Published Online First: November, 2024
https://doi.org/10.21123/bsj.2024.11424
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

On En- Prime Compactly Packed Acts over Monoid

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Received 21/04/2024, Revised 04/10/2024, Accepted 06/10/2024, Published Online First 20/11/2024

Abstract

This work studies the concept of En-prime compactly packed $(En-\mathcal{P}.c.\mathcal{P})$ modules. Some properties and characterizations have been studied. Put ₩ is an Я-module and every submodule is En-pure , then ₩ is $En-\mathcal{P}.c.\mathcal{P}$ if and only if each proper submodule ῳ of ₩ is cyclic, If ₩ is $En-\mathcal{P}.c.\mathcal{P}$. ₩ which has at least one maximal submodule then ₩ satisfies the ACC on En-p-radical submodule. The generalization of this idea has been given for S-Acts. if for each family $\{P_\alpha\}_{\alpha\in\lambda}$ of En-prime subact of Д with Ӂ $\subseteq \bigcup_{\alpha\in\lambda} P_\alpha$, Ӂ $\subseteq P_\beta$ for some $\beta \in \lambda$. An S-Act Д is $En-\mathcal{P}.c.\mathcal{P}$, if every subact is $En-\mathcal{P}.c.\mathcal{P}$. Various properties of $En-\mathcal{P}.c.\mathcal{P}$ modules and S-Acts have been studied, like, ₩ is an R-module and every submodule is En-pure, then ₩ is $En-\mathcal{P}.c.\mathcal{P}$ if and only if each proper submodule ῳ of ₩ is cyclic. The general is, if Д is $En-\mathcal{P}.c.\mathcal{P}$ S-Act which has at least one maximal subact then Д satisfies the ACC on En-p-radical subact.and suppose that Д is an $En-\mathcal{P}.c.\mathcal{P}$ S-Act. If the CST is satisfied for Д, then dim Д ≤ 1, and prove that, If Д is a multiplication S-Act that satisfies the ACC on En-p- radical subact, then for every proper subact Ӂ of Д there exists a finite number of minimal E_n-prime subact of Ӂ. Let f: Д → Д′ be an epimorphism. If Д is $En-\mathcal{P}.c.\mathcal{P}$ then so is Д′. The converse is true when Д is finitely generated or (multiplication) S-Act and ker f ⊆ rad{0}.

Keywords: En- Prime subacts, En-prime submodules, En-Pure subacts, En-prime compactly packed S-Act, Multiplication S-act.

Introduction

Let R be a commutative ring with 1 and let ₩ be a unitary R-module. An ideal I of R is said to be compactly packed if for each family $\{P_\alpha\}_{\alpha\in\lambda}$ of prime ideals with $I \subseteq \bigcup_{\alpha\in\lambda} P_\alpha$, there exists $\beta \in \lambda$, such that $I \subseteq P_\beta$. In [1] a ring Я in which every ideal is compactly packed is said to be compactly packed rings. A proper submodule ῳ of module ₩ is said to be En-prime if $f(x)R \in$ ῳ implies that either $x \in$ ₩ or f(₩) ⊆ ῳ, [2] An ideal I of Я is said to be $c.\mathcal{P}$ if for every family $\{P_\alpha\}_{\alpha\in\lambda}$ of prime ideals with $I \subseteq \bigcup_{\alpha\in\lambda} P_\alpha$, there is $\beta \in \lambda$, such that $I \subseteq P_\beta$. Thus we say that a proper submodule ῳ of ₩ is prime compactly packed if for each family $\{P_\alpha\}_{\alpha\in\lambda}$ of prime submodule of ₩ with ῳ $\subseteq \bigcup_{\alpha\in\lambda} P_\alpha$, ῳ $\subseteq P_\beta$ for some $\beta \in \lambda$ [3]. Generalize the concept of $En-\mathcal{P}.c.\mathcal{P}$ modules to $En-\mathcal{P}.c.\mathcal{P}.S.A$.

