

A New Algorithm for Finding Initial Basic Feasible Solution of Spherical Fuzzy Transportation Problem with Applications

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Abstract

In operation research, a specific area being analyzed in great depth is the transportation problem (TP). The key objective of this problem is to find the lowest total transportation costs for commodities to meet consumer requirements at destinations incorporating resources acquired at their points of origin. In this work, the spherical fuzzy transportation problem (SFTP) determines the lowest cost of carrying items from origin to destination. Most of the time, accurate data has been used, but these variables are actually inaccurate and ambiguous. According to the literature, several generalizations and expansions of fuzzy sets have been proposed and investigated. One of the most recent innovations in fuzzy sets is the spherical fuzzy sets (SFSs), which characterize not only membership and non-membership degrees but also neutral degrees. In this study, a novel approach is developed to derive the initial basic feasible solution (IBFS) for each of all three forms of the SFTP, and then obtain an optimal answer by applying the modified distribution (MODI) technique. For such frameworks, the proposed approach is illustrated by numerical examples. The conclusion and future scope are given at the end.

Keywords: Initial basic feasible solution, MODI method, Spherical fuzzy sets, Spherical fuzzy transportation problem, Transportation problem,

Introduction

Many real-world applications involve transportation-related issues. The objective of a transportation issue, which is a specific sort of linear programming problem (LPP), is to ascertain the best cost-effective and time-efficient method of delivering a commodity from a collection of sources to a set of desired locations¹. Fig 1 shows the network of transportation problems. A product's price is closely correlated with its transportation costs; that is, it will rise or fall in tandem with changes in transportation costs. For the same reason, a suitable way of delivering the product from different sources to different locations needs to be identified². It has long been believed that the transversal expenditures associated with supply and

demand should be expressed in exact quantities. However, these values are usually uncertain or nonspecific. The fuzzy set (FS) theory, which Zadeh³ created, reflects uncertain information by its membership level and can be used to manage data uncertainty in decision-making situations more effectively. A membership value and a non-membership value constitute the intuitionistic fuzzy set (IFS), which was first described by Atanassov⁴. As a result, experts have attempted to solve a variety of transportation-related issues in the fuzzy environment.

Fuzzy transportation problems were initially proposed by Chanas et al.⁵. Since then, multiple researchers have examined transportation problems in different fuzzy circumstances, such as triangular intuitionistic fuzzy⁶, fully intuitionistic multi-objective fractional⁷, fuzzy assignment⁸, fuzzy arithmetic data envelopment analysis⁹, interval-valued Intuitionistic fuzzy¹⁰, goal programming¹¹, efficient fuzzy goal programming¹², zero-point maximum allocation¹³, max-min average¹⁴, fuzzy zero-suffix¹⁵, irregular fuzzy variables¹⁶, modified Vogel's approximation¹⁷, stochastic fuzzy¹⁸, close interval approximation¹⁹, trisectional fuzzy²⁰, pareto-optimal²¹, type-2 fuzzy-random²², swarm optimization²³, fuzzy harmonic mean²⁴, fuzzy delphi²⁵ and so on. Although IFSs have vast applications in many fields, they cannot provide all the information²⁶. An instance when the total of membership and non-membership surpasses one might occur. An additional development of fuzzy concepts is pythagorean fuzzy sets (PyFS), which were suggested by Yager²⁷ as an effective extension of IFS. Both the membership level and the non-membership level, whose sum of squares is less than or equal to one, are further characteristics of PFS²⁸. PFS was later employed by other researchers to solve linear programming and multi-criteria decision-making challenges. Kumar et al.²⁹ provided two techniques for finding IBFS of pythagorean fuzzy transportation issues.

Neutral ratings, in addition to membership and non-membership levels, are often recommended in real-world situations. Fuzzy sets and IFS are not suitable for handling this type of unclear data. To get over this problem, Cuong³⁰ initially introduced the innovative idea of the picture fuzzy set (PFS). When a decision-maker is queried about a statement, the positive level is 0.6, the neutral level is 0.2, and the negative level is 0.1. A neutral function has been added to the picture FS development process, which provides a better solution to complicated situations. Using techniques such as similarity and distance metrics, among others, the idea of PFSs has been implemented to simulate a range of realistic decision-making issues³¹.

