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A Novel Approach to C_{exp} Average Assignments on Chain Graphs

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Abstract

In general, the exponential average of two positive numbers does not have to be an integer. Because of this, the exponential average needs to be an integer that takes into consideration the flooring or ceiling function. It has been defined that graphs can be labeled with an exponential average, where the flooring function or the ceiling function can apply labels to the edges. To establish the exponential average assignment on graphs, consider the edge labels that arise from the ceiling function alone. The vertex assignment function δ and edge assignment function δ^* are called a C_{exp} average assignment of the graph G with p vertices and q edges if δ is injective and $\delta^*$is bijective and the corresponding relations are $\delta: V \to N - \{q+2, \cdots, \infty\}$, $\delta^*: E \to N - \{1, q+2, \cdots, \infty\}$ and is defined by edge label δ^* is $\delta^*(uv) = \left\lceil \frac{1}{e}\left(\frac{X(v)}{X(u)}\right)^{\frac{1}{Y(u,v)}} \right\rceil$ where $X(u) = \delta(u)^{\delta(u)}, Y(u,v) = \delta(v) - \delta(u)$ and N is the set of all natural numbers. If the graph accepts a C_{exp} average assignment then it is called a C_{exp} average assignment graph. The C_{exp} average assignment of graphs is proposed in this paper, and its characteristics are explored on the cycle, the union of path and cycle, the union of T- graph and cycle, the graph G*, the graph G′, the graph $\widehat{G}$and tadpole.

Keywords: C_{exp} average assignment, C_{exp} average assignment graph, Chain graphs, Edge labeling, Vertex labeling.

Introduction

Graph labeling is the assignment of labels, generally represented by integers, to the edges and/or vertices of a graph. The average procedures with chain graph topologies are effective models in a wide range of applications, including circuit design, communications infrastructure addressing, astronomy, database management systems, and coding theory. The chain graph is essential in different applications which include electronics, electrical, and wireless communication. Mathematics is used in almost every aspect of computer science, including intelligent transportation, project management systems and tools, software architecture and design, multiprocessing, automated control, distributed and concurrent algorithms, and so on. Algorithm creation, implementation, and analysis are all aided by mathematics in scientific and technical applications. It also enhances the efficacy and application of current approaches and algorithms. The assignment of vertices and edges is important in many applications in the field of graph theory and optimization. Our paper introduces a new method for determining C_{exp} averages on chain graphs, a type of graph that is frequently found in various domains like network analysis, biology, channel assignment, routing and path selection, max-flow min-cut process, fault detection and localization process, resource allocation, interference mitigation, theoretical analysis or simulation, wireless network

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performance metrics, fuzzy decision making, and telecommunications. A graph is characterized as a finite, undirected, and simple graph throughout this work. Interpret the graph G(V, E), which has p endpoints and q linkages. West DB [1] provided concepts and notations for graphs and survey [2] provided a detailed analysis of graph labeling.

Materials and Methods

Path on n vertices is denoted by P_n and cycle on n vertices is denoted by C_n. The graph $K_{1,n}$ is called a star graph and it is denoted by S_n. The T-graph T_n is obtained by attaching a pendant vertex to a neighbor of the pendant vertex of a path on $n-1$ vertices. The graph Tadpoles $T(n,k)$ is obtained by identifying a vertex of the cycle C_n to an end vertex of the path P_k. Barrientos and Minion proposed a method that allows us to transform a special kind of graceful labeling into harmonious labeling for many families of graphs[3]. Durai Baskar et al. presented the F-geometric mean and developed its meanness for chain graphs including the graph G*, the graph G′, and the graph $\widehat{G}$ [4]. Uma Devi et al. delivered the odd Fibonacci edge irregular labeling [5] for some trees obtained from subdivision and vertex identification operations. The C-exponential meanness of graph [6] includes path, triangular tree of T_n, the cartesian product of two paths, a one-sided step graph, and a double-sided step graph was studied by Thamaraiselvi and Rajasekaran. Khan et al. studied the computational and topological properties of neural networks using graph-theoretic parameters[7]. Ashwini et al. determined the lucky number[8] of various graphs. Muthugurupackiam et al. determined the (a, d) -total edge irregularity strength of graphs[9]. The assignment calculations are based on the C_{exp} average across numerous ladders graphs[10] significantly the one-sided step graph, diamond ladder graph, and the meanness of graphs derived from various graph operations established by Rajesh Kannan et al. The vertex assignment function δ and edge assignment function δ^* are called a C_{exp} average assignment of the graph G with p vertices and q edges if δ is injective and δ^* is bijective and the corresponding relations are $\delta: V \to N - \{q+2, \cdots, \infty\}$, $\delta^*: E \to N - \{1, q+2, \cdots, \infty\}$ and is defined by edge label δ^* is $\delta^*(uv) = \left\lceil \frac{1}{e}\left(\frac{X(v)}{X(u)}\right)^{\frac{1}{Y(u,v)}} \right\rceil$ where $X(u) = \delta(u)^{\delta(u)}, Y(u,v) = \delta(v) - \delta(u)$ and N is the set of all natural numbers.

If $G(p,q)$ accepts C_{exp} average assignment then it is called a C_{exp} average assignment graph. Fig 1 offers an interesting memorable illustration of a C_{exp} average assignment with graph S_3.

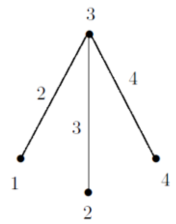

Figure 1. A C_{exp} average assignment of S_3.

A flow chart for C_{exp} average assignment graph is given in Fig 2.

