prominent research area. Additionally, Dubios and Prade [18], proposed overlapping relationships among fuzzy numbers to characterize a domain of possibility. Mahfouf M. et al. [19], highlight the application of fuzzy logic techniques in healthcare disciplines such as internal medicine and others. Kiptum et al. [20], evaluated the challenges associated with achieving sustainable urban development using a fuzzy approach. Besides, Alburaikan A. et al. [21] introduced a novel strategy for arranging tasks within a three-stage flow shop setting involving uncertain processing times. They presented two distinct methods: the first employs a ranking function, while the second utilizes a tight interval estimate of fuzzy numbers. In recent years, several researchers have made significant contributions to this literature. Zhou T et al. [22], examined three-machine n-job FSSP with fuzzy piecewise quadratic processing times. Zubair and Ahmed [23], devised an innovative set of operational guidelines and a ranking procedure for Single-Valued Neutrosophic Uncertain Linguistic Variables concerning linguistic scale functions. Akram et al. [24], introduced a new linear programming problem that incorporates LR-type Pythagorean fuzzy numbers. Zanjani B et al. [25], developed a multi-objective robust mixed-integer linear programming model considering real-world conditions where due dates and processing times are assumed to be uncertain. Edalatpanah et al.[26], utilized Cooperative Continuous Static Games with fuzzy cost functions that exhibit piecewise quadratic behavior. Gupta D. and Goyal B. [27], have created specialized structural models aimed at optimizing job waiting times in flow shop scheduling, with a focus on distinct setup times and the concept of job blocks. Here, a two-machine flow shop scheduling problem is presented, incorporating trapezoidal fuzzy processing times.

Recently, there has been a growing focus among researchers on the fuzzy FSSPs. Engin and Isler [28], developed a parallel greedy algorithm for the fuzzy hybrid FSSPs with setup time and lot size. Bahmani V et al. [29], examined the two-stage flow shop scheduling problem involving distribution through vehicle routing within a flexible timeframe. Rouhbakhsh R et al. [30], developed a lot-streaming algorithm for the hybrid flow shop scheduling problem considering transportation time. Jain et al. [31], presented a model for optimizing fuzzy inventory-transportation problems aimed at minimizing overall distribution costs. In this context, Goyal B and Kaur S [32], explored a triangular fuzzy number-based

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

algorithm for minimizing job waiting time in specific FSSPs, demonstrating its superiority in waiting time optimization compared to various heuristic approaches. To solve the same problem Goyal B and Kaur S [33], explored scheduling issues in a two-machine permutation flow shop, considering random processing times on both machines. In this article, an enhancement to the Goyal method is proposed. Specifically, a novel approach is introduced to thoroughly explore the potential relationships between fuzzy numbers, thereby leading to improved performance in obtaining optimal job sequences.

The rest of the article is structured in the following manner: Preliminary concepts that explain the fuzzy set and other principles, providing an overview of the problem statement. The fuzzy two-machine FSSPs are proposed. The result analysis discusses the numerical comparison employed to verify the computational efficiency of the proposed method. The article provides a summary and explores potential avenues for future research in the conclusion. The flow chart for the proposed algorithm is clearly outlined in Fig 1

Preliminary concepts

The preliminary concept contains a mathematical representation of job completion time and waiting time. Moreover, the assumptions applied in the problem setting were presented. The subsequent notations indicate the total waiting time for the fuzzy FSSP with n jobs and 2 machine problems.

Assumptions

The following assumptions form the foundation of the flow shop scheduling problems [34].

- At time zero, all machines should be operational.
- Each machine processes each job once, and they work independently.
- Predictable and consistent processing times exist.
- Machines are always accessible and do not malfunction throughout the operation.
- No machines can handle two or more tasks concurrently.
- Preemption of jobs is not authorized; the machine remains dedicated to the current job until completion.

The objective of the two-machine fuzzy FSSP is to determine the optimal sequencing of all jobs while minimizing the total waiting time. Table 1 provides a comprehensive list of symbols and their respective meanings utilized in two machine fuzzy FSSP.

Table .1 Notation

Characters	Descriptions
i	Index for jobs β_i, i= $1, 2, \ldots\ldots n$
M	Machines (M1 and M2)
$\tilde{P}_i^M$	Average Hesitant Fuzzy Set (AHR) score for the fuzzy processing time of the job i on machine M . This represented the average value of the fuzzy job time.
f_i^M	Fuzzy processing time of job i on machine M.
C_β^M	Completion time of job β on machine M.
U_β	Job $\beta's$ waiting time.
S_i	The starting time of the job i on machine M, denoted as W_t.
W_β	Waiting time of job β.
W_{time}	Total waiting time.

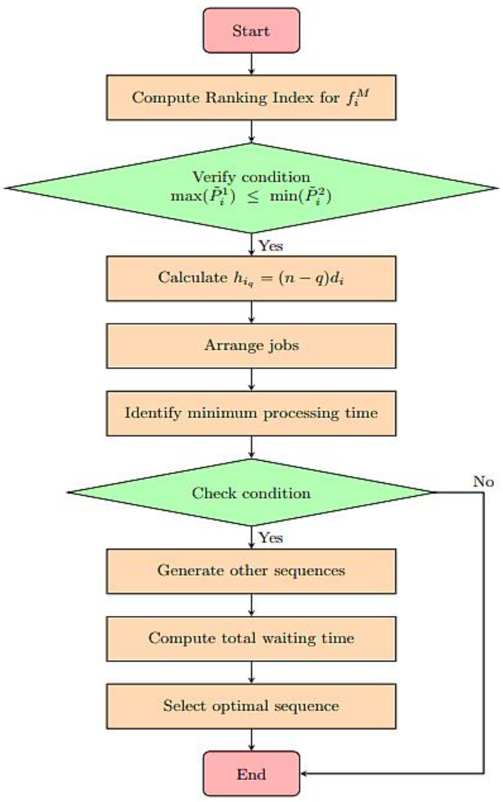

Figure 1. Flow chart for the proposed method

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Fuzzy flow shop scheduling problem

Problem statement

In this section, the formulation of the fuzzy FSSP has been investigated, where the processing times for a set of n jobs, sequenced across a pair of machines, are expressed as trapezoidal fuzzy numbers denoted as $\tilde{P}_i^M$. These fuzzy representations capture uncertainty, offering a range for each processing time rather than a precise value [35]. The primary objective is to devise an optimal job processing sequence on the two machines, emphasizing the minimization of total waiting time while accounting for the uncertainty introduced by trapezoidal fuzzy numbers. The duration for processing job i on machine M is represented by the trapezoidal fuzzy number $\tilde{P} = (e_1, e_2, e_3, e_4)$, where e_1, e_2, e_3, e_4 are parameters. For trapezoidal fuzzy numbers, the membership function is 1 or the maximum of $(e_2 - e_3)$, and it is zero at the corners e_1 and e_4 of the trapezoidal fuzzy number. Fig 2 illustrates these numbers. In this context, μ denotes the membership function, and x signifies the processing time. The membership function, employed in fuzzy logic, assesses the degree of membership or truth value of an element in a fuzzy set, considering its attributes like processing time. Additionally, a crucial condition $\max(\tilde{P}_i^1) \leq \min(\tilde{P}_i^2)$ ensures compatibility in processing time intervals on different machines, contributing to the feasibility of the scheduling solution.