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Results and discussion

En-prime Compactly Packed R-modules.

Definition 1: A proper submodule Ꞷ of R-module ₩ is said to be $En-\mathcal{P}.c.\mathcal{P}$ if whenever Ꞷ is contained in the union of a family of En-prime subact of ₩, then Ꞷ is included in one of the members of the family. And ₩ is $En-\mathcal{P}.c.\mathcal{P}$ R-module if every proper submodule of ₩ is $En-\mathcal{P}.c.\mathcal{P}$.

Let Ꞷ be a submodule of an R- module ₩, if there exists En-prime submodule that contains Ꞷ, then the intersection of all En-prime submodule containing Ꞷ is called the En-p- radical of Ꞷ and denoted by En-p- rad(Ꞷ). If there is no En-prime submodule containing Ꞷ, then En-p-rad (Ꞷ) = ₩. A submodule Ꞷ is called an En-p- radical submodule if En-p-rad(Ꞷ) = Ꞷ [4].

Theorem 1: Let ₩ be an R-module. The following statements are equivalent:

1-₩ is $En-\mathcal{P}.c.\mathcal{P}$.

2-For each proper submodule Ꞷ of ₩, there exists a∈ Ꞷ such that En-p-rad(Ꞷ) = En-p-rad(Ra).

3-For each proper submodule Ꞷ of ₩, if $\{ Ꞷ_\alpha \}_{(\alpha\in\lambda)}$ is a family of submodules of ₩ and Ꞷ $\subseteq \bigcup_{(\alpha\in\lambda)} Ꞷ_\alpha$ then Ꞷ ⊆ En-p-rad($Ꞷ_\beta$) for some β∈λ.

4-For each proper subact Ꞷ of ₩, if $\{ Ꞷ_\alpha \}_{(\alpha\in\lambda)}$ is a family of En-p-radical submodule of ₩ and Ꞷ $\subseteq \bigcup_{(\alpha\in\lambda)} Ꞷ_\alpha$ then Ꞷ ⊆ $Ꞷ_\beta$ for some β∈λ.

Proof: (1→2) Let Ꞷ be a proper submodule of ₩. Suppose En-p-rad Ꞷ ⊄ En-p-rad(Ra) for each a∈ Ꞷ, there exists an En-prime submodule P_a which contains Ra and Ꞷ ⊄P_a. But Ꞷ = $\bigcup_{(a\in\omega)} Ra \subseteq \bigcup_{(a\in\omega)} P_a$, that is ₩ is not $En-\mathcal{P}.c.\mathcal{P}$ which contradicts (a).

(2→3) Let Ꞷ be a proper submodule of ₩ and let $\{ Ꞷ_\alpha : (\alpha\in\lambda)\}$ be a family of submodule of ₩ such that Ꞷ ⊆ $\bigcup_{(\alpha\in\lambda)} Ꞷ_\alpha$. By (b) there exists a∈ Ꞷ such that En-p-rad(Ꞷ) = En-p-rad(Ra). Then a∈$\bigcup_{(\alpha\in\lambda)} Ꞷ_\alpha$ and hence a∈ $Ꞷ_\beta$ for some β∈λ, so that Ra ⊆ $Ꞷ_\beta$ and Ꞷ ⊆ En-p-rad(Ꞷ) = En-p-rad(Ra) ⊆ En-p-rad($Ꞷ_\beta$)

(3→4) & (4→1) are clear.

Proposition 1: Put ₩ is an R-module and every submodule is En-pure, then ₩ is $En-p.c.\mathcal{P}$ if and only if each proper submodule Ꞷ of ₩ is cyclic.

Proof: The sufficiency is clear. To prove the necessity, let Ꞷ be a proper submodule of ₩. Since ₩ is $En-\mathcal{P}.c.\mathcal{P}$ then by theorem 1, there exists a ∈ Ꞷ such that En-p- rad(Ꞷ) = En − p − rad(Ra) . But every submodule is En-pure, Ꞷ = Ra.