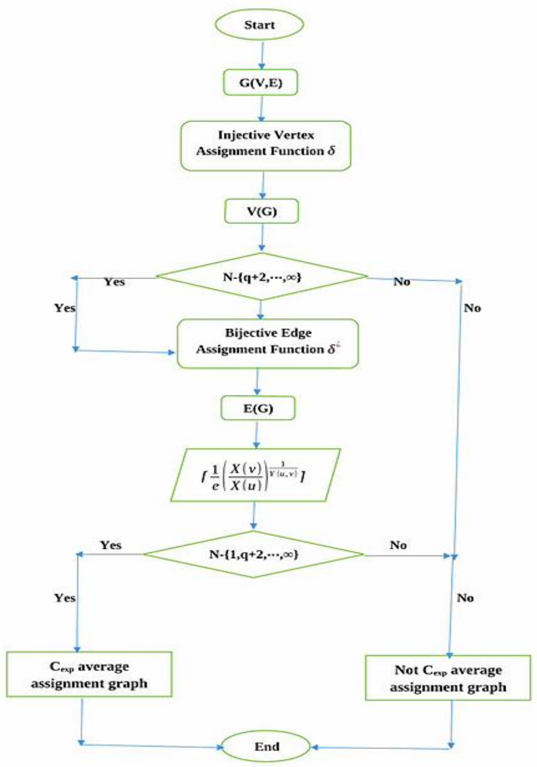

Figure 2. Flow chart for C_{exp} average assignment graph.

A C_{exp} average assignment for the graphs related to chains has been examined here.

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Results and Discussion

Based on the C_{exp} average assignment definition, the following theorems were proved.

Theorem 1: Every cycle C_n is a C_{exp} average assignment graph, for $n \geq 3$.

Proof: Let $\{v_a : 1 \leq a \leq n\}$ be the nodes of the cycle. Let $n \geq 4$. The point assignment of the cycle C_n is given in Table 1.

Table 1. The point assignment of the cycle C_n.

The point assignment δ is $\{1,2,3,\cdots,n+1\}$				
$\delta(v)$	$1 \leq a \leq 2$	$3 \leq a \leq \left\lfloor \frac{n}{2} \right\rfloor + 1$	$a = \left\lfloor \frac{n}{2} \right\rfloor + 2$	$\left\lfloor \frac{n}{2} \right\rfloor + 3 \leq a \leq n$
$\delta(v_a)$	$2a-1$	$2a-2$	$n+1$	$2n+5-2a$

The line assignment of the cycle C_n is given in Table 2.

Table 2. The line assignment of the cycle C_n.

Line Assignment δ^*					
$\delta^*(e)$	$1 \leq a \leq 2$	$3 \leq a \leq \left\lfloor \frac{n}{2} \right\rfloor + 1$	$a = 2 + \left\lfloor \frac{n}{2} \right\rfloor$ n is even	$a = 2 + \left\lfloor \frac{n}{2} \right\rfloor$ n is odd	$\left\lfloor \frac{n}{2} \right\rfloor + 3 \leq a \leq n-1$
$\delta^*(v_a v_{a+1}) = \left\lfloor \frac{1}{e}\left(\frac{X(v_{a+1})}{X(v_a)}\right)^{\frac{1}{Y(v_a,v_{a+1})}} \right\rfloor$	$2a$	$2a-1$	n	$n+1$	$2n+4-2a$

and $\delta^*(v_n v_1) = \left\lfloor \frac{1}{e}\left(\frac{X(v_1)}{X(v_n)}\right)^{\frac{1}{Y(v_n,v_1)}} \right\rfloor = 3$. A C_{exp} average assignment of C_{12} is given in Fig 3.

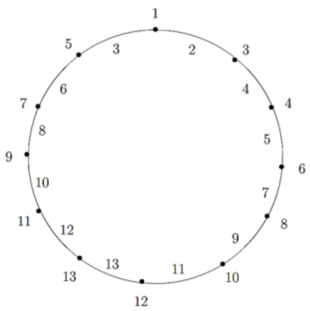

Figure 3. A C_{exp} average assignment of C_{12}.

For the special case, n=3 labeling is given in Fig 4.

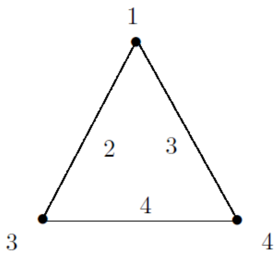

Figure 4. A C_{exp} average assignment of C_3.

Thus, every cycle C_n is a C_{exp} average assignment graph, for $n \geq 3$.

□

The cycle C_3 is one of the interesting and simple graph. In the above proof, a C_{exp} average assignment for cycle for $n \geq 4$ has been discussed. Also for n=3, the cycle c_3 and its assignment values have been represented.

Theorem 2: The graph $C_m \cup P_n$ is a C_{exp} average assignment graph, for $n \geq 2$.

Proof: Let $\{u_a, v_b : 1 \leq a \leq m, 1 \leq b \leq n\}$ be the nodes of the cycle and path. The point assignment $\delta(u_a)$ of the graph $C_m \cup P_n$ is given in Table 3.

Table 3. The point assignment $\delta(u_a)$ of the graph $C_m \cup P_n$.

The point assignment δ is $\{1,2,3,\cdots,n+m\}$			
$\delta(v)$	$1 \leq a \leq \left\lfloor \frac{m}{2} \right\rfloor$	$a = \left\lfloor \frac{m}{2} \right\rfloor + 1$	$\left\lfloor \frac{m}{2} \right\rfloor + 2 \leq a \leq m$
$\delta(u_a)$	$m+n+2-2a$	n	$-1+2a-m+n$

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The point assignment $\delta(v_b)$ of the graph $C_m \cup P_n$ is given in Table 4.

Table 4. The point assignment $\delta(v_b)$ of the graph $C_m \cup P_n$.

The point assignment δ		
$\delta(v)$	$1 \le b \le n-1$	$b = n$
$\delta(v_b)$	b	-
$\delta(v_n)$	-	$1+n$

The line assignment $\delta^*(u_a u_{a+1})$of the graph $C_m \cup P_n$ is given in Table 5.

Table 5. The line assignment $\delta^*(u_a u_{a+1})$ of the graph $C_m \cup P_n$.

Line Assignment δ^*				
$\delta^*(e)$	$1 \le a \le \left\lfloor \frac{m}{2} \right\rfloor$	$a = \left\lfloor \frac{m}{2} \right\rfloor + 1$		$\left\lfloor \frac{m}{2} \right\rfloor + 2 \le a \le m-1$
		m is odd	m is even	
$\delta^*(u_a u_{a+1}) = \left\lfloor \frac{1}{e}\left(\frac{X(u_{a+1})}{X(u_a)}\right)^{\frac{1}{Y(u_a,u_{a+1})}} \right\rfloor$	$m+n-2a+1$	$n+1$	$n+2$	$n-m+2a$

The line assignment $\delta^*(v_b v_{b+1})$ and $\delta^*(u_m u_1)$ of the graph $C_m \cup P_n$ are given in Table 6.

