Scheduling Job Families on a Single Machine

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Abstract:

The problem of scheduling n jobs on a single machine is considered, where the jobs are partitioned into several families and a set – up time is necessary between jobs of different families. The objective is to find a lower bound for the problem of minimizing the sum of completion times and the maximum tardiness. This paper uses a decomposition property to find a lower bound in order to incorporated in a branch and bound algorithm for constructing an optimal schedule.

Key words: Scheduling, Set-up time, Multiple objective

Introduction:

The problem of scheduling n jobs on a single machine is considered , where the jobs are divided into several families and a machine set-up is necessary for processing the first job in the schedule and also for processing jobs of different families . Each family f , for f =1,………., F contains n_f jobs which are labeled as (1 , f),….(n_f , f) .Job (i , f) for i =1 , …….n_f , f = l,……,F becomes available for processing at time zero , requires a positive processing time P_{if} and has a positive due date d_{if} . The set-up time of family f, f =1, ….., F is denoted by s_f . The machine can process at most one job at a time, and cannot perform any processing whilst undergoing a set-up. For each job i in family f we can calculate the completion time C_{if} and the tardiness $T_{if} = \max\{C_{if} - d_{if}, o\}$ and the objective function is $\sum_{f=1}^{F}\sum_{i=1}^{nf} Cif + T\max$

Bayati [1] used a branch and bound algorithm to find an optimal solution and derived a lower bound based on batches for $\sum_{f=1}^{F}\sum_{i=1}^{nf} cif$ and relaxed Tmax .Abdullah [2] used heuristic methods to get near optimal solutions. Many practical – scheduling problems involve sequencing number of jobs divided into several families. The range of application areas for scheduling theory goes beyond computers and manufacturing to include agriculture, hospitals, transport , etc. [3] . Since our problem consists of multiple objective, hence our aim is to find a sequence that does well on both criteria.

The remainder of this paper is organized as follows :

In the next two sections, we derive a lower bound based on decomposition property and provide an example to compute branch and bound algorithm. A final section contains some concluding remarks.

Derivation of a Lower Bound (LB):

Deriving a lower bound for a problem that has a multiple objective function is very difficult since it is not easy finding a sequence that have the minimum for the two objectives. Mason and Anderson's lower bound [4] for the $1|sf|\ \sum ci$ Problem is obtained by using objective splitting as follows :

1) They assume that $S_f = 0$ for each f , then they use shortest processing time (SPT) rule to get the sequence $\sigma 1$ to calculate min $\sum Ci(\sigma 1)$.

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2) Thereafter they consider $pi = 0$ for each job i and then they use weighted processing time (WPT) ratio Sf/Wf such that Wf = nf to order the families to get the sequence $\sigma 2$ to calculate min $\sum Wf Cf(\sigma 2)$

Now if σ is optimal for the problem $1|sf|\sum ci$, then $\sum Wf Cf(\sigma 2) + \sum_{i=1}^{n} Ci(\sigma 1) \geq \sum_{i=1}^{n} Ci(\sigma)$ = LBC , where LBC is a lower bound for the $1|sf|\sum ci$ problem.

A lower bound for the Tmax problem is obtained by applying the following steps:

Step (1): Consider each family as a composite job such that jobs within each family are sequenced in non-decreasing order of their due dates .

Step (2): Sequencing families (Composite jobs) in non- decreasing order of their due dates , where a due date for a family f (f=1,…..,F) is calculated as follows :

$$Df = \min \left\{ dif + \sum_{j=i+1}^{nf} pjf \quad if \in \{1,\dots..nf\} \right\}$$

Step(3) : For the above order compute the maximum tardiness T'max such that a setup time is required only one time for each family .

Hence by decomposition property [5] LB = LBC + T'max .

This lower bound is very good because it not neglects the contribution of set-up time, which is very important.

To illustrate the above procedure we present the following example :

Example: Consider the problem with 4 jobs .

Table (1) data for the problem

i	1	2	3	4
pi	5	3	6	8
di	10	12	20	17

Moreover assume the 4 jobs are divided into two families $f_1 = \{ 1,2 \}$ and $f_2 = \{ 3,4 \}$, and with set-up times are $S_1 = 2$ and $S_2 = 3$ for the families f_1 and f_2 respectively .

We order jobs in SPT rule, then

i	2	1	3	4
Ci	3	8	14	22

$$\sum ci = 47$$

We order **f**amilies in SWPT rule , where a processing time for a family f is $P_f = S_f$ and a weight for a family f is $w_f = n_f$.

f	f_1	f_2
C_f	2	5
W_f	2	2
w_fC_f	4	10

$\sum W_f C_f = 14$ Thus LBC = 47+14=61 .

We order jobs within each family by EDD rule .

i	1	2	4	3
pi	5	3	8	6
di	10	12	17	20

We find Df1 and Df2

$D_{f1} = \min \{10 + 3, 12\} = 12$

$D_{f2} = \min \{17 + 6, 20\} = 20$

We order families by EDD rule to get the following arrangement :

i	1	2	4	3
Ci	7	10	21	27
Ti'	0	0	4	7

T' max = 7 . Hence LB=61+7=68.

However, the optimal schedule obtained by BAB method is (2,1,4,3) and its optimal value is 70.

The Branch and Bound Algorithm (BAB)

The branch and bound is the most widely solution technique that is used to scheduling problems. We try to improve on the branch and bound procedure by using a good upper and lower bound at the root node of the search tree. We apply the SPT rule to yield an upper bound (UB) on the cost of an optimal schedule. Also at the root node of the search tree an efficient lower bound (LB) on the cost of an optimal schedule is obtained from section (2).

Thus, we should compute the sum of completion times and the maximum tardiness of each partial schedule, and work should proceed on whichever branch currently has the smallest total. The branch and bound (BAB) procedure continuous in a similar way by using a forward branching rule. If the branching ends at a complete schedule of jobs then this schedule is evaluated and if its value is less than the current UB, this UB is reset to take that value. The procedure is repeated until all nodes have been considered by using backtracking procedure[6] .

Conclusions:

This paper provides description of an algorithm for scheduling n jobs on a single machine, where the jobs are partitioned into several families and a set-up time is necessary between jobs of different families. For the problem of minimizing the sum of completion times and the maximum tardiness, we have the following:

1. our branch and bound (BAB) algorithm is effective since our lower bound (LB) is efficient .
2. There are some interesting research problems. One of the vexing issues is the derivation of dynamic programming algorithms of the sum of completion times and the maximum tardiness. Other fruitful research areas include the design and worst-case analysis of heuristics.

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جدولة عوائل من النتاجات على ماكنة واحدة

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الخلاصة :

إن هـــــذا البحث يتناول دراسة n من النتاجات (jobs) على ماكنة واحدة بحيـــــــث أن هذه النتاجات مقسمة إلى F من العوائل (Families) ، كل عائـــــلة (f=1 , ……..., F) تحتوي على f^n من النتاجات ، الهدف من هذه الدراسة هو إيجاد قيد ادنى لمسألة تصغير دالـــــة الهدف المركبة وهي مجمـــــوع أوقات الإتمام وأعظم تأخيـــر لاســـالب (The sum of completetion times and the maximum tardiness) .

في هذا البحث يتم إيجاد قيد أدنى (Lower bound) لغرض استخدامه في خوارزمية التقيد والتفرع للحصول على الجدول الأمثل .