Put ₩ is module. A submodule Ꞷ of ₩ is said to be En-pure in ₩ if for every endomorphism f, Ꞷ∩f(₩) = f(Ꞷ)[4]

Theorem 2: If ₩ is $En-\mathcal{P}.c.\mathcal{P}$ R-module which has at least one maximal submodule then ₩ satisfies the ACC on En-p-radical submodule.

Proof: let $Ꞷ_1 \subseteq Ꞷ_2 \subseteq \cdots$ be an ascending chain of En-p-radical submodule of ₩ and let $L = \bigcup_i Ꞷ_i$. If L = ₩ and Á is a maximal submodule of ₩, then Á ⊊ $\bigcup_i Ꞷ_i$. Since ₩ is $En-p.c.\mathcal{P}$ then Á ⊆ $Ꞷ_j$ for some j. Therefore Á ⊆ $Ꞷ_j$ and therefore $\bigcup_i Ꞷ_i \subseteq Ꞷ_j$, that is ₩ ⊆ $Ꞷ_j$ which is impossible. Thus L is a proper submodule of ₩. Thus L ⊆ $₩_j$ for some j and therefore $Ꞷ_1 \subseteq Ꞷ_2 \subseteq \cdots \subseteq Ꞷ_j = Ꞷ_{j+1} = Ꞷ_{j+2} = \cdots$, thus the ACC is satisfied for En-p-radical submodule.

Recall that a module ₩ is called a multiplication module if each submodule Ꞷ of ₩ has the form Ꞷ =I₩ for an ideal I of R. In fact Ꞷ = [Ꞷ: ₩]₩ [4]. Because every finitely generated module and every multiplication module has a proper maximal submodule [3] then the directly by theorem 2, the proof of the following result have been found:-

Corollary 1: If ₩ is finitely generated or multiplication $En-\mathcal{P}.c.\mathcal{P}$ module, then ₩ satisfies the ACC on En-prime radical submodule**.**

En-Prime Compactly Packed S-Acts ($En-\mathcal{P}.c.\mathcal{P}.S.A$)

Definition 2: A proper subact Ӿ of S-act Д is said to be $En-\mathcal{P}.c.\mathcal{P}.S.A$ if whenever Ӿ is contained in the union of a family of En-prime subact of Д, then Ӿ is included in one of the members of the family. And Д is $En-\mathcal{P}.c.\mathcal{P}$ S-act, if every proper subact of Д is $En-\mathcal{P}.c.\mathcal{P}.S.A$.

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Let Ӂ be a subact of an S-Act Д, if there exists En-prime subact that contains Ӂ, then the intersection of all En-prime subact containing Ӂ is called the En-prime radical of Ӂ and denoted by En-p-rad(Ӂ). If there is no En-prime subact containing Ӂ, then En-p-rad(Ӂ) = Д. A subact Ӂ is called an En-p-radical subact if En-p-rad(Ӂ) = Ӂ [5].

Theorem 3: Put Д is an S-Act. The following statements are equivalent:

a- Д is $En-\mathcal{P}.c.\mathcal{P}.S.A$.

b- For every proper (appropriate) subact Ӂ of Д, there is $a \in$ Ӂ such that En-p-rad(Ӂ) $=$ En $-$ p $-$ rad(Sa).

c- For every proper (appropriate) sub act Ӂ of Д, if $\{Ӂ_\alpha\}_{\alpha\in\lambda}$ is a family of sub act of Д and Ӂ $\subseteq \bigcup_{\alpha\in\lambda} Ӂ_\alpha$ then Ӂ $\subseteq En-p-\mathrm{rad}(Ӂ_\beta)$ for some $\beta \in \lambda$.

d- For every proper (appropriate) subact Ӂ of Д, if $\{Ӂ_\alpha\}_{\alpha\in\lambda}$ is a family of radical subact of Д and Ӂ $\subseteq \bigcup_{\alpha\in\lambda} Ӂ_\alpha$ then Ӂ $\subseteq Ӂ_\beta$ for some $\beta \in \lambda$.