Table 6. The line assignment $\delta^*(v_b v_{b+1})$and $\delta^*(u_m u_1)$ of the graph $C_m \cup P_n$.

Line Assignment δ^*	
$\delta^*(e)$	$1 \le b \le n-1$
$\delta^*(v_b v_{b+1})=\left\lfloor \frac{1}{e}\left(\frac{X(v_{b+1})}{X(v_b)}\right)^{\frac{1}{Y(v_b,v_{b+1})}} \right\rfloor$	$1+b$
$\delta^*(u_m u_1)=\left\lfloor \frac{1}{e}\left(\frac{X(u_1)}{X(u_m)}\right)^{\frac{1}{Y(u_m,u_1)}} \right\rfloor$	$n+m$

A C_{exp} average assignment of $C_{12} \cup P_7$is given in Fig 5.

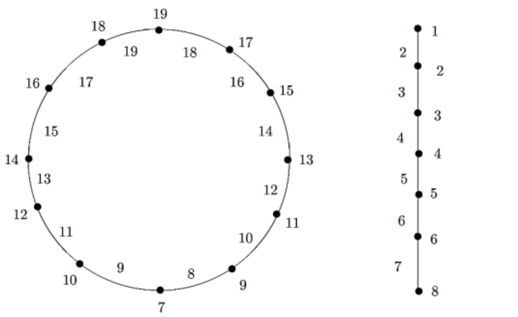

Figure 5. A C_{exp} average assignment of $C_{12} \cup P_7$.

Thus, the graph $C_m \cup P_n$ is a C_{exp} average assignment graph, for $n \ge 2$. □

The union operation preserves its C_{exp} average assignment property for C_m and P_n.

Theorem 3: For, $n \ge 2$ and $m \ge 3$, the graph $T_n \cup C_m$is a C_{exp} average assignment graph, where T_n is a T graph.

Proof: Let P_{n-1} be the path having nodes as $\{u_a: 1 \le a \le n-1\}$ and $\{v_a: 1 \le a \le m\}$ be the nodes of the cycle and u_n be the pendent vertex identified with u_2. The point assignment $\delta(v_a)$of the graph $T_n \cup C_m$ is given in Table 7.

Table 7. The point assignment $\delta(v_a)$ of the graph $T_n \cup C_m$.

The point assignment δ is $\{1,2,3,\cdots,n+m\}$			
$\delta(v)$	$1 \le a \le \left\lfloor \frac{m}{2} \right\rfloor$	$a = \left\lfloor \frac{m}{2} \right\rfloor + 1$	$\left\lfloor \frac{m}{2} \right\rfloor + 2 \le a \le m$
$\delta(v_a)$	$2+m-2a+n$	n	$-1+n+2a-m$

The point assignment $\delta(u_a)$ of the graph $T_n \cup C_m$ is given in Table 8.

Table 8. The point assignment $\delta(u_a)$ of the graph $T_n \cup C_m$.

$\delta(v)$	$1 \le a \le n-2$
$\delta(u_a)$	$a+1$

The point assignment $\delta(u_n)$ of the graph $T_n \cup C_m$ is given in Table 9.

Table 9. The point assignment $\delta(u_n)$ of the graph $T_n \cup C_m$.

$\delta(u_{n-1})$	$n+1$
$\delta(u_n)$	1

The line assignment $\delta^*(v_a v_{a+1})$ of the graph $T_n \cup C_m$ is given in Table 10.

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Table 10. The line assignment $\delta^*(v_a v_{a+1})$ of the graph $T_n \cup C_m$.

Line Assignment δ^*				
$\delta^*(e)$	$1 \le a \le \lfloor \frac{m}{2} \rfloor$	$a = \lfloor \frac{m}{2} \rfloor + 1$ m is odd	 m is even	$2 + \lfloor \frac{m}{2} \rfloor \le a \le m-1$
$\delta^*(v_a v_{a+1}) = \left\lceil \frac{1}{e} \left(\frac{X(v_{a+1})}{X(v_a)} \right)^{\frac{1}{Y(v_a, v_{a+1})}} \right\rceil$	$-2a + n + 1 + m$	$n + 1$	$n + 2$	$-m + 2a + n$

The line assignment $\delta^*(u_a u_{a+1})$ of the graph $T_n \cup C_m$ is given in Table 11.

Table 11. The line assignment $\delta^*(u_a u_{a+1})$ of the graph $T_n \cup C_m$.

$\delta^*(e)$	$1 \le a \le n-2$
$\delta^*(u_a u_{a+1}) = \left\lceil \frac{1}{e} \left(\frac{X(u_{a+1})}{X(u_a)} \right)^{\frac{1}{Y(u_a, u_{a+1})}} \right\rceil$	$a + 2$

The line assignment $\delta^*(u_2 u_n)$ and $\delta^*(u_2 u_n)$ of the graph $T_n \cup C_m$ are given in Table 12.

Table 12. The line assignment $\delta^*(u_2 u_n)$ and $\delta^*(v_1 v_m)$ of the graph $T_n \cup C_m$

$\delta^*(u_2 u_n) = \left\lceil \frac{1}{e} \left(\frac{X(u_n)}{X(u_2)} \right)^{\frac{1}{Y(u_2, u_n)}} \right\rceil$	2
$\delta^*(v_1 v_m) = \left\lceil \frac{1}{e} \left(\frac{X(v_m)}{X(v_1)} \right)^{\frac{1}{Y(v_1, v_m)}} \right\rceil$	$-1 + n + m$

A C_{exp} average assignment of $T_7 \cup C_6$ and $T_4 \cup C_3$ are given in Fig 6 and Fig 7 respectively.

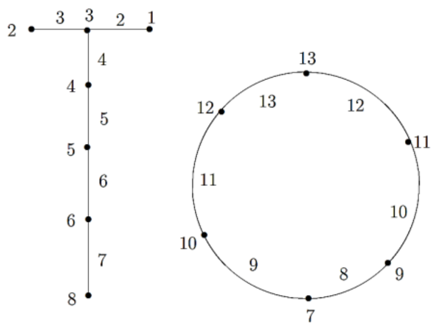

Figure 6. A C_{exp} average assignment of $T_7 \cup C_6$.