In actual life, some difficulties cannot be resolved with PFS, such as when $\mu_{P_F} + \eta_{P_F} + \nu_{P_F} > 1$. PFS

and PyFS are directly generalized into spherical fuzzy sets. An intriguing circumstance arose when the situation was beyond the capabilities of both PyFS and PFS. Spherical fuzzy sets are useful when opinion is not limited to yes or no but also includes some abstinence or rejection. A representative instance of a spherical fuzzy set is frequently encountered in decision-making processes, such as those in which four decision-makers evaluate candidates according to four distinct categories. An additional instance could involve the voting process, where four categories of voters exist: those who vote in favor, against, do not vote, or abstain from voting. The spherical fuzzy set is therefore required to address this circumstance³². As a further development of PFS, Ashraf et al.³³ present the concept of SFSs based on these situations. Membership degrees are improving the situation in SFS with the condition $0 < \mu_{S_F}^2 + \eta_{S_F}^2 + \nu_{S_F}^2 < 1$. They examined the fundamental spherical fuzzy set operations in their suggested work and used those aggregating operators to create multi-attribute decision-making challenges. By employing spherical fuzzy prioritized weighted aggregating operators, Akram et al.³⁴ developed an approach for resolving group decision-making with multiple criteria challenges. Using the idea of a spherical fuzzy difference, Garg and Sharaf³⁵ presented a new spherical aggregation algorithm. Spherical fuzzy information-based outranking algorithms were combined by Akram et al.³⁶ for the digitization of the transit system. In the sense that threshold values were used to observe the outranking relationships among the options before making the decision. A unique method for selecting an advanced manufacturing setup that integrates AHP and TOPSIS underneath spherical fuzzy concepts was presented by Mathew et al.³⁷. To address various criterion group decision-making difficulties, Donyatalab et al.³⁸ expanded the traditional linear assignment approach to a spherical fuzzy linear assignment method. The physician selection difficulty was addressed by Sarucan et al.³⁹ using the spherical fuzzy TOPSIS approach. To aid in decision-making, Ajay et al.⁴⁰ established new exponential as well as Einstein exponential operational laws for spherical fuzzy collections, along with the matching aggregation operators. Similarity metrics of spherical uncertain concepts were derived by Wei et al.⁴¹ and utilized for pattern

recognition and medical diagnostics using the cosine function. A spherical fuzzy transportation issue was studied by Kumar et al.⁴² by employing three models.

As previously discussed, limited study has been done on the spherical fuzzy transportation problem. Additionally, to solve the spherical fuzzy transportation problem, the researchers employed software and vogel's approximation method (VAM). According to the literature review, there are no unique methods for solving SFTP. This motivates the authors to devise a novel method for deriving the IFBS of SFTP and its optimal value without any mathematical tools. The major purpose of this research is to reduce total transportation expenses. The following is the paper's key contribution: Without utilizing any mathematical tools,

- (i) A novel algorithm for determining the IBFS of three types (I, II, and III) is introduced.
- (ii) Applied random values of the three types to the suggested algorithm to validate it.
- (iii) Both balanced and unbalanced issues are validated using the suggested technique.

The scheduled phases for this research are as follows: The following section reports the preliminary findings and related mathematical operations; Section 3 lays the groundwork for the SFTP's mathematical structure. Section 4 offers an approach to obtaining IBFS, and Practical instances are provided in Section 5. Section 6 contains the findings and discussion. And the conclusion and future study are discussed in Section 7.

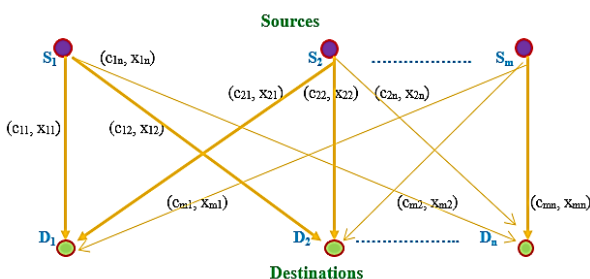


Figure 1. Simplified form of the transportation problem.

