L-pre- and L-semi-P- compact Spaces

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Abstract:

The purpose of this paper is to study a new types of compactness in the dual bitopological spaces. We shall introduce the concepts of L-pre- compactness and L-semi-P- compactness .

Key words: L-pre-compact ,L-semi-p-compact ,L-pre-open,L-semi-p-open.

Introduction:

The concepts of bitopological space was initiated by Kelly[1].A set X equipped with two topologies τ_1 and τ_2 is called a bitopological space denoted by (X, τ_1, τ_2). Navalagi [2] introduced the concepts of pre-open and semi-P-open sets. A subset A of a topological space (X, τ) is said to be "pre-open" set if and only if $A \subseteq \operatorname{int} cl(A)$, the family of all pre open subsets of X is denoted by PO(X).The complement of a pre-open set is called pre-closed set, the family of all pre- closed subsets of X is denoted by PC(X) [2].The smallest pre- closed subset of X containing A is called "pre-closure of A" and is denoted by pre-cl(A)[3].

Let (X, τ) be a topological space, a subset A of X is said to be "semi-P-open" set if and only if there exists a pre-open subset U of X such that $U \subseteq A \subseteq pre-cl(U)$, the family of all semi –p-open subsets of X is denoted by SPO(X).The complement of a semi-p-open set is called "semi-p-closed" set, the family of all semi-p-closed subsets of X is denoted by SPC(X). The smallest semi-p-closed set containing A is called semi-p-closure of A denoted by semi-p-cl(A)[4].[3]shows that every open set is a pre-open and the union of any family of pre-open subsets of X is a pre-open set, but the intersection of any two pre-open subsets of X need not be apre-open set.[4] shows that every pre-open set is a semi –p-open and consequentiy every open set is a semi-p-open. Also she shows that the union of any family of semi-p-open subsets of X is a semi-p-open set,but the intersection of any two semi-p-open subsets of X need not be a semi-p-open set.

L-open set was studied by Al-Talkhany [5], asubset G of a bitopological space (X, τ_1, τ_2) is said to be "L –open" set if and only if there exists a τ_1-open set U such that $U \subseteq G \subseteq cl\tau_2(U)$,the family of all L-open subset of X is denoted by L-O(X).The complement of an L-open set is called "L-closed" set,the family of all L-closed subsets of X is denoted by L-C(X).In a bitopological space (X, τ_1, τ_2) every τ_1-open set is an L-open set[5].The union of any family of L-open subsets of X is an L-open set, but the intersection of any two L-open

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subsets of X need not be L-open set[5].

A collection of sets is said to have the finite intersection property (FIP) if and only if the intersection of each finite subcollection of it is non empty.[6]

2-L-pre - and L-semi-p - compact spaces

In this section we shall introduce a new typ of compactness namely L-pr – (L-semi-p-) compactness. We start with definition of L-pre-(L-semi-p-) open set.

Definition (2.1):

Let (X, τ_1, τ_2) be a bitopological space and let G be a subset of X. then G is said to be:

1- " L-pre-open" set if and only if there exists a τ_1-pre-open set U such that $U \subseteq G \subseteq cl\tau_2(U)$.The family of all L-pre-open sub sets of X is denoted by $L-PO(X)$.

2- " L-semi-P-open" set if and only if there exists a τ_1- semi-P-open setU such that $U \subseteq G \subseteq cl\tau_2(U)$.The family of all L- semi-P-open sub sets of X is denoted by $L-SPO(X)$.

Definition (2.2):

Let (X, τ_1, τ_2) be a bitopological space and let A be a subset of X .

1. By an "L-open cover of A" we mean a subcollection of the family L-O(X) which covers A .

2. By an "L -pre-open cover of A" we mean a subcollection of the family L-PO(X) which covers A.

3. By an "L -semi-p-open cover of A" we mean a subcollection of the family L-SPO(X) which covers A.

Remark (2.3):

1- Every L-open cover is an L- pre-open.

2- Every L-pre-open cover is an L-semi-P-open.

3- Every L-open cover is an L-semi-P-open.

The converse of each case of remark (2.3) is not true in general as the following example shows:

Example (2.4):

Let $X = \{a, b, c, d\}$

$\tau_1 = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

$\tau_2 = D$=The discrete topology=The power set of X

$L-O(X) = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

$$L-PO(X) = \begin{Bmatrix} X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \\ \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \end{Bmatrix}$$

$L-SPO(X) = L-PO(X) \cup$
$\{\{a, d\}, \{c, d\}, \{b, c, d\}\}$

Let $C = \{\{c\}, \{a, b, d\}\}$ and $B = \{\{a, d\}, \{b, c\}\}$, clear that B is an L -semi-p- open cover, but it is neither L-pre-open nor L-open, and C is an L-pre-open cover,but it is not L-open cover.