Preliminaries

The initial introduction of fuzzy mathematical programming at a broad level originated within the framework of fuzzy FSSP proposed by Behnamian J [36]. Next, essential definitions are provided.

Fuzzy set [37]

A fuzzy set $\tilde{P}$ maps elements from the universe of discourse $\tilde{X}$ to the unit interval. Let $\tilde{P}$ be represented as $\tilde{P} = \{(\text{x}, \mu_{\tilde{P}}(x)/\text{x} \in \tilde{X})\}$, where $\mu_{\tilde{P}}$ is the membership function that assigns a value to each element x belonging to X in the fuzzy set $\tilde{P}$. $\mu_{\tilde{P}}(x)$ yields the degree to which x belongs to the fuzzy set $\tilde{P}$. $\mu_{\tilde{P}}$ maps $\tilde{X}$ to the interval [0,1], and the fuzzy set $\tilde{P}$ can be represented as $\tilde{P}: \tilde{X} \rightarrow [0,1]$.

Fuzzy number [37, 38]

A fuzzy number, denoted as $\tilde{P}$, is defined based on the following criterion

- $\mu_{\tilde{P}}(x): R \rightarrow [0,1]$ is continuous.
- $\mu_{\tilde{P}}(x) = 1$ for all $x \epsilon [e_1, e_4]$ where $e_1 < e_2 < e_3 < e_4$.
- $\mu_{\tilde{P}}(x)$ strictly increasing on $[e_1, e_2]$ and strictly decreasing on $[e_3, e_4]$.
- $\mu_{\tilde{P}}(x) = 0$ for all $x \epsilon (-\infty, e_1) \cup (e_4, +\infty)$.

Trapezoidal Fuzzy Number: A trapezoidal fuzzy number $\tilde{P} = (e_1, e_2, e_3, e_4)$ is defined by its membership function $\mu_{\tilde{P}}(\text{x})$ as follows:

$$\mu_{\tilde{P}}(\text{x}) = \begin{cases} \frac{x-e_1}{e_2-e_1}, & if \quad e_1 < x < e_2 \\ 1, & if \quad e_2 < x < e_3 \\ \frac{e_4-x}{e_4-e_3} & if \quad e_3 < x < e_4 \\ 0, & if \quad \text{otherwise} \end{cases} \qquad 1$$

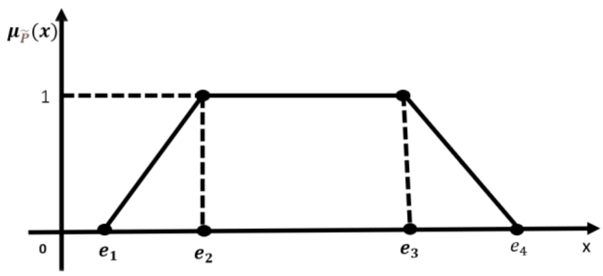

Figure 2. Trapezoidal membership function

Defuzzification method [39]

Finding the singleton value (crisp value), which is the average value of the trapezoidal fuzzy numbers, is the process of defuzzification. Due to its simplicity and accuracy, Robust's Ranking approach is utilized in this case to defuzzify trapezoidal fuzzy numbers.

Robust ranking technique

If $\tilde{P}$ is a trapezoidal fuzzy number, then the ranking method is given by

$R(\tilde{P}) = \int_0^1 \alpha(b_\alpha^L, b_\alpha^R)\, d\alpha$, where $(b_\alpha^L, b_\alpha^R) = \{[\alpha(e_2 - e_1) + e_1, -\alpha(e_4 - e_3) + e_4]\}$ the $\alpha -$level cut off the fuzzy numbers $\tilde{P}$.

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

$$R(\tilde{P}) = \int_0^1 [e_1 + \alpha(e_2 - e_1)]\alpha d\alpha + \int_0^1 [e_4 - \alpha(e_4 - e_3)]\alpha d\alpha \quad 2$$

$$R(\tilde{P}) = \int_0^1 [e_1\alpha + \alpha^2(e_2 - e_1)]d\alpha + \int_0^1 [e_4\alpha - \alpha^2(e_4 - e_3)]d\alpha$$

$$R(\tilde{P}) = \left[e_1\frac{\alpha^2}{2} + (e_2 - e_1)\frac{\alpha^3}{3}\right]_0^1 + \left[e_4\frac{\alpha^2}{2} - \frac{\alpha^3}{3}(e_4 - e_3)\right]_0^1$$

$$R(\tilde{P}) = \frac{e_1 + 2(e_2 + e_3) + e_4}{6} \quad 3$$

$R(\tilde{P})$ is the ranking index for fuzzy number $\tilde{P}$

Significance and novelty of the proposed model

The importance of the proposed objective resonates across every service provider organization and industry because client satisfaction is of paramount importance to every executive. In today's rapidly advancing world, where time is at a premium, everyone seeks services that minimize waiting times. Therefore, service executives consistently strive to ensure timely service delivery, avoiding extended wait periods for clients. Most of the previous research has focused on achieving different objectives, including minimizing elapsed time and reducing the rental cost of machines. The objective introduced in this study has previously garnered little attention from researchers in the context of trapezoidal fuzzy processing numbers. The novelty of this research lies in delving into this objective and presenting an algorithm aimed at minimizing job waiting times for flow shop scheduling problems with randomly generated trapezoidal fuzzy processing times.

Total waiting time for flow shop scheduling

Consider the problem of a flow-shop with n jobs and two machines, where the jobs are processed sequentially on machines M1 and M2, following the order M1M2, without allowing any passing. Furthermore, let S_i^1 and C_i^1 denote the starting and completion times of job i on machine M1, and S_i^2 and C_i^2 represent the starting and completion times of job i on machine M2, where $i = 1, 2 \dots, n$. The waiting time for job i on machine M2, referred to as W_i, is determined as $S_i^2 - C_i^1$ within a schedule 'S' involving n jobs ($S = \beta_1, \beta_2, \dots \dots \beta_n$). The total completion time of all jobs in a two-machine flow shop problem is the completion time of the last job on the second machine, denoted as $C_{\beta_n}^2$

$$C_{\beta_1}^1 = \tilde{P}_1^1 \quad 4$$

$$C_{\beta_i}^1 = C_{\beta_{i-1}}^1 + \tilde{P}_{\beta_i}^1, \quad i = 2, \dots, n \quad 5$$

$$C_{\beta_i}^2 = max\{C_{\beta_{i-1}}^2, C_{\beta_i}^1\} + \tilde{P}_{\beta_i}^2, \quad i = 2, \dots, n \quad 6$$

The total completion time is then $C_{\beta_n}^2$. The objective is to find a schedule that minimizes the total waiting time W_{time}, where $W_{time} = \sum_{i=1}^{n} W_i$. The mathematical details of this problem can be found in Table 2.