Proof: (a→b) Put Ӂ is a proper subact of Д. Suppose En-p-rad(Ӂ) ⊄ En-p-rad(Sa) for each a∈Ӂ, there is an En- prime subact P_a which contains Sa and Ӂ ⊄ P_a. But Ӂ $= \bigcup_{a\in Ӂ} S_a \subseteq \bigcup_{a\in Ӂ} P_a$, that is Д is not En-prime compactly packed which contradicts (a).

(b→c) Put Ӂ is a proper subact of Д and let $\{Ӂ_\alpha\}_{\alpha\in\lambda}$ be a family of subact of Д such that Ӂ $\subseteq \bigcup_{\alpha\in\lambda} Ӂ_\alpha$. By (b) there is $a \in$ Ӂ such that En-p-rad(Ӂ) = En-p-rad(Sa). Then $a \in \bigcup_{\alpha\in\lambda} Ӂ_\alpha$ and hence $a \in Ӂ_\beta$ for some $\beta \in \lambda$, so that Sa $\subseteq Ӂ_\beta$ and Ӂ ⊆ En-p-rad(Ӂ) = En-p-rad(Sa) ⊆ En-p- rad($Ӂ_\beta$)

(c→d) & (d→a) are evident.

Recall that an S-Act Д is called a multiplication S-Act if each subact Ӂ of Д has the form Ӂ=IД for an ideal I of S. In fact Ӂ = [Ӂ: Д]Д [6].

Recall that if Д is a multiplication S-Act and Ӂis a maximal subact of Д, then Ӂ is En-prime, therefore Ӂ is prime [6] with Ӂ $\subseteq \bigcup_{\alpha\in\lambda} Ӂ_\alpha$, where λ is a finite set, then Ӂ $\subseteq Ӂ_\beta$ for some $\beta \in \lambda$ [7]. If Д is a multiplication S-Act containing finite number of En-prime subact then Д is $En-\mathcal{P}.c.\mathcal{P}$.

The example that follows provides an S-Act that isn't $En-\mathcal{P}.c.\mathcal{P}.S.A$

Example 1: Put Ɔ be an infinite set. Let S be the commutative Boolean monoid (P(Ɔ),∆,∩), where P(Ɔ) is the power set of Ɔ, and the operation ∆ is the usual operation. Let Ù = {Á: Á is finite set of Ɔ}. Since S is commutative Boolean monoid , then for each Á∈ Ù, ⟨Á⟩ is radical ideal, therefore is En-prime radical [6], then ⟨Á⟩ = ⋂{ Ꞷ: Ꞷ is En-prime ideal containing Á}. Because Ù is not principal ideal then Ù ⊄ ⟨Á⟩, that is, there exists an En-prime $Ꞷ_{Á}$ containing Á and Ù ⊄ $Ꞷ_{Á}$, but Ù $= \bigcup_{Á\in Ù}\langle Á\rangle \subseteq \bigcup_{Á\in Ù} Ꞷ_{Á}$, then Ù is not En-prime compactly packed and hence S is not $En-\mathcal{P}.c.\mathcal{P}.S.A$.

Put Д is an S-Act. A subact Ӂ of Д is said to be En-pure in Д if for every endomorphism f, Ӂ∩f(Д) = f(Ӂ)[4].

Proposition 2: Put Д is an S-Act and every subact is En-pure, then Д is $En-\mathcal{P}.c.\mathcal{P}.S.A$ iff each proper subact Ӂ of Д is cyclic.

Proof: The sufficiency is clear. To prove the necessity, let Ӂ be a proper subact of Д. Since Д is $En-\mathcal{P}.c.\mathcal{P}.S.A$ then by theorem 3, there exists a∈ Ӂ such that En-p- rad(Ӂ) $=$ En $-$ p $-$ rad(Sa) . But every subact is En-pure, then by [8], Ӂ = Sa.