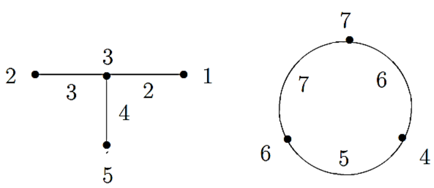

Figure 7. A C_{exp} average assignment of $T_4 \cup C_3$.

Thus, for, $n \ge 2$ and $m \ge 3$, the graph $T_n \cup C_m$ is a C_{exp} average assignment graph. □

Even though C_m and T_n are C_{exp} average assignment graphs, their union preserves its C_{exp} average assignment property.

The graph $G^*(p_1, p_2, \dots, p_n)$ is obtained from n cycles of length $p_1, p_2, \dots, p_n$ by identifying consecutive cycles at a vertex as follows. If the b^{th} cycle is of odd length, then its $\left(\frac{p_b+3}{2}\right)^{th}$ vertex is identified with the first vertex of $(b+1)^{th}$ cycle and if the b^{th} cycle is of even length, then its $\frac{p_j+2}{2}$ vertex is identified with the first vertex of $(b+1)^{th}$ cycle.

Theorem 4: The graph $G^*(p_1, p_2, \dots, p_n)$ is a C_{exp} average assignment graph for any p_b, for $1 \le b \le n$.
Proof: Let the points of cycles of frequency n be $\{v_a^{(b)}: 1 \le b \le n, 1 \le a \le p_b\}$ and for $1 \le b \le n-1$, the b^{th} cycle and $(b+1)^{th}$ cycle are identified by a vertex $v_{\frac{p_b+3}{2}}^{(b)}$ and $v_1^{(b+1)}$ while p_b is odd and $v_{\frac{p_b+2}{2}}^{(b)}$ and $v_1^{(b+1)}$ while p_b is even. The point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$ is given in Table 13.

Table 13. The point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$

The point assignment δ is $\{1,2,3, \dots, \sum_{b=1}^{n} p_b + 1\}$		
$\delta(v)$	$1 \le a \le \lfloor \frac{p_1}{2} \rfloor + 1$	$\lfloor \frac{p_1}{2} \rfloor + 2 \le a \le p_1$
$\delta(v_a^{(1)})$	$-1 + 2a$	$2p_1 + 4 - 2a$

The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$ is given in Table 14.

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Table 14. The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$.

$\delta(v)$	$2 \le b \le n$			
	$2 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor + 1$	$a = \left\lfloor \frac{p_b}{2} \right\rfloor + 2$		$\left\lfloor \frac{p_b}{2} \right\rfloor + 3 \le a \le p_b$
		p_b is odd	p_b is even	
$\delta(v_a^{(b)})$	$\sum_{c=1}^{b-1} p_c + 2a - 1$	$\sum_{c=1}^{b-1} p_c + 2a - 2$	$\sum_{c=1}^{b-1} p_c + 2a - 4$	$\sum_{c=1}^{b-1} p_c + 2p_a - 2a + 4$

The line assignment $\delta^*\left(v_a^{(1)} v_{a+1}^{(1)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$ is given in Table 15.

Table 15. The line assignment $\delta^*\left(v_a^{(1)} v_{a+1}^{(1)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$.

Line Assignment δ^*				
$\delta^*(e)$	$1 \le a \le \left\lfloor \frac{p_1}{2} \right\rfloor$	$a = \left\lfloor \frac{p_1}{2} \right\rfloor + 1$		$\left\lfloor \frac{p_1}{2} \right\rfloor + 2 \le a \le p_1 - 1$
		p_1 is odd	p_1 is even	
$\delta^*(v_a^{(1)} v_{a+1}^{(1)}) = \left\lfloor \frac{1}{e} \left(\frac{X(v_{a+1}^{(1)})}{X(v_a^{(1)})} \right)^{\frac{1}{Y(v_a^{(1)}, v_{a+1}^{(1)})}} \right\rfloor$	$2a$			$2p_1 - 2a + 3$

The line assignment $\delta^*\left(v_a^{(b)} v_{a+1}^{(b)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$ is given in Table 16.

Table 16. The line assignment $\delta^*\left(v_a^{(b)} v_{a+1}^{(b)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$.

$\delta^*(e)$	$2 \le b \le n$			
	$1 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor$	$a = \left\lfloor \frac{p_b}{2} \right\rfloor + 1$		$\left\lfloor \frac{p_b}{2} \right\rfloor + 2 \le a \le p_b - 1$
		p_b is odd	p_b is even	
$\delta^*(v_a^{(b)} v_{a+1}^{(b)}) = \left\lfloor \frac{1}{e} \left(\frac{X(v_{a+1}^{(b)})}{X(v_a^{(b)})} \right)^{\frac{1}{Y(v_a^{(b)}, v_{a+1}^{(b)})}} \right\rfloor$	$\sum_{c=1}^{b-1} p_c + 2a$			$\sum_{c=1}^{b-1} p_c + 2p_b - 2a + 3$

The line assignment $\delta^*\left(v_{p_1}^{(1)} v_1^{(1)}\right)$ and $\delta^*\left(v_{p_b}^{(b)} v_1^{(b)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$ are given in Table 17.

Table 17. The line assignment $\delta^*\left(v_{p_1}^{(1)} v_1^{(1)}\right)$ and $\delta^*\left(v_{p_b}^{(b)} v_1^{(b)}\right)$ of the graph $G^*(p_1, p_2, \dots, p_n)$.

$\delta^*(e)$	$1 \le a \le m_1 - 1$
$\delta^*(v_{p_1}^{(1)} v_1^{(1)}) = \left\lfloor \frac{1}{e} \left(\frac{X(v_1^{(1)})}{X(v_{p_1}^{(1)})} \right)^{\frac{1}{Y(v_{p_1}^{(1)}, v_1^{(1)})}} \right\rfloor$	3
$\delta^*(v_{p_b}^{(b)} v_1^{(b)}) = \left\lfloor \frac{1}{e} \left(\frac{X(v_1^{(b)})}{X(v_{p_b}^{(b)})} \right)^{\frac{1}{Y(v_{p_b}^{(b)}, v_1^{(b)})}} \right\rfloor$	$\sum_{c=1}^{b-1} p_c + 3$

A C_{exp} average assignment of $G^*(10,9,12,4,5)$ is given in Fig 8.