Remark (2.5):

Every τ_1-pre-open(τ_1-semi- p-open) cover of a sub set of a bitopological space (X, τ_1, τ_2) is an L -pre-open "L-semi-p-open" respectively.

The opposite direction of remark (2.5) is not true in general as the following example show:

Example (2.6):

Let

$X = \{a, b, c, d\}$ $\tau_1 = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

$\tau_2 = I$ =the indiscrete topology

$$\tau_1 - po(X) = \begin{Bmatrix} X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\} \\ , \{a, c\}, \{a, b, d\}, \{a, c, d\}. \{a, b, c\} \end{Bmatrix}$$

$\tau_1 - SPO(X) = \tau_1 - PO(X) \cup \{\{a, d\}, \{b, c, d\}\}$

$L-PO(X) = \tau_1 - PO(X) \cup$
$\{\{a, d\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$

$L-SPO(X) = L-PO(X)$

If $C=\{\{a,c\},\{b,d\}\}$, then C is an L -pre-open and L -semi-p-open cover, but it is neither τ_1-pre-open nor τ_1 -semi-p-open cover.

Definition (2.7):

A bitopological space (X,τ_1,τ_2) is said to be :

1- "L-pre-compact space " if and only if every L-pre-open cover of X has a finite sub cover.

2- "L-semi-p-compact space " if and only if every L-semi-p-open cover of X has a finite sub cover.

Proposition (2.8):

1- Every L-semi-p-compact space is an L- pre-compact.

2- Every L-pre- compact space is an L- compact.

3- Every L-semi-p-compact space is an L –compact.

Proof:

Follows from remark (2.3).

Remark (2.9):

The opposite direction of each case in proposition (2.8) is not true in general. As the following two examples show:

1- Let X be an infinite set with two topologies $\tau_1=I$ and $\tau_2=D$

$L-O(X)=\{X,\phi\}$, $L-PO(X)=$ ℙ(X) and $L-SPO(X)=$ ℙ(X)

Note that (X,τ_1,τ_2) is an L-compact space but it is neither L-pre-compact space nor L-semi-p-compact.

Let X=N with two topologies $L-SPO(N)=$ ℙ(N)

Note that (N,τ_1,τ_2) is an L-pre-compact space , but it is not L-semi-p-compact.

Proposition (2.10):

Let (X,τ_1,τ_2) be abitopological

$\tau_1=\{u\subseteq N:2\not\in u\}\cup\{N\}$

$\tau_2=D$

$L-O(N)=\tau_1$

$L-PO(N)=\tau_1$

space. If

1- X is an L-pre- compact space ,than (X,τ_1) is pre- compact space .

2- X is an L-semi-p-compact space, then (X,τ_1) is semi-p-compact space .

Proof :

follows from remark (2.5).

Remark (2.11):

The opposite direction of each case in proposition (2.10) is not true in general.

As the following example show:

Let $X=$N = The set of natural numbers

$\tau_1=\{u\subseteq N:1\not\in u\}\cup\{N\}$

$\tau_2=I$

$\tau_1-PO(N)=\tau_1$

$L-PO(N)=$ ℙ(N)\{1}

Note that (N,τ_1) is pre- compact space, but (N,τ_1,τ_2) is not L-pre-compact space .

Proposition (2.12):

An L-pre-closed (L-semi-p-closed) subset of an L-pre- compact(L-semi-p-compact) space is an L-pre-compact(L-semi-p-compact) set respectively

Proof:

Let A be an L-pre-(L-semi-p-) closed subset of an L-pre- (L-semi-p-) compact space (X,τ_1,τ_2) and let $\{G_\alpha:\alpha\in\Lambda\}$ be an L-pre-(L-semi-p-) open cover of A .Then $\{G_\alpha:\alpha\in\Lambda\}\cup A^c$ forms an L-pre-(L-semi-p-) open cover of X which is L-pre- (L-semi-p-) compact space. So there are finitely many elements $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that $X = \bigcup_{i=1}^{n} G_{\alpha_i} \cup A^c$, it follows that $A \subseteq \bigcup_{i=1}^{n} G_{\alpha_i}$. Hence A is an L-pre-(L-semi-p-) compact.

Corollaries (2.13):

1- An L-pre-closed (L-semi-p-closed) subset of an L-pre- compact(L-semi-p-compact) space is an L- compact.

2- An L-semi-p-closed subset of an L-semi -p- compact space is an L-pre-compact.