Table 2. Representation of the problem description in matrix form

Jobs(i)	$M_1(f_i^1)$	$M_2(f_i^2)$
1	$(e_{11}^1, e_{21}^1, e_{31}^1, e_{41}^1)$	$(e_{11}^2, e_{21}^2, e_{31}^2, e_{41}^2)$
2	$(e_{12}^1, e_{22}^1, \theta_{32}^1, e_{42}^1)$	$(e_{12}^2, e_{22}^2, e_{32}^2, e_{42}^2)$
3	$(e_{13}^1, e_{23}^1, e_{33}^1, e_{43}^1)$	$(e_{13}^2, e_{23}^2, e_{33}^2, e_{43}^2)$
.	.	.
.	.	.
.	.	.
n	$(e_{1n}^1, e_{2n}^1, e_{3n}^1, e_{4n}^1)$	$(e_{1n}^2, e_{2n}^2, e_{3n}^2, e_{4n}^2)$

The two-machine specialized flow-shop scheduling problem arises when the processing times of the n jobs on machines M1 and M2 adhere to the condition expressed as $max\ \tilde{P}_i^1 \leq \min \tilde{P}_i^2$, transforming the problem into a specialized scheduling problem.

Theorem 1:
Let n-jobs, indexed from 1 through n, undergo processing on two machines (M1 and M2) in a flow shop environment, excluding any transient operations. Suppose these jobs adhere to the structural condition:

$$max\ \tilde{P}_i^1 \leq min\ \tilde{P}_i^2 \quad 7$$

where $\tilde{P}_i^M$- represents the alpha cut ranking index (Eq 3) value of the equivalent fuzzy processing time required by the job i on machine M ($M = 1,2$), and (W_{time}), the total waiting time of jobs, is determined as:

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Under this constraint, the total waiting time of jobs (W_{time}) is mathematically represented as:

$$W_{time} = n \cdot \tilde{P}^1_{\beta_1} + \sum_{j=1}^{n-1} (n-q)\, d_{\beta_q} - \sum_{i=1}^{n=1} \tilde{P}^1_{\beta_i} \quad 8$$

where:

$$d_{\beta_q} = (\tilde{P}^2_{\beta_q} - \tilde{P}^1_{\beta_q}) \quad 9$$

Proof:

Initially, the evaluation begins with determining the completion time, denoted as C^M_β, for orders β on machine M, considering the sequence, $S = \beta_1, \beta_2, \beta_3, \ldots \beta_k \ldots, \beta_n$.

Claim:

$$C^2_{\beta_n} = \tilde{P}^1_{\beta_1} + \tilde{P}^2_{\beta_1} + \tilde{P}^2_{\beta_2} + \cdots + \tilde{P}^2_{\beta_n} \quad 10$$

Applying mathematical induction to n,

let $P(n)$ denote:

$$C^2_{\beta_n} = \tilde{P}^1_{\beta_1} + \tilde{P}^2_{\beta_1} + \tilde{P}^2_{\beta_2} + \cdots + \tilde{P}^2_{\beta_n} \quad 11$$

Now, for $n = 1$,

$$C^2_{\beta_1} = \tilde{P}^1_{\beta_1} + \tilde{P}^2_{\beta_1} \quad 12$$

Now, assuming $P(k)$ to be true for $n = k$,

Then for $P(k + 1)$, utilizing Eq.7.

$$C^2_{\beta_{k+1}} = max\left(C^1_{\beta_{k+1}}, C^2_{\beta_k}\right) + \tilde{P}^2_{\beta_{k+1}} \quad 13$$

Proving,

$$C^2_{\beta_n} = \tilde{P}^1_{\beta_1} + \tilde{P}^2_{\beta_1} + \tilde{P}^2_{\beta_2} + \cdots + \tilde{P}^2_{\beta_k} + \tilde{P}^2_{\beta_{k+1}} \quad 14$$

Next, U_β will be evaluated, representing the time consumed by the job β while waiting.

Claim: For the sequence $S = \beta_1, \beta_2, \beta_3, \ldots \beta_k \ldots, \beta_n$ of the jobs

Next, the waiting time U_β for the job, β is analyzed for the sequence $U_{\beta_k} = \beta_1, \beta_2, \beta_3, \ldots, \beta_k \ldots, \beta_n$ of the jobs:

Clearly,

$$U_{\beta_1} = 0 \quad 15$$

and

$$U_{\beta_k} = S^2_{\beta_1} - C^1_{\beta_k} \qquad k = 2, 3 \ldots, n \quad 16$$

Implicitly,

$$U_{\beta_k} = \max\left(C^2_{\beta_{k-1}}, C^1_{\beta_k}\right) - C^1_{\beta_k}, \quad k = 2, 3 \ldots, n \quad 17$$

Condition (Eq 7) of the proposed model specifies a requirement that must be satisfied, expressed as:

$$U_{\beta_k} = \tilde{P}^1_{\beta_1} + \sum_{q=1}^{n-1} d_{\beta_q} + \sum_{i=1}^{n} \tilde{P}^1_{\beta_k}, \; k = 2,3 \ldots, n \quad 18$$

Approaching the major proof of the theorem:

$$W_{time} = U_{\beta_1} + U_{\beta_2} + U_{\beta_3} + \cdots, + U_{\beta_n} \quad 19$$

$$W_{time} = n \,.\, \tilde{P}^1_{\beta_1} + \sum_{q=1}^{n-1} (n-q) d_{\beta_q} + \sum_{i=1}^{n} \tilde{P}^1_{\beta_i}, \; k = 2,3 \ldots, n \quad 20$$

Theorem 2:

Given a natural number k and real numbers $r_1, r_2, \ldots, r_k$, among all possible linear combinations of the form

$$\sum_{i=0}^{k-1} (k-i)\, r_{i+1}$$

the minimum value is attained when $r_1 \leq r_2 \leq \cdots, \leq r_k$.

Proof:

Applying the induction hypothesis on k,

the result holds trivially for $k = 1$.

Assume that the result holds for less than k real numbers.

Now, considering the ordered sequence

$$r_1 \leq r_2 \leq \cdots \leq r_k$$

$$kr_1 + (k-1)r_2 + (k-2)r_3 + \cdots + 2r_{k-1} + r_k$$

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

$$= (k-1)r_1 + (k-2)r_2 + (k-3)r_3 + \cdots + r_{k-1} + \sum_{i=1}^{k} r_i$$

Since the last term $\sum_{i=1}^{k} r_i$ is constant,

the hypothesis assumption implies that

$$kr_1 + (k-1)r_2 + (k-2)r_3 + \cdots + 2r_{k-1} + r_k$$

is minimized.

Remark:

Based on the result from Theorem 2,

it is evident that for an n-job sequence $S = \beta_1, \beta_2, \beta_3, \dots \beta_k \dots, \beta_n$ the term

$$\sum_{q=1}^{n} (n-q)\, d_{\beta_q}$$

in Eq.8 will be minimized if the n -jobs in sequence S are arranged in the non-decreasing order of the values

d_{β_q}, while $\sum_{i=1}^{n} \tilde{P}_{\beta_i}$

remains constant for every sequence of jobs. Bearing these observations in mind, an exact method is proposed to minimize the total waiting time W_{time} for two-machine specially structured flow-shop scheduling problems.

Proposed algorithm

The proposed algorithm entails the following steps:

Step 1: Calculate the Ranking Index value of fuzzy processing time $f_i^M = (e_1, e_2, e_3, e_4)$ for all jobs j_i, where $i = 1, 2, 3 \dots, n$, using alpha cut Ranking Index (Eq 3).