Preliminaries

The terminology and score function used in the study are presented in this section. \bar{U} denotes the universal set throughout this study.

Picture Fuzzy Set (PFS)²⁶

A PFS P_F is defined by

$$P_F = \{ \langle u_0, \mu_{P_F}(u_0), \eta_{P_F}(u_0), \nu_{P_F}(u_0) \rangle : u_0 \in \bar{U} \}$$

where $\mu_{P_F}(u_0) \in [0,1]$ is the level of membership in truth, $\eta_{P_F}(u_0) \in [0,1]$ is the level of membership in neutral and $\nu_{P_F}(u_0) \in [0,1]$ is the level of membership in false provided that

$$0 \leq \mu_{P_F}(u_0) + \eta_{P_F}(u_0) + \nu_{P_F}(u_0) \leq 1.$$

Spherical Fuzzy Set (SFS)³⁴

A SFS S_F is defined by

$$S_F = \{ \langle u_0, \mu_{S_F}(u_0), \eta_{S_F}(u_0), \nu_{S_F}(u_0) \rangle : u_0 \in \bar{U} \}$$

where $\mu_{S_F}(u_0) \in [0,1]$ is the level of membership in truth, $\eta_{S_F}(u_0) \in [0,1]$ is the level of membership in neutral and $\nu_{S_F}(u_0) \in [0,1]$ is the level of membership in false provided that

$$0 \leq \mu_{S_F}^2(u_0) + \eta_{S_F}^2(u_0) + \nu_{S_F}^2(u_0) \leq 1.$$

Arithmetic Operations³⁴

Let $A_{S_F} = (\mu_{S_{F_1}}, \eta_{S_{F_1}}, \nu_{S_{F_1}})$ and $B_{S_F} = (\mu_{S_{F_2}}, \eta_{S_{F_2}}, \nu_{S_{F_2}})$ be two SFSs. The following Eqs.1-4 describes the fundamental operations of SFSs.

$$A_{S_F} \oplus B_{S_F} = \left\{ \left(\mu_{S_{F_1}}^2 + \mu_{S_{F_2}}^2 - \mu_{S_{F_1}}^2 \mu_{S_{F_2}}^2 \right)^{\frac{1}{2}}, \eta_{S_{F_1}} \eta_{S_{F_2}}, \left((1 - \mu_{S_{F_2}}^2) \nu_{S_{F_1}}^2 + (1 - \mu_{S_{F_1}}^2) \nu_{S_{F_2}}^2 - \nu_{S_{F_1}}^2 \nu_{S_{F_2}}^2 \right)^{\frac{1}{2}} \right\} \quad 1$$

$$A_{S_F} \otimes B_{S_F} = \left\{ \mu_{S_{F_1}} \mu_{S_{F_2}}, \left(\eta_{S_{F_1}}^2 + \eta_{S_{F_2}}^2 - \eta_{S_{F_1}}^2 \eta_{S_{F_2}}^2 \right)^{\frac{1}{2}}, \left((1 - \eta_{S_{F_2}}^2) v_{S_{F_1}}^2 + (1 - \eta_{S_{F_1}}^2) v_{S_{F_2}}^2 - v_{S_{F_1}}^2 v_{S_{F_2}}^2 \right)^{\frac{1}{2}} \right\} \quad 2$$

$$\lambda \cdot A_{S_F} = \left\{ \left(1 - \left(1 - \mu_{A_{S_{F_1}}}^2 \right)^\lambda \right)^{\frac{1}{2}}, \eta_{S_{F_1}}^\lambda, \left((1 - \mu_{A_{S_{F_1}}}^2)^\lambda - \left(1 - \mu_{A_{S_{F_1}}}^2 - v_{A_{S_{F_1}}}^2 \right)^\lambda \right)^{\frac{1}{2}} \right\} \quad 3$$

$$A_{S_F}^\lambda = \left\{ \mu_{S_{F_1}}^\lambda, \left(1 - \left(1 - \eta_{A_{S_{F_1}}}^2 \right)^\lambda \right)^{\frac{1}{2}}, \left((1 - \eta_{A_{S_{F_1}}}^2)^\lambda - \left(1 - \eta_{A_{S_{F_1}}}^2 - v_{A_{S_{F_1}}}^2 \right)^\lambda \right)^{\frac{1}{2}} \right\}, \lambda > 0 \quad 4$$