Proof :

follows from propositions (2.12) and (2.8).

Corollaries (2.14):

1- An L-pre-closed (L-semi-p-closed) subset of an L-pre- compact (L-semi-p-compact) space is a τ_1-pre-compact(τ_1 -semi-p-compact) respectively.

2- An L-semi-p-closed subset of an L-semi -p- compact space is a τ_1-pre-compact.

3- An L-pre-closed (L-semi-p-closed) subset of an L-pre- compact(L-semi-p-compact) space is a τ_1- compact.

Proof :

follows from proposition (2.12),remarks (2.3) and (2.5).

Definition (2.15):

A bitopological space (X, τ_1, τ_2) is said to be :

1. "L-T_2 -space" if and only if for each pair of distinct points x and y in X,there exist two disjoint L-open subset G and H of X such that $x \in G$ and $y \in H$.[5]

2. "L-pre-T_2-space" if and only if for each pair of distinct points x and y,there are two disjoint L-pre-open subsets U and V of X such that $x \in U$ and $y \in V$.

3. "L-semi-p-T_2-space" if and only if for each pair of distinct points x and y,there are two disjoint L-semi-p-open subsets U and V of X such that $x \in U$ and $y \in V$.

Remark (2.16):

An L-pre- compact subset of an L-pre -T_2-space need not be L-pre-closed.

For example:

$X = \{1,2,3\}$

$\tau_1 = \{X, \phi, \{1,2\}\}$

$\tau_2 = \{X, \phi, \{1\}, \{3\}, \{1,3\}\}$

$L - O(X) = \{X, \phi, \{1,2\}\}$

$L - PO(X) = L - O(X)$

$\cup \{\{1\}, \{2\}, \{2,3\}, \{1,3\}\}$

Note that (X, τ_1, τ_2) is an L-pre -T_2-space.

Let $A = \{1,2\}$, clear that A is an L-pre- compact subset of X, but it is not L-pre-closed.

Remark (2.17):

An L-semi -p- compact subset of an L-semi-p-T_2-space need not be L-semi-p-closed.

For example:

Note that (X, τ_1, τ_2) is an L-semi-p -T_2-space.

Let $A = \{1,2,4\}$, clear that A is an L-semi-p- compact subset of X, but it is not L -semi-p-closed.

Definition (2.18):

$X = \{1,2,3,4\}$
$\tau_1 = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$
$\tau_2 = D$
$L - O(X) = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$
$L - PO(X) = \{X, \phi, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}\}$
$L - SPO(X) = L - PO(X) \bigcup$
$\{\{2,3,4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,3,4\}\}$

Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1', \tau_2')$ be any function, then f is said to be:

1. "L-continuous" function if and only if the inverse image of any L-open subset of Y is an L-open subset of X.[5]

2. "L-pre-irresolute" function if and only if the inverse image of an L-pre-open subset of Y is an L-pre-open subset of X .

3. "L-semi-p-irresolute" function if and only if the inverse image of an L-semi-p-open subset of Y is an L-semi-p-open subset of X .

Proposition (2.19):

The L-pre-irresolute (L-semi-p-irresolute) image of an L-pre-compact (L-semi-p-compact) space is an L-pre-compact (L-semi-p-compact) respectively.

Proof:

Suppose that $f: (X, \tau_1, \tau_2) \to (Y, \tau_1, \tau_2)$ is an L-pre-(L-semi-p-) irresolut and onto function and X is an L-pre-(L-semi-p-) compact space. Let $\{G_\alpha : \alpha \in \Delta\}$ be an L-pre-(L-semi-p-) open cover of Y,it follows that $\{f^{-1}(G_\alpha) : \alpha \in \Delta\}$ is an L-pre-(L-semi-p-) open cover of X which is L-pre-(L-semi-p-) compact.So there are finitely many elements $\alpha_1, \alpha_2, \alpha_3, \dots\dots, \alpha_n$ such that $X = \bigcup_{i=1}^{n} f^{-1}(G_{\alpha_i}) = f^{-1}(\bigcup_{i=1}^{n} G_{\alpha_i})$ .Therefore $Y = \bigcup_{i=1}^{n} G_{\alpha_i}$ Hence $Y$is an Lpre-(L-semi-p-) compact.

Proposition(2.20):

The L-continuous image of an L-compact space is an L-compact.