Step 2. Verify the structural condition, i.e., $max\ \tilde{P}_i^1 \leq min\ \tilde{P}_i^2$

Step 3. Calculate $h_{i_q} = (n-q)d_i$, where $d_i = \tilde{P}_i^2 - \tilde{P}_i^1$ for $i = 1,2,3, \dots, n-1$, and present the computed entries in the following tabulated format:

Step 4. Arrange the jobs in ascending order of d_i and obtain the sequence $S_1 = \beta_1, \beta_2, \beta_3, \dots, \beta_n$.

Step 5. Identify the minimum processing time of machine 1 and denote it as $\tilde{P}_x^1$. Then, verify the condition

$$\tilde{P}_x^1 = \tilde{P}_{\beta_1}^1,$$

If this condition is satisfied, then the sequence obtained in the previous step is optimal; otherwise, proceed to the next step.

Step 6. Generate other sequences S_i, where $i = 2,3,4, \dots, n$, by exchanging the i^{th} job with the first one of the sequences S_{i-1} while keeping the remaining job sequence unchanged.

Step 7. Compute the total waiting time W_{time} for all sequences $S_1, S_2, S_3, \dots, S_n$ using the formula defined in Eq.8.

Step 8. Select the sequence with the minimum total waiting time from the list mentioned in the previous step; this sequence represents the optimal solution.

Illustration

A mathematical representation of a problem involving 10 jobs and two machines is presented in Table 3, as illustrated in the adjusted algorithm [37].

Table 3. Trapezoidal fuzzy processing times

Jobs	M_1	M_2
i	f_i^1	f_i^2
1	(65, 69, 77, 93)	(75, 89, 97, 112)
2	(61, 72, 83, 93)	(80, 92, 104, 106)
3	(65, 69, 77, 93)	(81, 86, 97, 112)
4	(64, 71, 79, 94)	(76, 89, 99, 107)
5	(57, 75, 78, 88)	(79, 83, 98, 107)
6	(54, 71, 76, 92)	(82, 87, 102, 113)
7	(65, 72, 85, 89)	(76, 85, 103, 110)
8	(60, 70, 80, 92)	(80, 87, 98, 112)
9	(58, 69, 78, 90)	(76, 84, 94, 106)
10	(63, 69, 84, 87)	(75, 89, 94, 107)

The representation of the ranking method (Eq 3) for the previously mentioned fuzzy processing times is shown in Table 4.

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Table 4. Crisp values of $\widetilde{P}_j^M$

Jobs	M_1	M_2
j	$\tilde{P}_i^1$	$\tilde{P}_i^2$
1	75.00	93.16
2	77.33	96.33
3	76.66	91.50
4	76.33	93.16
5	75.16	91.33
6	73.33	95.50
7	78.00	93.66
8	75.33	93.66
9	73.66	89.66
10	76.00	91.33

It is clear from the above table that the max $max(\tilde{P}_i^1) \leq min(\tilde{P}_i^2)$.

Therefore, the structural criterion has been fulfilled. Then with the aid of step 4, the subsequent sequences have been obtained.

$$S_1 = \beta_3, \beta_{10}, \beta_7, \beta_9, \beta_5, \beta_4, \beta_1, \beta_8, \beta_2, \beta_6$$

As a result, $P_x^1 \neq P_{\beta_1}^1$, all potential sequences according to step 6 are

$$S_2 = \beta_{10}, \beta_3, \beta_7, \beta_9, \beta_5, \beta_4, \beta_1, \beta_8, \beta_2, \beta_6$$

$$S_3 = \beta_7, \beta_3, \beta_{10}, \beta_9, \beta_5, \beta_4, \beta_1, \beta_8, \beta_2, \beta_6$$

$$S_4 = \beta_9, \beta_3, \beta_{10}, \beta_7, \beta_5, \beta_4, \beta_1, \beta_8, \beta_2, \beta_6$$

$$S_5 = \beta_5, \beta_3, \beta_{10}, \beta_7, \beta_9, \beta_4, \beta_1, \beta_8, \beta_2, \beta_6$$

$$S_6 = \beta_4, \beta_3, \beta_{10}, \beta_7, \beta_9, \beta_5, \beta_1, \beta_8, \beta_2, \beta_6$$

$$S_7 = \beta_1, \beta_3, \beta_{10}, \beta_7, \beta_9, \beta_5, \beta_4, \beta_8, \beta_2, \beta_6$$

$$S_8 = \beta_8, \beta_3, \beta_{10}, \beta_7, \beta_9, \beta_5, \beta_4, \beta_1, \beta_2, \beta_6$$

$$S_9 = \beta_2, \beta_3, \beta_{10}, \beta_7, \beta_9, \beta_5, \beta_4, \beta_1, \beta_8, \beta_6$$

$$S_{10} = \beta_6, \beta_3, \beta_{10}, \beta_7, \beta_9, \beta_5, \beta_4, \beta_1, \beta_8, \beta_2$$

Therefore, the optimal job sequence S_4, consisting of $\beta_9, \beta_3, \beta_{10}, \beta_7, \beta_5, \beta_4, \beta_1, \beta_8, \beta_2, \beta_6$ had achieved the minimum total waiting time W_{time} of 702.16 units, as had been indicated in Table 5.

Table 5. The optimal job schedules

Sequences	Total Waiting Time (W_{time})
S_1	730.00
S_2	723.83
S_3	744.50
S_4	702.16
S_5	717.83
S_6	732.83
S_7	727.50
S_8	732.00
S_9	757.33
S_{10}	745.83

After analyzing the outcomes of the aforementioned methods, it becomes evident that the proposed approach outperforms the Goyal method, exhibiting the most favorable optimal value. This comparison is detailed in Table 6.

Table 6. Comparison for optimal sequence

S. No	Ranking Method	Optimal sequence	Waiting time
1	B. Goyal & S. Kaur [34]	$\beta_9, \beta_3, \beta_{10}, \beta_4, \beta_7, \beta_1, \beta_5, \beta_2, \beta_8, \beta_6$	708.25 units
2	Ranjith. K and Karthikeyan [40]	$\beta_9, \beta_7, \beta_{10}, \beta_3, \beta_4, \beta_1, \beta_5, \beta_2, \beta_8, \beta_6$	706.34 units
3	Proposed algorithm	$\beta_9, \beta_3, \beta_{10}, \beta_7, \beta_5, \beta_4, \beta_1, \beta_8, \beta_2, \beta_6$	702.16 units

Pseudo code for the proposed algorithm

The pseudocodes of the proposed algorithm steps are mentioned below

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Pseudo code for Proposed algorithm

Require: Fuzzy processing time matrix f_i^M, Job processing times $\tilde{P}_i^1, \tilde{P}_i^2$
Ensure: Optimal job sequence S_i, minimum total waiting time W_{time}
Initialization
While $max\ \tilde{P}_i^1 \le min\ \tilde{P}_i^2$ do
Compute the alpha Ranking Index for f_i^M
  if processing times satisfy the condition, then
    Compute difference matrix $h_{i_q} = (n - q)V_i$, where $V_i = \tilde{P}_i^2 - \tilde{P}_i^1$
    Arrange jobs in ascending order based on differences
    Find minimum processing time $\tilde{P}_x^1$
    Obtain alternative job sequences S_i
    Calculate the total waiting time W_i for each sequence
    Select a sequence with a minimum total waiting time S_i
    Set W^* to minimum W_i
  else
    Print "Structural condition not met."
  end if
end while