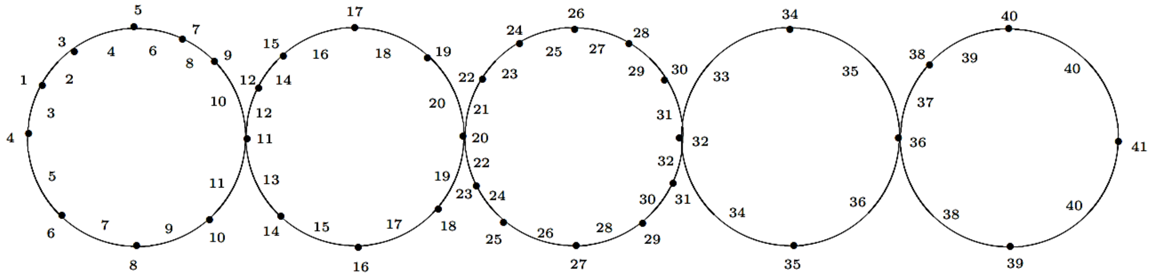

Figure 8. A C_{exp} average assignment of $G^*(10,9,12,4,5)$.

Thus, the graph $G^*(p_1,p_2,\dots,p_n)$ is a C_{exp} average assignment graph for any p_b, for $1 \le b \le n$. □

The graph $G'(p_1,p_2,\dots,p_n)$ is obtained from n cycles of length $p_1,p_2,\dots,p_n$ by identifying consecutive cycles at an edge as follows: The $\left(\frac{p_b+3}{2}\right)^{th}$ edge of b^{th} cycle is identified with the first edge of $(b+1)^{th}$ cycle when b is odd and the $\left(\frac{p_j+1}{2}\right)^{th}$ edge of the b^{th} cycle is identified with the first edge of $(b+1)^{th}$ cycle when b is even.

Theorem 5: The graph $G'(p_1,p_2,\dots,p_n)$ is a C_{exp} average assignment graph if all p_b's are even or all p_b's are odd, for $1 \le b \le n$.

Proof: The points of G' *be* $\{v_a^{(b)}; 1 \le b \le n, 1 \le a \le p_b$. Assume that all p_b is odd. For $1 \le b \le n-1$, the b^{th} and $(b+1)^{th}$ cycles are identified by the edges $v^{(b)}_{\frac{p_{b+1}}{2}} v^{(b)}_{\frac{p_{b+3}}{2}}$ and $v_1^{(b+1)} v^{(b+1)}_{p_{b+1}}$ while b is odd and $v^{(b)}_{\frac{p_{b-1}}{2}} v^{(b)}_{\frac{p_{b+1}}{2}}$ and $v_1^{(b+1)} v^{(b+1)}_{p_{b+1}}$ while b is even. Take $t = \sum_{c=1}^{b-1} p_c$. The point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $G'(p_1,p_2,\dots,p_n)$ is given in Table 18.

Table 18. The point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $G'(p_1,p_2,\dots,p_n)$.

The point assignment δ is $\{1,2,3,\dots,\sum_{b=1}^{n} p_b - n + 2\}$			
$\delta(v)$	$a = 1$	$2 \le a \le \left\lfloor\frac{p_1}{2}\right\rfloor + 1$	$\left\lfloor\frac{p_1}{2}\right\rfloor + 2 \le a \le p_1$
$\delta\left(v_a^{(1)}\right)$	1	$2a$	$2p_1 + 3 - 2a$

The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $G'(p_1,p_2,\dots,p_n)$ is given in Table 19.

Table 19. The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $G'(p_1,p_2,\dots,p_n)$.

$\delta(v)$	$2 \le b \le n$			
	b is even		b is odd	
	$2 \le a \le \left\lfloor\frac{p_b}{2}\right\rfloor$	$\left\lfloor\frac{p_b}{2}\right\rfloor + 1 \le a \le p_b - 1$	$2 \le a \le \left\lfloor\frac{p_b}{2}\right\rfloor + 1$	$\left\lfloor\frac{p_b}{2}\right\rfloor + 2 \le a \le p_b - 1$
$\delta\left(v_a^{(b)}\right)$	$t - b + 2a + 2$	$-2a + t - b + 2p_b + 3$	$t - b + 2a + 1$	$t - 2a - b + 2p_b + 4$

The line assignment $\delta^*\left(v_a^{(1)} v_{a+1}^{(1)}\right)$ of the graph $G'(p_1,p_2,\dots,p_n)$ is given in Table 20.

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Table 20. The line assignment $\delta^*\left(v_a^{(1)}v_{a+1}^{(1)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$.

Line Assignment δ^*		
$\delta^*(e)$	$1 \le a \le \left\lfloor \frac{p_1}{2} \right\rfloor$	$\left\lfloor \frac{p_1}{2} \right\rfloor + 1 \le a \le p_1 - 1$
$\delta^*\left(v_a^{(1)}v_{a+1}^{(1)}\right) = \left\lceil \frac{1}{e}\left(\frac{X(v_{a+1}^{(1)})}{X(v_a^{(1)})}\right)^{\frac{1}{Y(v_a^{(1)},v_{a+1}^{(1)})}} \right\rceil$	$2a + 1$	$2p_1 + 2 - 2a$

The line assignment $\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$ is given in Table 21.

Table 21. The line assignment $\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$.

$\delta^*(e)$	$2 \le b \le n$			
	b is even		b is odd	
	$1 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor$	$\left\lfloor \frac{p_b}{2} \right\rfloor + 1 \le a \le p_b - 1$	$1 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor$	$\left\lfloor \frac{p_b}{2} \right\rfloor + 1 \le a \le p_b - 1$
$\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right) = \left\lceil \frac{1}{e}\left(\frac{X(v_{a+1}^{(b)})}{X(v_a^{(b)})}\right)^{\frac{1}{Y(v_a^{(b)},v_{a+1}^{(b)})}} \right\rceil$	$t - b + 2a + 3$	$t - 2a + 2p_b + b + 2$	$t - b + 2a + 2$	$t - 2a + 2p_b + b + 3$

Assume that all p_b is even. Take $t = \sum_{c=1}^{b-1} p_c$ and for $1 \le b \le n-1$, the b^{th} and $(b+1)^{th}$ circuits are merged by $v_{\frac{p_b}{2}}^{(b)} v_{\frac{p_b+2}{2}}^{(b)}$ and $v_1^{(b+1)} v_{p_{b+1}}^{(b+1)}$. Take $t = \sum_{c=1}^{b-1} p_c$. Then, the point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$ is given in Table 22.