Score and Accuracy Functions³⁵

Let S_F be the SFS. The definitions of the accuracy and score functions are as follows:

$$\text{Score}(S_F) = (\mu_{S_F} - \eta_{S_F})^2 - (v_{S_F} - \eta_{S_F})^2, S(S_F) \in [-1, 1] \quad 5$$

$$\text{Accuracy}(S_F) = \mu_{S_F}^2 + \eta_{S_F}^2 + v_{S_F}^2, \text{Acc.}(S_F) \in [0, 1]$$

A relationship of order between two SFSs S_{F_1} and S_{F_2} is stated as

$S_{F_1} < S_{F_2}$ iff $\text{Score}(S_{F_1}) < \text{Score}(S_{F_2})$ or $\text{Score}(S_{F_1}) = \text{Score}(S_{F_2})$ and $\text{Accuracy}(S_{F_1}) < \text{Accuracy}(S_{F_2})$

Assume, that A = (0.46, 0.48, 0.36) and B = (0.57, 0.35, 0.36) be two spherical fuzzy numbers³⁵. Using the score and accuracy function in (5), If $\text{Score}(A) =$

-0.014 and $\text{Score}(B) = 0.0483$ then $\text{Accuracy}(A) < \text{Accuracy}(B)$.

Mathematical Structure of Spherical Fuzzy Transportation Problem

Consider "m" providers and "n" spots. The distribution network tries to reduce the expense of moving things from those providers to the spots; meanwhile, the accessibility and demand for items are specified using a few presumptions and constraints. The mathematical expressions for a spherical fuzzy TP are as follows in Eqs 6-9 and Table 1:

i - entire source index for m

j - entire destination index for n

x_{ij} - amount of goods transported in units from the point of origin to the destination

$$\text{Minimize } \bar{z}^{S_F} = \sum_{i=1}^m \sum_{j=1}^n \bar{c}_{ij}^{S_F} \cdot x_{ij} \quad 6$$

Subject to constraints,

$$\sum_{j=1}^n x_{ij} = \bar{a}_i^{S_F}, i = 1 \text{ to } m, \quad 7$$

$$\sum_{i=1}^m x_{ij} = \bar{b}_j^{S_F}, j = 1 \text{ to } n, \quad 8$$

$$x_{ij} \geq 0 \text{ for each } i, j \quad 9$$

where,

$\bar{c}_{ij}^{S_F}$ - spherical fuzzy expense of moving one unit of a given good supplier i to recipient j ,

$\bar{a}_i^{S_F}$ - spherical fuzzy units of supply to be carried between n places,

$\bar{b}_j^{S_F}$ - spherical fuzzy number of demand units needed at endpoints.

Table 1. Spherical fuzzy transportation setting.

		<i>Destinations</i>				
		D_1	D_2	D_n	Supply
<i>Sources</i>	s_1	c_{11}^{SF}	c_{12}^{SF}	c_{1n}^{SF}	a_1^{SF}
	s_2	c_{21}^{SF}	c_{22}^{SF}	c_{2n}^{SF}	a_2^{SF}
	⋮	⋮	⋮	⋮	⋮	⋮
	s_m	c_{m1}^{SF}	c_{m2}^{SF}	c_{mn}^{SF}	a_m^{SF}
Demand		b_1^{SF}	b_2^{SF}	b_n^{SF}	

Proposed Algorithm for Solving Spherical Fuzzy Transportation Problem

The following is a description of the suggested algorithm's steps.

Step 1: Under the Spherical fuzzy environment, choose the transportation problem.

Step 2: Spherical fuzzy values should be converted into crisp values using the recommended scoring function in Eq 5.

Step 3: Determine whether the problem at hand is balanced or not after crisping them.

- (i) If the given problem is balanced, proceed to step 5.
- (ii) If the given problem is unbalanced, proceed to step 4

Step 4: Add a dummy row or column to balance total demand and supply.