The proof of the following theorem by the same way of theorem (2)

Theorem 4: If Д is $En-\mathcal{P}.c.\mathcal{P}.S.A$ which has at least one maximal subact then Д satisfies the ACC on En-p-radical subact.

Because every finitely generated S-Act and every multiplication S-Act has a proper maximal subact, [8,9] thus:-

Corollary 2: If Д is finitely generated or multiplication $En-\mathcal{P}.c.\mathcal{P}.S.A$, then Д satisfies the ACC on En-prime radical subact.

Definition 3: An En- prime subact Ɔ of an S-Act Д is called a minimal En-prime subact of a sub act Ӂ if Ӂ ⊆ Ɔ and there exist no smaller En-prime sub act with this property.

Remember that Every En-prime subact is prime subact, therefore, if Д is an S-Act that satisfies the ACC on En-p-radical subact then the En- p-radical of any proper subact Ӂ of Д is the intersection of a finite number of minimal En-prime sub act of Ӿ[10,11,12]

We require the following lemma in order to derive another corollary:

Lemma 1: If Д be a multiplication S-Act that satisfies the ACC on En-p- radical subact, then for every proper subact Ӿ of Д there exists a finite number of minimal prime sub act of Ӿ.

Proof: let Ӿ be a proper sub act of Д, then En-p-rad(Ӿ) is the intersection of a finite number of minimal En-prime subact of Д say $Ɔ_1, Ɔ_2, \ldots, Ɔ_n$. We shall prove that these $Ɔ_i$'s are the only minimal En-prime subact of Ṅ. Suppose Ù is a minimal En-prime sub act. It is clear that En-p--rad(Ӿ) ⊆ Ù that is $\bigcap_{i=1}^{n} Ɔ_i \subseteq Ù$ and hence $\bigcap_{i=1}^{n} [Ɔ_i : Д] = [\bigcap_{i=1}^{n} Ɔ_i : Д] \subseteq [Ù_i : Д]$. And $[Ù : Д]$ is En-prime ideal [8] then there exists $j \in \{1,2,\ldots,n\}$ such that$[Ɔ_j : Ṁ] \subseteq [Ù : Д]$, but Д is a multiplication S-Act thus $Ɔ_j \subseteq Ù$ because Ù is minimal prime subact.

Corollary 3: If Д is a multiplication $En - \mathcal{P}.c.\mathcal{P}.S.A$, then for every proper subact Ӿ of Д there exist a finite number of minimal En-prime subact of Ӿ.

Definition 4: Let ꞵ be a En-prime subact of an S-Act Д. The height of ꞵ equals n (denoted by $\mathrm{ht}(ꞵ) = \mathrm{n}$) if there exists a chain of distinct En- prime subact of $ꞵ_i$ of Д of the form $ꞵ = ꞵ_0 \supset ꞵ_1 \supset \cdots \supset ꞵ_n$ and it is the longest chain such that $ꞵ = ꞵ_0$.

Theorem 5: Put Д is an S-act and every finitely generated subact is cyclic. If Д satisfies the ACC on En-p-radical subact, then Д is $En - \mathcal{P}.c.\mathcal{P}.S.A$.

Proof: Put Ӿ is a proper subact of Д. By [2], there exists a finitely generated subact Ù of Д such that En-p-rad(Ӿ) = En-p-rad(Ù) and hence Ù is cyclic sub act, by theorem 3 Д is $En - \mathcal{P}.c.\mathcal{P}.S.A$

Definition 5: An S-Act Д is said to be satisfy the Cyclic Subact Condition (CSC) if for each x∈ Д and each En-prime subact Ķ of Д minimal over S therefore ht(K) ≤ 1.

Proposition 3: Suppose that Д is an $En - \mathcal{P}.c.\mathcal{P}.S.A$. If the CST is satisfied for Д, then $\dim Д \leq 1$.