Table 22. The point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$.

The point assignment δ			
$\delta(v)$	$a = 1$	$2 \le a \le \left\lfloor \frac{p_1}{2} \right\rfloor$	$\left\lfloor \frac{p_1}{2} \right\rfloor + 1 \le a \le p_1$
$\delta\left(v_a^{(1)}\right)$	1	$2a$	$2p_1 + 3 - 2a$

The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$ is given in Table 23.

Table 23. The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$.

$\delta(v)$	$2 \le b \le n$	
	$2 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor$	$\left\lfloor \frac{p_b}{2} \right\rfloor + 1 \le a \le p_b - 1$
$\delta\left(v_a^{(b)}\right)$	$1 - b + t - b + 2a$	$-2a + t + 2p_b - b + 4$

The line assignment $\delta^*\left(v_a^{(1)}v_{a+1}^{(1)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$ is given in Table 24.

Table 24. The line assignment $\delta^*\left(v_a^{(1)}v_{a+1}^{(1)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$.

Line Assignment δ^*		
$\delta^*(e)$	$1 \le a \le \left\lfloor \frac{p_1}{2} \right\rfloor$	$\left\lfloor \frac{p_1}{2} \right\rfloor + 1 \le a \le p_1 - 1$
$\delta^*\left(v_a^{(1)}v_{a+1}^{(1)}\right) = \left\lceil \frac{1}{e}\left(\frac{X(v_{a+1}^{(1)})}{X(v_a^{(1)})}\right)^{\frac{1}{Y(v_a^{(1)},v_{a+1}^{(1)})}} \right\rceil$	$2a + 1$	$2p_1 + 2 - 2a$

The line assignment $\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$ is given in Table 25.

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Table 25. The line assignment $\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$.

Line Assignment δ^*		
$\delta^*(e)$	$2 \le b \le n$	
	$1 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor$	$\left\lfloor \frac{p_b}{2} \right\rfloor + 1 \le a \le p_b - 1$
$\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right) = \left\lfloor \frac{1}{e}\left(\frac{X(v_{a+1}^{(b)})}{X(v_a^{(b)})}\right)^{\frac{1}{Y(v_a^{(b)}, v_{a+1}^{(b)})}} \right\rfloor$	$t - b + 2a + 2$	$t + 2p_b + 3 - b - 2a$

The line assignment $\delta^*\left(v_{p_1}^{(1)}v_1^{(1)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$ is given in Table 26.

Table 26. The line assignment $\delta^*\left(v_{p_1}^{(1)}v_1^{(1)}\right)$ of the graph $G'(p_1, p_2, \dots, p_n)$.

$\delta^*\left(v_{p_1}^{(1)}v_1^{(1)}\right) = \left\lfloor \frac{1}{e}\left(\frac{X(v_1^{(1)})}{X(v_{p_1}^{(1)})}\right)^{\frac{1}{Y(v_{p_1}^{(1)}, v_1^{(1)})}} \right\rfloor$	2

Fig 9, displays a C_{exp} average assignment of $G'(7,5,9,13)$. A C_{exp} average assignment of $G'(4,8,10,6)$ is represented in Fig 10.

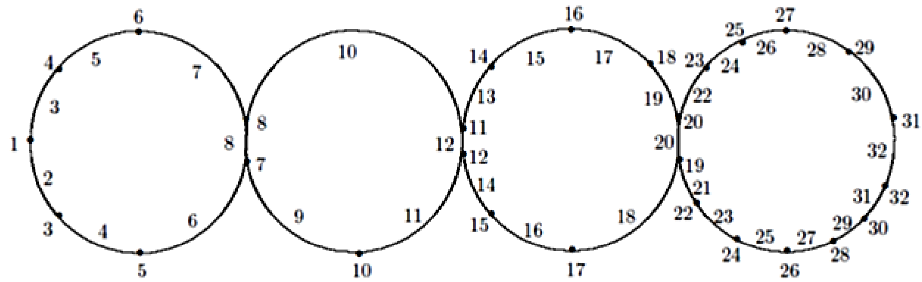

Figure 9. A C_{exp} average assignment of $G'(7, 5, 9, 13)$.

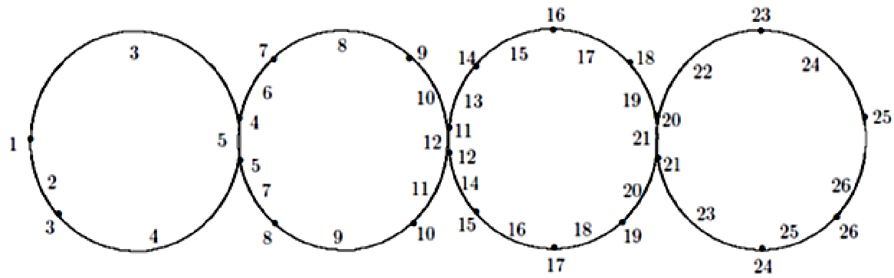

Figure 10. A C_{exp} average assignment of $G'(4, 8, 10, 6)$.

Thus, the graph $G'(p_1, p_2, \dots, p_n)$ is a C_{exp} average assignment graph if all p_b's are even or all p_b's are odd, for $1 \le b \le n$. □

The graph $\widehat{G}(p_1, m_1, \dots, m_{n-1}, p_n)$ is obtained from n cycles of length $p_1, p_2, \dots, p_n$ and $(n-1)$ paths on $m_1, m_2, \dots, m_{n-1}$ vertices respectively by identifying a cycle and a path at a vertex alternatively as follows: If the b^{th} cycle is of odd length, then its $\left(\frac{p_b+3}{2}\right)^{th}$ vertex is identified with a pendant vertex of the b^{th} path and if the b^{th} cycle is of even length, then its $\left(\frac{p_b+2}{2}\right)^{th}$ vertex is identified with a pendant vertex of the b^{th} path while the other pendant vertex of the b^{th} path is identified with the first vertex of the $(b+1)^{th}$ cycle.