Step 5: Select the highest cost from the cost table.

Step 6: To every value in the cost table, add the maximum cost value.

Step 7: Calculate the penalty by subtracting the smallest from the next smallest in each row and column.

Step 8: To the corresponding row penalty, add the corresponding supply value, and to the

corresponding column penalty, add the corresponding demand value.

Step 9: Among the rows and columns, find the maximum penalty. Select anyone if it appears more than once.

Step 10: Choose the lowest cost value in the relevant row or column of the maximum penalty.

Step 11: Allocate as much as possible among the supply and demand for the chosen cell.

Step 12: When there is no demand or supply to assign, delete the entire row or column.

Step 13: Steps 5-12 must be repeated until all units of supply and demand are met.

Step 14: After determining IBFS, adopt the MODI approach to get an optimal solution.

Numerical examples

This section presents the suggested algorithm using six problems representing three different models.

Existing Data

Example 1: To determine the minimal transportation expense, the costs in Table 2 of SFTP for model I are stated in spherical fuzzy, while the supply and demand are expressed in crisp.

Step 1: Under the Spherical fuzzy environment, choose the transportation problem.



Table 2. Data for SFTP of model I⁴²

	A ₁	A ₂	A ₃	A ₄	Availability
Television	(0.9,0.1,0.1)	(0.6,0.4,0.4)	(0.91,.03,.02)	(0.99,.05,.02)	26
Air Cooler	(0.89,.08,.03)	(0.74,.16,.1)	(0.5,0.5,0.5)	(0.7,0.3,0.3)	24
Geyser	(0.99,.05,.02)	(0.73,.15,.08)	(0.73,.12,.08)	(0.68,.26,.06)	30
Requirement	17	23	28	12	80

Step 2: Spherical fuzzy values should be converted into crisp values using the recommended ranking function that was represented in Eq. 1.

Step 3: For Table 3, as the chosen problem is balanced, there is no need for an additional dummy row or column.

Table 3. Defuzzified values⁴²

	A ₁	A ₂	A ₃	A ₄	Availability
Television	0.64	0.04	0.792	0.94	26
Air Cooler	0.73	0.40	0	0.16	24
Geyser	0.94	0.41	0.42	0.34	30
Requirement	17	23	28	12	80

Step 4: Select the highest cost from the cost table from Table 3.

Step 5: To every value in the cost table, add the maximum cost value. Table 4 illustrates this.

Table 4. Added maximum cost value

	A ₁	A ₂	A ₃	A ₄	Availability
Television	1.58	0.98	1.732	1.88	26
Air Cooler	1.67	1.34	0.94	1.1	24
Geyser	1.88	1.35	1.36	1.28	30
Requirement	17	23	28	12	

Step 6: Calculate the penalty by subtracting the smallest from the next smallest in each row and column shown in Table 5.

Table 5. Row and Column penalties

	A ₁	A ₂	A ₃	A ₄	Availability	Row Penalty
Television	1.58	0.98	1.732	1.88	26	0.6
Air Cooler	1.67	1.34	0.94	1.1	24	0.16
Geyser	1.88	1.35	1.36	1.28	30	0.07
Requirement	17	23	28	12		
Column Penalty	0.09	0.36	0.42	0.18		

Step 7: To the corresponding row penalty, add the corresponding supply value, and to the corresponding column penalty, add the

corresponding demand value. Actual penalties are displayed in Table 6.

Table 6. Actual penalties

	A ₁	A ₂	A ₃	A ₄	Availability	Row Penalty
Television	1.58	0.98	1.732	1.88	26	26.6
Air Cooler	1.67	1.34	0.94	1.1	24	24.16
Geyser	1.88	1.35	1.36	1.28	30	30.07
Requirement	17	23	28	12		
Column Penalty	17.09	23.36	28.42	12.18		

Step 8: Among the rows and columns, find the maximum penalty in Table 6.

Step 10: Allocate as much as possible among the supply and demand for the chosen cell in Table 7.

Step 9: Choose the lowest cost value in the relevant row or column of the maximum penalty.