Proof: Put Ķ be a maximal subact of Д, then by Theorem (3), there exists a∈Д such that Ķ = En-p-rad(Sa). This implies that Ķ is minimal En-prime sub act over Sa. By CSC, ht(K) ≤ 1, therefore dim Д ≤ 1.

Proposition 4: Let f: Д → Д′ be an S-epimorphism. If Д is $En - \mathcal{P}.c.\mathcal{P}.S.A$ then so is Д′. The converse is true when Д is finitely generated or (multiplication) S-Act and $\ker \mathrm{f} \subseteq \mathrm{rad}\{0\}$**.**

Conclusion

In this work, the concepts of $En - \mathcal{P}.c.\mathcal{P}$ modules and $En - \mathcal{P}.c.\mathcal{P}.S.A$ have been introduced and prove some properties which related to these concepts, proving that a- Д is $En - \mathcal{P}.c.\mathcal{P}.S.A$. b- For every appropriate subact Ӿ of Д, there is a∈Ӿ such that En-p-rad(Ӿ)=En-p-rad(Sa). c-For every appropriate subact Ӿ of Д, if $\{Ӿ_\alpha\}_{(\alpha\in\lambda)}$ is a family of subact of Д and $Ӿ \subseteq \bigcup_{(\alpha\in\lambda)} Ӿ_\alpha$ then Ӿ ⊆ En-p-rad($Ӿ_\beta$) for some β∈λ. d-For every appropriate subact Ӿ of M, if $\{Ӿ_\alpha\}_{(\alpha\in\lambda)}$ is a family of radical subact of Д and $Ӿ \subseteq \bigcup_{(\alpha\in\lambda)} Ӿ_\alpha$ then $Ӿ \subseteq Ӿ_\beta$ for some β∈λ are equivalent

Author's Declaration

- Conflicts of Interest: None.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Baghdad.

Published Online First: November, 2024
https://doi.org/10.21123/bsj.2024.11424
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Author's Contribution

This work described in this study was performed in collaboration among the authors S. N. K.. Proposed the concept of $En-\mathcal{P}.c.\mathcal{P}$ modules. U. S. A. and M. J. M. A.

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الآثار الاولية من النمط En المرصوصة المكتضة

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الخلاصة

في هذا العمل تم دراسة المقاسات الاولية من النمط Enالمرصوصة المكتضة ودراسة تعيم هذا المفهوم الى مفهوم الآثار الاولية من النمط Enالمرصوصة المكتضة حيث تم دراسة بعض العلاقات والتشخيصات الخاصة بهذه المفاهيم حيث تم برهنت العلاقات الخاصة بمفهوم المقاسات الاولية من النمط Enالمرصوصة المكتضة ليكن ₩ مقاس و كل مقاس جزئي (ɑ) هو اولي من النمط En فان المقاس ₩ هو مقاس اولي من النمط En مرصوص مكتض اذا و فقط اذا كل مقاس جزئي دائري و كذلك تمت برهنت اذا كان المقاس ₩ اولي من النمط En مرصوص مكتض يمتلك على الاقل مقاس جزئي اعظم فان ₩ يحقق خاصية ACC على المقاس الجزئي الاولي من النمط En-p-radical. قد تم دراسة مفهوم الاثار الاولية من النمط En المرصوصة المكتضة والتي هي تعميم لمفهوم المقاسات الاولية من النمط Enالمرصوصة المكتضة حيث تم برهنت بعض الخواص وبعض التشخيصات الخاصة بمفهوم الآثار الاولية من النمط En المرصوصة المكتضة , اذا كان الاثر∏ اولي من النمط En مرصوص مكتض ويمتلك على الاقل اثر جزئي اعظم فان الاثر ∏ يحقق خاصية ACC على الاثر من النمط En-p-radical

الكلمات المفتاحية: الآثار الجزئية الاولية من النمط En والمقاسات الجزئية الاولية من النمط Enوا الآثار الجزئية المخلصة من النمط En، الآثار الاولية من النمط Enالمرصوصة المكتضة و الآثار الجدائية.