Theorem 6: The graph $\widehat{G}(p_1, m_1, \dots, m_{n-1}, p_n)$ is a C_{exp} average assignment graph for any p_b's and m_b's.

Proof: Let $\{v_a^{(b)}; 1 \le b \le n, 1 \le a \le p_b\}$ and $\{u_a^{(b)}; 1 \le b \le n-1,\ 1 \le a \le m_b\}$ be the circles and paths of frequency n and n-1. If $1 \le b \le n-1$, the b^{th} cycle and the b^{th} path is identified by a vertex $v_{\frac{p_b+2}{2}}^{(b)}$ and $u_1^{(b)}$ while p_b is even and $v_{\frac{p_b+3}{2}}^{(b)}$ and

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$u_1^{(b)}$ while p_b is odd and the b^{th} path and $(b+1)^{th}$ cycle are identified by a vertex $u_{m_b}^{(b)}$ and $v_1^{(b+1)}$. The point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$ is given in Table 27.

Table 27. The point assignment $\delta\left(v_a^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$.

The point assignment δ is $\{1,2,3,\dots,\sum_{b=1}^{n-1}(p_b + m_b) + p_n - n + 2\}$		
$\delta(v)$	$1 \le a \le \left\lfloor \frac{p_1}{2} \right\rfloor + 1$	$\left\lfloor \frac{p_1}{2} \right\rfloor + 2 \le a \le p_1$
$\delta(v_a^{(1)})$	$2a -`1$	$2p_1 + 4 - 2a$

Take $t = \sum_{c=1}^{b-1}(p_c + m_c)$. The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$ is given in Table 28.

Table 28. The point assignment $\delta\left(v_a^{(b)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$.

The point assignment δ				
$\delta(v)$	$2 \le b \le n$			
	$2 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor + 1$	$a = \left\lfloor \frac{p_b}{2} \right\rfloor + 2$		$\left\lfloor \frac{p_b}{2} \right\rfloor + 3 \le a \le p_b$
		p_b is odd	p_b is even	
$\delta(v_a^{(b)})$	$-b + t + 2a$	$-1 + t + 2a - b$	$t + 2a - b - 3$	$t + 2p_b - 2a - b + 5$

The point assignment $\delta\left(u_a^{(b-1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$ is given in Table 29.

Table 29. The point assignment $\delta\left(u_a^{(b-1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$.

The point assignment δ	
$\delta(v)$	$3 \le b \le n, 2 \le a \le m_b - 1$
$\delta(u_a^{(b-1)})$	$\sum_{c=1}^{b-2}(p_c + m_c) + p_{b-1} + a + 2 - b$

The line assignment $\delta^*\left(v_a^{(1)} v_{a+1}^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$ is given in Table 30.

Table 30. The line assignment $\delta^*\left(v_a^{(1)} v_{a+1}^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$.

Line Assignment δ^*				
$\delta^*(e)$	$1 \le a \le \left\lfloor \frac{p_1}{2} \right\rfloor$	$a = \left\lfloor \frac{p_1}{2} \right\rfloor + 1$		$\left\lfloor \frac{p_1}{2} \right\rfloor + 2 \le a \le p_1 - 1$
		p_1 is odd	p_1 is even	
$\delta^*(v_a^{(1)} v_{a+1}^{(1)}) = \left\lceil \frac{1}{e} \left(\frac{X(v_{a+1}^{(1)})}{X(v_a^{(1)})} \right)^{\frac{1}{Y(v_a^{(1)}, v_{a+1}^{(1)})}} \right\rceil$	$2a$			$2p_1 - 2a + 3$

The line assignment $\delta^*\left(u_a^{(1)} u_{a+1}^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p}_1, \mathrm{m}_1, \dots, \mathrm{m}_{\mathrm{n}-1}, \mathrm{p}_\mathrm{n})$ is given in Table 31.

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Table 31. The line assignment $\delta^*\left(u_a^{(1)}u_{a+1}^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$.

Line Assignment δ^*	
$\delta^*(e)$	$1 \le a \le m_1 - 1$
$\delta^*\left(u_a^{(1)}u_{a+1}^{(1)}\right)$	$p_1 + a + 1$

The line assignment $\delta^*\left(v_{p_1}^{(1)}v_1^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$ is given in Table 32.

Table 32. The line assignment $\delta^*\left(v_{p_1}^{(1)}v_1^{(1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$.

Line Assignment δ^*	
$\delta^*\left(v_{p_1}^{(1)}v_1^{(1)}\right)$	3

The line assignment $\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$ is given in Table 33.

Table 33. The line assignment $\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$.

Line Assignment δ^*				
$\delta^*(e)$	$2 \le b \le n$			
	$1 \le a \le \left\lfloor \frac{p_b}{2} \right\rfloor$	$a = \left\lfloor \frac{p_b}{2} \right\rfloor + 1$		$\left\lfloor \frac{p_b}{2} \right\rfloor + 2 \le a \le p_b - 1$
		p_b is odd	p_b is even	
$\delta^*\left(v_a^{(b)}v_{a+1}^{(b)}\right) = \left\lfloor \frac{1}{e}\left(\frac{X(v_{a+1}^{(b)})}{X(v_a^{(b)})}\right)^{\frac{1}{Y(v_a^{(b)},v_{a+1}^{(b)})}} \right\rfloor$	$t + 1 + 2a - b$			$2p_b - 2a - b + 4 + t$

The line assignment $\delta^*\left(v_{p_b}^{(b)}v_1^{(b)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$ is given in Table 34.

Table 34. The line assignment $\delta^*\left(v_{p_b}^{(b)}v_1^{(b)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$.

Line Assignment δ^*	
$\delta^*(e)$	$1 \le a \le m_1 - 1$
$\delta^*\left(v_{p_b}^{(b)}v_1^{(b)}\right)$	$t - b + 4$

The line assignment $\delta^*\left(u_a^{(b-1)}u_{a+1}^{(b-1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$ is given in Table 35.

Table 35. The line assignment $\delta^*\left(u_a^{(b-1)}u_{a+1}^{(b-1)}\right)$ of the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$.