Table 7. First allocation

	A ₁	A ₂	A ₃	A ₄	Availability	Row Penalty
Television	1.58	0.98	1.732	1.88	26	26.6
Air Cooler	1.67	1.34	0.94	1.1	24	24.16
Geyser	1.88	1.35	1.36	1.28 ¹²	30	30.07
Requirement	17	23	28	12		
Column Penalty	17.09	23.36	28.42	12.18		

Step 11: When there is no demand or supply to assign, delete the entire row or column. The adjusted matrix of transportation costs is shown in Table 8.

Table 8. Second allocation

	A ₁	A ₂	A ₃	Availability	Row Penalty
Television	1.58	0.98	1.732	26	26.6
Air Cooler	1.67	1.34	0.94 ²⁴	24	24.4
Geyser	1.88	1.35	1.36	18	18.01
Requirement	17	23	28		
Column Penalty	17.09	23.36	28.42		

Step 12: Repeat the previous steps until all allocations are met. The processes are repeated and are displayed in Tables 9 and 10.

Table 9. Third allocation

	A ₁	A ₂	A ₃	Availability	Row Penalty
Television	1.58	0.98 ²³	1.732	26	26.6
Geyser	1.88	1.35	1.36	18	18.01
Requirement	17	23	4		
Column Penalty	17.3	23.37	4.372		

Table 10. Final allocations

	A ₁	A ₃	Availability	Row Penalty
Television	1.58 ³	1.732	3	3.152
Geyser	1.88 ¹⁴	1.36 ⁴	18	18.52
Requirement	17	4		
Column Penalty	17.3	4.372		

The allocations made with the suggested approach are displayed in Tables 7-10. Moreover, the allocations are shown as superscripts in Table 7-10. The optimal value becomes 21.76 after proceeding with the modified distribution method.

Example 2: To determine the minimal transportation expense, consider the SFTP Table 11 for model II, where the supply and demand are represented in spherical fuzzy form, but the expenses are expressed in crisp form.

Table 11. Data for SFTP of model II ⁴²

	L ₁	L ₂	L ₃	L ₄	Availability
Water Purifier	0.64	0.04	0.792	0.94	(0.9,0.1,0.1)
Dishwasher	0.73	0.4	0	0.16	(0.89,.08,.03)
Air Fryer	0.94	0.41	0.42	0.34	(0.99,.05,.02)
Requirement	(0.9,0.1,0.1)	(0.6,0.4,0.4)	(0.91,.03,.02)	(0.99,.05,.02)	

Obtained IBFS using the proposed approach, and the optimal solution becomes 0.7052.

Example 3: To determine the minimal transportation expense, consider the SFTP Table 12 for model III, where the costs, demand, and supply are all spherical fuzzy.

Table 12. Data for SFTP of model III ⁴²

	B ₁	B ₂	B ₃	B ₄	Availability
Vacuum Cleaner	(0.61,.46,.34)	(0.74,.27,.28)	(0.7,0.3,0.3)	(0.62,.39,.39)	(0.9,0.1,0.1)
Refrigerator	(0.81,0.2,.23)	(0.55,.47,.43)	(0.5,0.5,0.5)	(0.7,0.3,0.3)	(0.89,.08,.03)
Chimney	(0.99,.05,.02)	(0.73,.15,.08)	(0.73,.12,.08)	(0.68,.26,.06)	(0.99,.05,.02)
Requirement	(0.9,0.1,0.1)	(0.6,0.4,0.4)	(0.91,.03,.02)	(0.99,.05,.02)	

Obtained IBFS using the proposed approach, and the optimal solution becomes 0.4307.

Example 4: To determine the minimal transportation expense, consider the following random values from SFTP of Table 13 for model I, where the costs are spherical fuzzy and the demand and supply are crisp.

Random Data

Table 13. Random values for SFTP of model I

	1	2	3	4	Availability
M	(0.7,0.3,0.3)	(0.87,.34,.52)	(0.5,0.1,0.3)	(0.74,.16,0.1)	14
N	(0.6,0.4,0.4)	(0.49,.09,.18)	(0.63,.21,.48)	(0.5,0.5,0.5)	18
O	(0.4,.09,.17)	(0.8,0.2,0.6)	(0.73,.15,.08)	(0.68,.26,.06)	20
Requirement	10	16	19	17	

Obtained IBFS using the proposed approach, and the optimal solution becomes 4.674.