Results and Discussion

The algorithms were employed in MATLAB and performed on computers running Windows 10 Professional, each equipped with 4 GB of RAM and Intel Core i5-3770 processors operating at 3.10 GHz. To assess the proposed algorithm, experiments were carried out by testing it on 10 different cases of the fuzzy two-machine FSSPs. These cases were randomly generated [33], resulting in a total of 14 combinations for 2-machine n-jobs problems, with the number of machines (m) held constant at 2 and the number of jobs (n) varying within the range of n= {10, 20, 30, 40, 50, 60, 80, 90, 100, 120, 200, 250, 300, 500}. The processing time $\tilde{P} = e_1, e_2, e_3, e_4$ for job i on machine M1 and M2 was determined as follows: $\tilde{P}_i^1$ was assigned a random value between 65 and 90, while $\tilde{P}_i^2$ was randomly generated within the range of 90 to 115. Each of the algorithms requires an equal amount of computation time, resulting in a waiting time of $n \times 2$ seconds. For each group, the mean total waiting time for each problem generated by the proposed algorithm is compared with the mean makespan values of existing approaches such as the Palmer algorithm [2], Johnson algorithm [1], NEH algorithm [41,] and Goyal B. and Kaur S. [34]. These comparisons are visualized in the graph presented in Fig 3. These results were obtained using both the proposed algorithm and existing heuristics, as depicted in Table 7.

The examination of the experiment involved applying a multi-factor Analysis of Variance (ANOVA) technique [42], with n-jobs and 2-machine considered as uncontrollable factors. To execute ANOVA, it is crucial to verify the primary hypotheses, specifically focusing on the normality and independence of residuals. Normality can be assessed through methods such as a Quantile–Quantile plot (Fig 4) of the residuals, or by evaluating their fit to a theoretical normal distribution. Additionally, statistical tests such as the chi-square test or the Kolmogorov–Smirnov test for normality can be employed in this context. The results of the ANOVA analysis are presented in Table 10. Fig 5, illustrates the results of different statistical tests used to evaluate the adherence of our data to a normal distribution. A high p-value (exceeding 0.05) indicates that the data does not significantly deviate from normality.

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Table 7. Average mean of total waiting time for FSSPs

Jobs	Palmer's Method [2]	Johnson's Method [1]	NEH Algorithm [41]	B. Goyal & S. Kaur [34]	Proposed algorithm
10	1001.78	910.12	856.56	882.11	840.43
20	4465.84	4324.52	3865.56	3986.13	3799.23
30	10605.92	9905.92	9205.92	9428.81	9280.59
40	19205.52	18005.52	16585.52	16916.67	16374.47
50	29535.36	28635.36	25535.36	26564.62	25398.47
60	41708.53	38952.36	35069.27	37217.52	35049.66
80	75259.35	70523.23	66925.52	66984.44	65923.66
90	92124.52	89365.42	83245.25	83799.91	83009.82
100	112235.3	111235.3	104235.3	104957.6	103945.3
120	150873.36	158973.67	149883.85	148873.62	148873.6
200	428004.26	412984.26	398984.26	398004.18	397984.26
250	701661.88	681661.88	661661.88	651661.88	651735.97
300	953894.94	933894.94	923894.94	935718.69	923894.94
500	2652455.01	2552455.01	2489542.66	2587262.75	2417305.66

In Table 8, descriptive statistics outline the mean, standard deviation, and standard error for various algorithms, including the Palmer, Johnson, NEH, Goyal, and Kaur, and the proposed method. Notably, the mean values for the proposed method are lower than those of the other algorithms, indicating potential performance improvements. Specifically, the proposed method exhibits the lowest mean among all algorithms, suggesting its superior performance in the evaluated metric. However, drawing definitive conclusions requires considering the associated standard deviations and standard errors, which offer insights into the variability and precision of the measurements. The lower mean in the proposed method category, coupled with an assessment of the associated variability, implies that this algorithm may represent the most advantageous choice among the options considered in this analysis.

As mentioned in Table 9, the proposed method was compared to the Johnson, Palmer, NEH, and B. Goyal and S. Kaur algorithms. The mean deviations were -27829.67929, -16315.10357, -6148.27071, and -13488.7764 respectively. The significance level (p-value) was 1.000, and the confidence intervals ranged from -27829.67929 to -13488.7764. Our experiment involved a non-parametric analysis, including ANOVA and multiple comparisons, to determine whether there are statistically significant differences between the algorithms. Surprisingly, varying levels of parameter T do not yield statistically significant differences. This suggests that the proposed algorithm exhibits robustness across different values of T. As previously mentioned, there is no evident statistically significant disparity among the various levels of T. The high p-values indicate no statistically significant differences between the means of the proposed method and the other algorithms, as shown in Table 10.

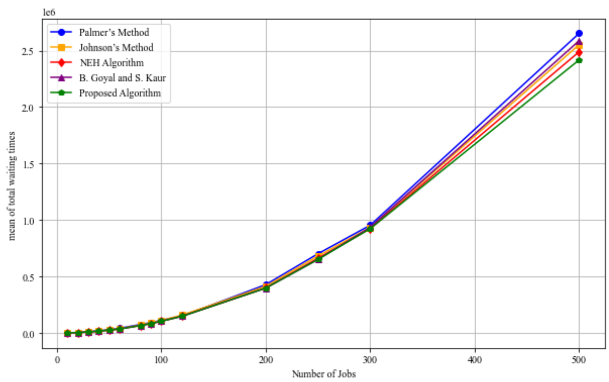

Figure 3. Comparison of mean of total waiting time for FSSPs

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Table 8. Descriptives of mean comparison

Algorithms	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Palmer's Method [2]	14	376645.11	717233.64	191688.75	-37473.26	790763.49	1001.78	2652455.01
Johnson's Method [1]	14	365130.54	691114.24	184708.05	-33906.95	764168.02	910.12	2552455.01
NEH Algorithm[41]	14	354963.70	675471.15	180527.26	-35041.73	744969.13	856.56	2489542.66
B. Goyal &S. Kaur [34]	14	362304.21	699562.45	186965.93	-41611.13	766219.54	882.11	2587262.75
Proposed algorithm	14	348815.43	657684.54	175773.59	-30920.31	728551.18	840.43	2417305.66
Total	70	361571.79	668327.41	79880.40	202214.75	520928.84	840.43	2652455.06

As discussed earlier, there is no clear statistically significant distinction among the various T levels. However, setting T to 0.5 appears to yield superior outcomes compared to a setting of 0.0, where only enhanced solutions are accepted. Further experiments with higher T levels did not result in additional improvements.

Considering uncontrollable factors such as n-jobs and 2-machine setups, Further investigation into the optimal combinations of destruction and temperature factors for each of the 14 groups of instances can be conducted. While such analysis could potentially fine-tune the algorithm, it also runs the risk of over-tuning, complicating its implementation. Therefore, all algorithms are executed under identical conditions.