Line Assignment δ^*	
$\delta^*(e)$	$2 \le b \le n, 1 \le a \le m_{b-1} - 1$
$\delta^*\left(u_a^{(b-1)}u_{a+1}^{(b-1)}\right) = \left\lfloor \frac{1}{e}\left(\frac{X(u_{a+1}^{(b-1)})}{X(u_a^{(b-1)})}\right)^{\frac{1}{Y(u_a^{(b-1)},u_{a+1}^{(b-1)})}} \right\rfloor$	$\sum_{c=1}^{b-2}(p_c + mc) + p_{b-1} + a + 3 - b$

An example of a representation of a C_{exp} average assignment is shown in Fig 11.

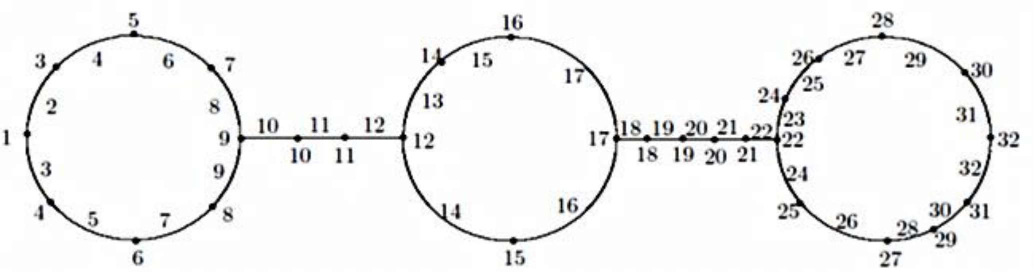

Figure 11. A C_{exp} average assignment of $\widehat{\mathrm{G}}(8, 4, 5, 6, 10)$.

Thus, the graph $\widehat{\mathrm{G}}(\mathrm{p_1}, \mathrm{m_1}, \dots, \mathrm{m_{n-1}}, \mathrm{p_n})$ is a C_{exp} average assignment graph for any $\mathrm{p_b}$'s and $\mathrm{m_b}$'s. □

Corollary 1: Tadpole $\mathrm{T(n, k)}$ and triangular snake graphs are C_{exp} average assignment graph.

Conclusion

This paper has discussed a novel approach to C_{exp} average assignments on chain graphs, leveraging the concept of chain graphs to efficiently compute average assignments. The approaches of C_{exp} average assignment of the cycle, the union of path and cycle, the union of T- graph and cycle, the graph G^*, the graph G', the graph $\widehat{G}$ and tadpole have been given. This labeling scheme aims to encode the structure and properties of the chain graph into the numerical values assigned to its vertices and edges. The assigned values should respect the structural constraints imposed by the chain graph, ensuring that the average values are consistent with the underlying dependencies among variables. By integrating techniques from graph theory, optimization, and statistical analysis, it was developed a method that offers significant advantages over existing approaches in terms of computational complexity and accuracy. By utilizing C_{exp} average assignment, researchers can effectively capture the characteristics of chain graphs and use them for various purposes, such as inference, optimization, or analysis of complex systems. This approach provides a powerful tool for modeling and understanding relationships in datasets where variables exhibit both direct and indirect dependencies. Our results demonstrate the effectiveness and scalability of the proposed approach across various datasets and scenarios. Moving forward, there are several avenues for future research. Further investigation could explore enhancements to the algorithm to handle larger datasets or to incorporate additional constraints. Examining the C_{exp} average assignment of several chain graphs would be quite interesting. The C_{exp} average assignment of other graph classes is still being open, and it will be done in the future. One can also look at the unique uses of a C_{exp} average assignment graph in real-world challenges.

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Authors’Declaration

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Furthermore, any Figures and images, that are not mine, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee of Mepco Schlenk Engineering College (Autonomous), Sivakasi-626005, Tamil Nadu, India.

Authors’ Contribution Statement

This work was carried out solely by A.R.K., who contributed to the design and implementation of the research, to the analysis of the results and the writing of the manuscript, Conceptualization, investigation, methodology, writing-original draft, review, and editing. Also, the author read and approved the final manuscript.

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نهج جديد لتخصيصات Cexp المتوسطة على الرسوم البيانية المتسلسلة

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الخلاصة

بشكل عام، ليس من الضروري أن يكون المتوسط الأسي لعددين موجبين عددًا صحيحًا. ولهذا السبب، يجب أن يكون المتوسط الأسي عددًا صحيحًا يأخذ في الاعتبار دالة الأرضية أو السقف. لقد تم تعريفها بحيث يمكن تسمية الرسوم البيانية بمتوسط أسي، حيث يمكن لدالة الأرضية أو دالة السقف تطبيقها على تسميات الحواف. لتأسيس تعيين المتوسط الأسي على الرسوم البيانية، سوف يتم وضع تسميات الحواف التي تنشأ من دالة السقف وحدها في الاعتبار. تُسمى دالة تعيين قمة الرأس δ ودالة تعيين الحافة δ^* بتخصيص متوسطC_{exp} للرسم البياني G مع رؤوس p وحواف q إذا كانت δ شاملة و δ^* متباينة وتكون العلاقات المكافئة

$\delta: V \to N - \{q+2, \cdots, \infty\}, \delta^*: E \to N - \{1, q+2, \cdots, \infty\}$ ويتم تعريفه بواسطة تسمية الحافة δ* كما يلي:

$\delta^*(uv) = \left[\frac{1}{e}\left(\frac{X(v)}{X(u)}\right)^{\frac{1}{Y(u,v)}}\right]$ ، حيث $X(u) = \delta(u)^{\delta(u)}, Y(u,v) = \delta(v) - \delta(u)$ و N هي مجموعة جميع الأعداد الطبيعية.

إذا كان الرسم البياني يقبل تعيين متوسطC_{exp} ، فإنه يطلق عليه رسم بياني لمتوسط تخصيص C_{exp}. يُقترح في هذه الورقة متوسط تخصيص الرسوم البيانية لـC_{exp} ، ويتم استكشاف خصائصه في الدورة، واتحاد المسار والدورة، واتحاد الرسم البياني T والدورة، والرسم البياني G*، والرسم البياني'G ، والرسم البياني Ĝ والشرغوف.

الكلمات المفتاحية:ك متوسط تخصيصC_{exp}، الرسم البياني لمتوسط تخصيصC_{exp}، الرسوم البيانية المتسلسلة، وضع العلامات على الحواف، وضع العلامات على الحواف.