SFTP of Table 14 for model II in which the costs are expressed in crisp form whereas the demand and supply are in spherical fuzzy.

Example 5: To determine the minimal transportation expense, consider the following random values from

Table 14. Random values for SFTP of model II

	Q ₁	Q ₂	Q ₃	Q ₄	Availability
V ₁	(0.7,0.3,0.3)	(0.87,.34,.52)	(0.5,0.1,0.3)	(0.74,.16,0.1)	(0.72,.25,.36)
V ₂	(0.6,0.4,0.4)	(0.49,.09,.18)	(0.63,.21,.48)	(0.5,0.5,0.5)	(0.62,.17,.25)
V ₃	(0.4,.09,.17)	(0.8,0.2,0.6)	(0.73,.15,.08)	(0.68,.26,.06)	(0.8,0.2,0.1)
Requirement	(0.5,.32,.42)	(0.89,.08,.03)	(0.9,0.1,0.3)	(0.6,.05,.02)	

Obtained IBFS using the proposed approach, and the optimal solution becomes 0.0926.

Example 6: To determine the minimal transportation expense, consider the following random values from SFTP of Table 15 for model III in which the costs, demand, and supply are in spherical fuzzy.

Table 15. Random values for SFTP of model III

	P ₁	P ₂	P ₃	P ₄	Availability
1	(0.6,0.4,0.5)	(0.5,0.2,0.3)	(0.8,0.4,0.4)	(0.73,.15,.08)	(0.83,.23,.33)
2	(0.9,0.3,0.5)	(0.7,.27,.45)	(0.89,.35,.49)	(0.55,.47,.43)	(0.9,0.3,0.5)
3	(0.78,.17,.34)	(0.65,.04,.19)	(0.68,.42,.44)	(0.99,.05,.02)	(0.8,0.1,0.3)
Requirement	(0.53,.15,.26)	(0.67,.09,.18)	(0.92,.37,.56)	(0.9,0.1,0.1)	

Obtained IBFS using the proposed approach, and the optimal solution becomes 0.1136.

Results and Discussion

This paper presents a novel algorithm to determine the IBFS of SFTP. It is evident from Table 16 and Fig 2 that the suggested IBFS method yields superior outcomes. To verify the correctness of the proposed technique, random problems are taken into account for each of the three types, along with the existing data. The three SFTP models in the available data⁴² are balanced. On the other hand, the random data used in this study are imbalanced problems from each of the three models to test the suggested technique. The suggested approach produces similar results to the traditional approach for both balanced and unbalanced issues. In most transportation

situations, vogel's approximation method is considered traditional and effective for cost optimization. In this paper, based on a comparison investigation, the suggested algorithm gives better results than VAM and, in certain situations, produces the same outcomes as VAM. There were no difficulties found in the intended investigation when the suggested method was demonstrated on the mathematical instances of three distinct models. The purpose of this study is to reduce total transportation costs in a spherical fuzzy environment. Thus, achieved the objective of the transportation problem by the proposed algorithm.

Table 16. Comparison of results with existing method

Examples	Types	VAM	Proposed IBFS	Optimum
Existing data				
1	I	21.76	21.76	21.76
2	II	0.7292	0.7146	0.7052
3	III	0.4308	0.4307	0.4307
Random data				
4	I	5.99	5.32	4.674
5	II	0.1539	0.1539	0.0926
6	III	0.1168	0.1168	0.1136