Table 9. Multiple Comparisons

Dependent Variable:

Tukey HSD

(I) Algorithm		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Palmer's Method [2]	Johnson's Method	11514.57571	260234.00707	1.000	-718656.2734	741685.4248
	NEH Algorithm	21681.40857	260234.00707	1.000	-708489.4406	751852.2577
	B. Goyal & S. Kaur	14340.90286	260234.00707	1.000	-715829.9463	744511.7520
	Proposed algorithm	27829.67929	260234.00707	1.000	-702341.1698	758000.5284
Johnson's Method [1]	Palmer's Method	-11514.57571	260234.00707	1.000	-741685.4248	718656.2734
	NEH Algorithm	10166.83286	260234.00707	1.000	-720004.0163	740337.6820
	B. Goyal & S. Kaur	2826.32714	260234.00707	1.000	-727344.5220	732997.1763
	Proposed algorithm	16315.10357	260234.00707	1.000	-713855.7456	746485.9527
NEH Algorithm [41]	Palmer's Method	-21681.40857	260234.00707	1.000	-751852.2577	708489.4406
	Johnson's Method	-10166.83286	260234.00707	1.000	-740337.6820	720004.0163
	B. Goyal & S. Kaur	-7340.50571	260234.00707	1.000	-737511.3548	722830.3434
	Proposed algorithm	6148.27071	260234.00707	1.000	-724022.5784	736319.1198
B. Goyal & S. Kaur [34]	Palmer's Method	-14340.90286	260234.00707	1.000	-744511.7520	715829.9463
	Johnson's Method	-2826.32714	260234.00707	1.000	-732997.1763	727344.5220
	NEH Algorithm	7340.50571	260234.00707	1.000	-722830.3434	737511.3548
	Proposed algorithm	13488.77643	260234.00707	1.000	-716682.0727	743659.6256
Proposed algorithm	Palmer's Method	-27829.67929	260234.00707	1.000	-758000.5284	702341.1698
	Johnson's Method	-16315.10357	260234.00707	1.000	-746485.9527	713855.7456
	NEH Algorithm	-6148.27071	260234.00707	1.000	-736319.1198	724022.5784
	B. Goyal & S. Kaur	-13488.77643	260234.00707	1.000	-743659.6256	716682.0727

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Based on the provided ANOVA results, it does not appear that there is a statistically significant difference in the performance of the proposed algorithm. The high p-values for the F-statistic in both the Between Groups comparisons suggest that any observed variations are likely due to random chance rather than systematic differences. The analysis of the mean differences between the proposed algorithm and the comparison algorithms (Palmer, Johnson, NEH, B. Goyal, and S. Kaur) reveals that, on average, there are no statistically significant variations. The mean differences and associated confidence intervals suggest that any observed distinctions in performance are likely attributable to random chance rather than inherent differences in the algorithms.

Table 10. ANOVA for the experiment on the parameter of the proposed algorithm.

	Total waiting time				
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	6255166517.299	4	1563791629.325	0.003	1.000
Within Groups	30813390987352.500	65	474052169036.192		
Total	30819646153869.800	69			

These values represent the differences in means, precision, significance, and confidence intervals around those differences. The high p-values suggest that there are no statistically significant differences between the means of the proposed method and each of the other algorithms. The mean comparisons are clearly displayed in Fig 4. As discussed earlier, there is no discernible statistically significant distinction among the various T levels. Nevertheless, it appears that setting T to 0.5 yields superior outcomes compared to a setting of 0.0, where only enhanced solutions are accepted. Further experiments with higher T levels did not result in any additional improvements.

Our experiment employed a non-parametric analysis, incorporating ANOVA and multiple comparisons, to assess the presence of statistically significant differences among the algorithms. ANOVA Table 10 suggests no overall significance, yet mean comparisons reveal a slight variance in the total waiting time of jobs between the proposed and existing algorithms. Notably, one of the suggested heuristics consistently emerges as the most effective across various distributions, yielding an ANOVA below one. This conclusion is supported by both visual and statistical analyses, including Tukey's ANOVA and the Kolmogorov-Smirnov signed rank test. Future research could explore integrating machine setup time, using trapezoidal fuzzy numbers, and extending the algorithm's application to scheduling problems with three or more machines.

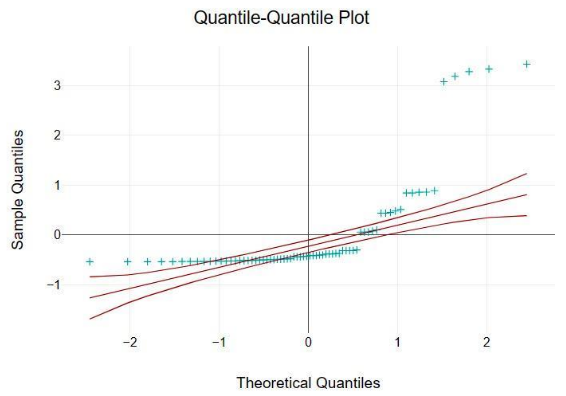

Figure 4. Quantile-Quantile Plot

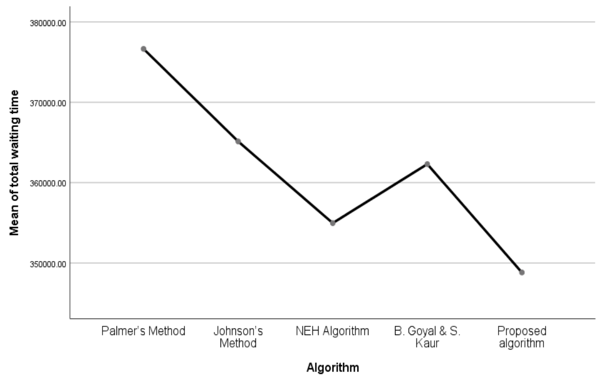

Figure 5. The statistical mean difference for FSSPs

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Conclusion

In this paper, the two-machine flow-shop scheduling problem with trapezoidal fuzzy processing times is addressed, aiming to minimize the total waiting time. A novel algorithm with a specially structured model is proposed. A defuzzification function is utilized to rank fuzzy numbers. The proposed method is then compared with existing algorithms, including NEH, Palmer, Johnson, and B. Goyal & Kaur all re-implemented for this study. Results from exact algorithms indicate that the approach outperforms the existing algorithms in both solution quality and computational effort. The efficiency of the proposed algorithm for jobs 10, 20, 30, 40, 50, 60, 80, 90, 100, 120, 150, 200, 250, 300, and 500 is demonstrated, statistically outperforming other heuristics. Additionally, there is an aim to integrate uncertainty into distributed scheduling to enhance the practicality of scheduling outcomes. Furthermore, a key focus of future work is to merge artificial intelligence techniques like reinforcement learning with intelligent optimization algorithms to tackle combinatorial optimization challenges.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, Vellore, India.

Authors' Contribution Statement

K.R. and K.K. were responsible for the design and implementation of the research. K.R. performed drafting the manuscript, conducted the analysis, acquired data, interpreted the results, and contributed to the writing of the manuscript. K.K. did the revision and interpretation. All authors read and agreed upon the published version of the manuscript.