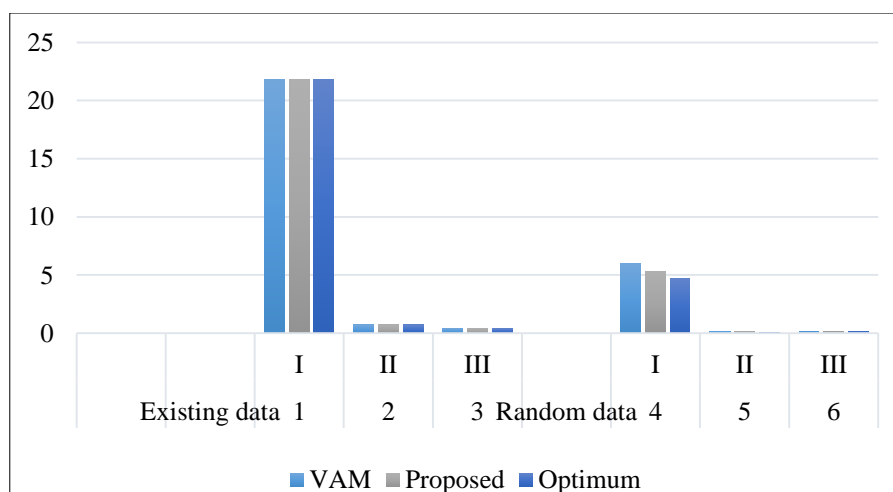


Figure 2. Comparison graph of the proposed and existing method.

Conclusion

In today's highly competitive market, institutions are under increasing pressure to identify better means for distributing commodities to clients. As a result, various institutions strive to deliver goods to consumers in the most cost-effective or time-efficient manner, and the transportation model offers a strong foundation to address this issue. This article uses spherical fuzzy sets to quantify inaccurate, partial, and vague information. This work has created a novel algorithm for the IBFS of SFTP. Since SFSs are more important in characterizing uncertain information. The score function has been utilized in these models to transform the ambiguous data into a clear transportation problem. The aforementioned

method was demonstrated in six numerical examples, and the goals of the intended study were satisfied while no weaknesses in the technique were found. The effectiveness of the proposed algorithms was determined by examining the existing methods. This illustrates the usefulness and effectiveness of our proposed algorithm. The suggested algorithm offers a fresh approach to dealing with uncertainty in real-world transportation issues. Assignment and bi-objective transportation problems cannot be solved with this proposed work. Fuzzy bi-objective transportation problems will be solved by expanding this work in the future.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, Vellore, India.

Authors' Contribution Statement

KH wrote the manuscript. VB wrote and edited the manuscript with a revision idea. All authors read and approved the final manuscript.

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خوارزمية جديدة لإيجاد حل أولي أساسي وممكن لمشكلة النقل الكروي الضبابي مع تطبيقات

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الخلاصة

في بحوث العمليات، هناك منطقة معينة يتم تحليلها بعمق كبير وهي مشكلة النقل (TP). الهدف الرئيسي لهذه المشكلة هو العثور على أقل تكاليف نقل إجمالية للسلع لتلبية متطلبات المستهلك في الجهات التي تتضمن الموارد المكتسبة في نقاطها الأصلية. في هذا العمل، تحدد مشكلة النقل الضبابي الكروي (SFTP) أقل تكلفة لنقل العناصر من الأصل إلى الوجهة. وفي معظم الأحيان، يتم استخدام بيانات دقيقة، ولكن هذه المتغيرات في الواقع غير دقيقة وغامضة. وفقاً للمصادر، تم اقتراح وحساب العديد من التعميمات والتوسعات للمجموعات الضبابية. واحدة من أحدث الابتكارات في المجموعات الضبابية هي المجموعات الكروية الضبابية (SFSS)، والتي تتميز ليس فقط درجات العضوية وغير العضوية ولكن أيضاً الدرجات المحايدة. في هذه الدراسة، تم تطوير نهج جديد لاستخلاص الحل الأساسي الأولي الممكن (IBFS) لكل من الأشكال الثلاثة لـ SFTP، ومن ثم الحصول على الإجابة المثلى من خلال تطبيق تقنية التوزيع المعدل (MODI). بالنسبة لمثل هذه الأطر، يتم توضيح النهج المقترح من خلال الأمثلة العددية. في النهاية تم إعطاء الاستنتاج والمقترحات للعمل المستقبلي.

الكلمات المفتاحية: الحل الأساسي الأولي الممكن، طريقة MODI، المجموعات الغامضة الكروية، مشكلة النقل الكروية الغامضة، مشكلة النقل.