References

1. Johnson SM. Optimal Two- and Three-Stage Production Schedules with Setup Times Included. Nav Res Logist Q. 1954 Mar; 1(1): 61–68. https://doi.org/10.1002/nav.3800010110
2. Palmer DS. Sequencing Jobs Through a Multi-Stage Process in the Minimum Total Time—A Quick Method of Obtaining a Near Optimum. J Oper Res Soc. 1965 Mar; 16(1): 101-107. https://doi.org/10.1057/jors.1965.8
3. Gupta JND. A Functional Heuristic Algorithm for the Flowshop Scheduling Problem. J Oper Res Soc. 1971; 22(1): 39–47. https://doi.org/10.1057/jors.1971.18
4. Campbell HG, Dudek RA, Smith ML. A Heuristic Algorithm for the n Job, m Machine Sequencing Problem. Manag Sci. Jun 1970; 16(10): B630-B637. https://www.jstor.org/stable/2628231
5. Lee ES, Li R-J. Comparison of Fuzzy Numbers Based on the Probability Measure of Fuzzy Events. Comput Math Appl. 1988 Jan 1; 15(10): 887-896. https://doi.org/10.1016/0898-1221(88)90124-1
6. Liu GS, Tu M, Tang YS, Ding TX. Energy-Aware Optimization for the Two-Agent Scheduling Problem with Fuzzy Processing Times. Int J Interact Des Manuf. 2023 Feb 1; 17(1): 237–248. https://doi.org/10.1007/s12008-022-00927-9
7. Amat S, Ortiz P, Ruiz J, Trillo JC, Yanez DF. Geometric Representation of the Weighted Harmonic Mean of n Positive Values and Potential Applications. Indian J Pure Appl Math. 2023 Apr 30; 55(2): 794–804. https://doi.org/10.1007/s13226-023-00409-y
8. Abduljabbar IA, Abdullah SM. An Evolutionary Algorithm for Solving Academic Courses Timetable Scheduling Problem. Baghdad Sci J. 2022 Apr 1; 19(2): 399–408. https://doi.org/10.21123/bsj.2022.19.2.0399

9. Shen J, Shi Y, Shi J, Dai Y, Li W. An Uncertain Permutation Flow Shop Predictive Scheduling Problem with Processing Interruption. Phys. A: Stat Mech Appl. 2023 Feb 1; 611: 1-15. https://doi.org/10.1016/j.physa.2023.128457
10. Allahverdi M, Allahverdi A. Minimizing Total Completion Time for Flowshop Scheduling Problem with Uncertain Processing Times. RAIRO–Oper Res. 2021; 55: S929 - S946. https://doi.org/10.1051/ro/2020022
11. Mehdizadeh E, Soleimaninia F. A Vibration Damping Optimization Algorithm to Solve Flexible Job Shop Scheduling Problems with Reverse Flows. Int J Res Ind Eng. 2023 Dec 1; 12(4): 431-449. https://doi.org/10.22105/riej.2023.383451.1363
12. Bagheri M, Meybodi NB, Enzebati AH. Modeling and Optimizing a Multi-Objective Flow Shop Scheduling Problem to Minimize Energy Consumption, Completion Time and Tardiness. J Oper Res Decis. 2018 Nov 22; 3(3): 204-222. https://doi.org/10.22105/dmor.2018.81214
13. Allahverdi M. An Improved Algorithm to Minimize the Total Completion Time in a Two-Machine No-Wait Flow-Shop with Uncertain Setup Times. J Proj Manag. 2022; 7(1): 1-12. https://doi.org/10.5267/j.jpm.2021.9.001
14. AL-jabr M, Diab A, AL-Diab J. Distributed Heuristic Algorithm for Migration and Replication of Self-Organized Services in Future Networks. Baghdad Sci J. 2022 Dec 1; 19(6): 1335-1345. https://doi.org/10.21123/bsj.2022.6338
15. Asif MK, Alam ST, Jahan S, Arefin MR. An Empirical Analysis of Exact Algorithms for Solving Non-Preemptive Flow Shop Scheduling Problem. Int J Res Ind Eng. 2022 Sep 1; 11(3): 306-321. http://dx.doi.org/10.22105/riej.2022.350120.1324
16. McCahon CS, Lee ES. Job Sequencing with Fuzzy Processing Times. Computers Math Applic. 1990 Jan 1; 19(7): 31–41. https://doi.org/10.1016/0898-1221(90)90191-L
17. Abdullah S, Abdolrazzagh-Nezhad M. Fuzzy Job-Shop Scheduling Problems: A review. Inf Sci. 2014 Sep 10; 278: 380–407. https://doi.org/10.1016/j.ins.2014.03.060
18. Dubois D, Prade H. Ranking Fuzzy Numbers in the Setting of Possibility Theory. Inf Sci. 1983 Sep 1; 30(3): 183–224. DOI: https://doi.org/10.1016/0020-0255(83)90025-7
19. Mahfouf M, Abbod MF, Linkens DA. A Survey of Fuzzy Logic Monitoring and Control Utilisation in Medicine. Artif Intell Med. 2001 Jan 1; 21(1-3): 27-42. https://doi.org/10.1016/S0933-3657(00)00072-5
20. Kiptum CK, Bouraima MB, Badi I, Zonon BIP, Ndiema KM, Qiu Y. Assessment of the Challenges to Urban Sustainable Development Using an Interval-Valued Fermatean Fuzzy Approach. Syst Anal. 2023 Aug 29; 1(1): 11-26. https://doi.org/10.31181/sa1120233
21. Alburaikan A, Garg H, Khalifa HA. A Novel Approach for Minimizing Processing Times of Three-Stage Flow Shop Scheduling Problems Under Fuzziness. Symmetry. 2023 Jan 2; 15(1): 1-13. https://doi.org/10.3390/sym15010130
22. Zhou W, Chen F, Ji X, Li H, Zhou J. A Pareto-Based Discrete Particle Swarm Optimization for Parallel Casting Workshop Scheduling Problem with Fuzzy Processing Time. Knowl.-Based Syst. 2022 Nov 28; 256: 1-13. https://doi.org/10.1016/j.knosys.2022.109872
23. Zubair SAM. Single-Valued Neutrosophic Uncertain Linguistic Set Based on Multi-Input Relationship and Semantic Transformation. J Fuzzy Ext Appl. 2023 Oct 1; 4(4): 257–270. https://doi.org/10.22105/jfea.2023.423474.1319
24. Akram M, Ullah I, Allahviranloo T, Edalatpanah SA. LR-Type Fully Pythagorean Fuzzy Linear Programming Problems with Equality Constraints. J Intell Fuzzy Syst. 2021; 41(1): 1975–1992. https://doi.org/10.3233/JIFS-210655
25. Zanjani B, Amiri M, Hanafizadeh P, Salahi M. Robust Multi-Objective Hybrid Flow Shop Scheduling. J Appl Res. Ind Eng. 2021 Mar 1; 8(1): 40-55. https://doi.org/10.22105/jarie.2021. 252651.1202
26. Edalatpanah SA, Khalifa HA, Keyser RS. A New Approach for Solving Fuzzy Cooperative Continuous Static Games. Soft Comput Fusion Appl. 2024 Jan 15; 1(1): 19-26.
27. Gupta D, Goyal B. Specially Structured Flow Shop Scheduling in Two Stage with Concept of Job Block and Transportation Time to Optimize Total Waiting Time of Jobs. Int J Eng Technol. 2018 Oct-Nov; 10(5): 1273–1284. https://doi.org/10.21817/ijet/2018/v10i5/181005022
28. Engin O, Isler M. An Efficient Parallel Greedy Algorithm for Fuzzy Hybrid Flow Shop Scheduling with Setup Time and Lot Size: A Case Study in Apparel Process. J Fuzzy Ext Appl. 2022 Jul 1; 3(3): 249–262. https://doi.org/10.22105/jfea.2021.314312.1169
29. Bahmani V, Adibi MA, Mehdizadeh E. Integration of Two-Stage Assembly Flow Shop Scheduling and Vehicle Routing Using Improved Whale Optimization Algorithm. J Appl Res Ind Eng. 2023 Mar 1; 10(1): 56–83. https://doi.org/10.22105/jarie.2022.329251.1450
30. Rouhbakhsh R, Mehdizadeh E, Adibi MA. Presenting a Model for Solving Lot-Streaming Hybrid Flow Shop Scheduling Problem by Considering Independent Setup Time and Transportation Time. J Decis Oper Res. 2023; 8(2): 307–332. https://doi.org/10.22105/dmor.2022.296154.1450
31. Jain C, Saini RK, Sangal A. Application of Trapezoidal Fuzzy Numbers in the Inventory Problem of Decision Science. Multicriteria Algo Appl. 2024

Mar 12; 3: 1–14. https://doi.org/10.61356/j.mawa.2024.311461

32. Goyal B, Kaur S. Flow Shop Scheduling-Especially Structured Models Under Fuzzy Environment with Optimal Waiting Time of Jobs. Int J Design Engineering. 2022 Nov; 11(1): 47–60. https://doi.org/10.1504/IJDE.2022.127075
33. Goyal B, Kaur S. Comparative Performance Analysis of Heuristics with Bicriterion Optimization for Flow Shop Scheduling. In: Das S, Saha S, Coello Coello CA, Bansal JC, editors. Advances in Data-Driven Computing and Intelligent Systems. ADCIS 2022. Lecture Notes in Networks and Systems. Singapore: Springer. 2023; 698: p.653–669. https://doi.org/10.1007/978-981-99-3250-4_50.
34. Goyal B, Kaur S. Specially Structured Flow Shop Scheduling Models with Processing Times as Trapezoidal Fuzzy Numbers to Optimize Waiting Time of Jobs. In: Tiwari A, Ahuja K, Yadav A, Bansal JC, Deep K, Nagar AK, editors. Soft Computing for Problem Solving. Advances in Intelligent Systems and Computing. Singapore: Springer; 2021; 1393: p. 27-42. https://doi.org/10.1007/978-981-16-2712-5_3
35. Zhang W, Li C, Gen M, Yang W, Zhang G. A Multiobjective Memetic Algorithm with Particle Swarm Optimization and Q-Learning-Based Local Search for Energy-Efficient Distributed Heterogeneous Hybrid Flow-Shop Scheduling Problem. Expert Syst Appl. 2023 Mar; 237(Part C): 1-20. https://doi.org/10.1016/j.eswa.2023.121570
36. Behnamian J. Survey on Fuzzy Shop Scheduling. Fuzzy Optim Decis Making. 2016 Sep; 15: 331-366. https://doi.org/10.1007/s10700-015-9225-5
37. Zimmerman H-J. Fuzzy Set Theory and its Applications. 4th edition. New York: Springer Seience+Business Media; 2001. Chapter 1, Introduction to Fuzzy Sets; p.1-8. https://doi.org/10.1007/978-94-010-0646-0_1
38. Zadeh LA. Fuzzy Sets Inf Control. 1965; 8(3): 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X
39. Elizabeth S, Sujatha L. Project Scheduling Method Using Triangular Intuitionistic Fuzzy Numbers and Triangular Fuzzy Numbers. Appl Math Sci. 2015; 9(4): 185–198. http://dx.doi.org/10.12988/ams.2015.410852
40. Ranjith K, Karthikeyan K. New Algorithm for Two-Machine Fuzzy Flow Shop Scheduling Problem with Trapezoidal Fuzzy Processing Time. J Intell Fuzzy Syst. 2024; 46: 1-14. https://doi.org/10.3233/JIFS-235526
41. Nawaz M, Enscore EE, Ham I. A Heuristic Algorithm for the m-Machine, n-Job Flow-Shop Sequencing Problem. Omega. 1983; 11(1): 91-95. https://doi.org/10.1016/0305-0483(83)90088-9
42. Paraveen R, Khurana MK. A Comparative Analysis of SAMP-Jaya and Simple Jaya Algorithms for PFSSP. Soft Comput. 2023 Aug 1; 27(15): 10759–10776. https://doi.org/10.1007/s00500-023-08261-2

Published Online First: October, 2024
https://doi.org/10.21123/bsj.2024.10784
P-ISSN: 2078-8665 - E-ISSN: 2411-7986

Baghdad Science Journal

تقليل إجمالي وقت الانتظار لمشكلة جدولة متجر التدفق الغامض لجهازين مع وقت معالجة غير مؤكد

ك.رانجيث ، ك. كارثيكيان

قسم الرياضيات، كلية العلوم المتقدمة، معهد فيلور للتكنولوجيا، فيلور، الهند.

الخلاصة

تتضمن الجدولة تخصيص الموارد للوظائف ضمن قيود محددة مع مرور الوقت. تقليديا، كان وقت المعالجة لكل مهمة يعتبر قيمة ثابتة. ومع ذلك، في السيناريوهات العملية، يمكن أن تتقلب أوقات معالجة الوظائف ديناميكيًا بناءً على الظروف السائدة. في هذه المقالة، يتم تقديم أسلوب جديد لمشكلة جدولة متجر التدفق الضبابي (FSSP) في بيئة غامضة، حيث يتم تمثيل أوقات معالجة الوظيفة بأرقام غامضة شبه منحرفة. بالإضافة إلى ذلك، تم اقتراح وتطوير خوارزمية جديدة تعتمد على نهج جويال لـ FSSP المكون من جهازين مع أوقات معالجة غامضة. وفي الوقت نفسه، تم تعزيز الخوارزمية الموجودة لتحسين أدائها في هذا السياق المحدد. تهدف الدراسة إلى تقليل إجمالي وقت الانتظار للوظائف. يتم استخدام وظيفة إزالة الضبابية لترتيب الأرقام الغامضة، بهدف نهائي هو تقليل إجمالي وقت الانتظار. علاوة على ذلك، يقوم المقال بتقييم أداء هذه الأساليب من حيث جودة الحلول، وذلك باستخدام مشاكل الاختبار مع 10، 20، 30، 40، 50، 60، 80، 90، 100، 120، 200، 250، 300، و 500 وظيفة، جنبا إلى جنب مع 2 آلات. تتم مقارنة متوسط إجمالي وقت الانتظار بالخوارزميات الموجودة، بما في ذلك خوارزميةNEH ، وطريقةPalmer ، وطريقةJohnson ، و B. Goyalو B. Kaur، والخوارزمية المقترحة. بالإضافة إلى ذلك، فإن النتائج التي تم الحصول عليها، إلى جانب اختبار ANOVA جيد التنظيم، تسلط الضوء على فعالية الطريقة المقترحة في معالجة مشكلة الجدولة قيد التحقيق. توضح النتائج التجريبية أن الخوارزمية المقترحة يمكنها تقليل وقت الانتظار بشكل فعال في FSSP ثنائي الجهاز وتحقيق نتائج متفوقة بالمقارنة مع الخوارزميات المختلفة الموجودة.

الكلمات المفتاحية: مشكلة جدولة متجر التدفق الغامض، المنطق الضبابي، الخوارزمية الإرشادية، التسلسل الأمثل، الرقم شبه المنحرف (TrFN)، إجمالي وقت الانتظار.