Proof:

Suppose that $f: (X, \tau_1, \tau_2) \to (Y, \tau_1, \tau_2)$ is an L-continuous and onto functionand X is an L-compact space. Let $\{G_\alpha : \alpha \in \Delta\}$ be an L-open cover of Y,it follows that $\{f^{-1}(G_\alpha) : \alpha \in \Delta\}$ is an L-open cover of X which is L-compact.So there are finitely many elements $\alpha_1, \alpha_2, \alpha_3, \dots\dots, \alpha_n$ such that $X = \bigcup_{i=1}^{n} f^{-1}(G_{\alpha_i}) = f^{-1}(\bigcup_{i=1}^{n} G_{\alpha_i})$ .Therefore $Y = \bigcup_{i=1}^{n} G_{\alpha_i}$,hence $Y$is an L-compact.

Proposition (2.21):

The L-continuous image of an L-pre-compact (L-semi-p-compact) space is an L-pre-compact

Proof:

follows from proposition (2.20) and (2.8).

Proposition (2.22) :

The L-pre-irresolute image of an L-semi-p-compact space is an L-pre-compact.

Proof:

follows from proposition (2.19) and (2.8).

Theorem (2.23):

Let (X, τ_1, τ_2) be abitopological space and let A be a subset of X.A point x in X is an L-pre-closure (L-semi-p-closure) point of A if and only if every L-pre-neighbourhood (L-semi-p- neighbourhood) of x intersects A.

Proof:

Assum that x is an L-pre-closure (L-semi-p-closure) of A , then

$$x \in \Im = \bigcap \left\{ \begin{array}{l} F \subseteq X : A \subseteq F \\ and\ F\ is\ an\ L-pre-closed \\ (L-semi-p-closed) \end{array} \right\}.$$

Suppose that there exists an L-pre-neighbourhood (L-semi-p-neighbourhood) M of x such that $M \cap A = \phi$, that is, there exists an L-pre-open(L-semi-p -open) set G such that $x \in G \subseteq M$,then such that $A \subseteq M^c \subseteq G^c$,but G^c is an L-pre-closed (L-semi-p-closed) with $x \notin G^c$. Therefor $x \notin \Im$ which is a contradiction hence every L-pre-neighbourhood (L-semi-p-neighbourhood) of x must intersects A.

Conversely

Assume that every L-pre-neighbourhood (L-semi-p-neighbourhood) of x intersects A, and suppose that x is notL -pre-closure (L-semi-p-closure) point of A,then $x \notin \Im$,that is, there exists an L-pre-closed (L-semi-p -closed) subset F of X with $A \subseteq F$ such that $x \notin F$,it follows that $x \in F^c$ which is an L-pre-open(L-semi-p -open) set. Now there is an L-pre-neighbourhood (L-semi-p-neighbourhood) F^c of x with $A \cap F^c = \phi$.that implies to contradiction with our assumption.Hence x must be an L-pre-(L-semi-p-) closure point of A

Theorem (2.24):

Let (X, τ_1, τ_2) be abitopological space. Asubset A of X is an L-pre-(L-semi-p-) closed if and only if $A = L-Pcl(A)(L-SPcl(A))$.

Proof:

Suppose that $A \in L-PC(X)(L-SPC(X))$ and $A \neq L-Pcl(A)(L-SPcl(A))$. Since $A \subseteq L-Pcl(A)(L-SPcl(A))$, so $L-Pcl(A)(L-SPcl(A)) \not\subset A$,that is,there exists an element $r \in L-Pcl(A)(L-SPcl(A))$and $r \notin A$,it follows that $r \in A^c$ which is an L-pre-(L-semi-p-) open set. Then by theorem (2.23) $A \cap A^c \neq \phi$ which is a contradiction with the fact $A \cap A^c = \phi$.Hence $A = L-Pcl(A)(L-SPcl(A))$

Conversly

Assume that $A = L-Pcl(A)(L-SPcl(A))$, but $L-Pcl(A)(L-SPcl(A))$ is an L-pre-(L-semi-p-) closed subset of X by definition of L-pre-(L-semi-p-)closur of a set A wich is the intersection of all L-pre-(L-semi-p-)closed subsets of X containing A. So A is an L-pre-(L-semi-p-) closed set.

Definition (2.25):

Let (X, τ_1, τ_2) be abitopological space and let $(f, X, A, \geq)$ be a net in X. Then f is said to be:

1- "L-pre-convergent" to a point x_o in X if and only if for each L-pre-nhd.M of x_o there exists an element $a_o \in A$ such that $f_a \in N$ for each $a \geq a_o$.

2- "L-semi-p-convergent" to a point x_o in X if and only if for each L-semi-p-nhd. M of x_o there exists an element $a_o \in A$ such that $f_a \in N$ for each $a \geq a_o$.

Definition (2.26):

Let (X, τ_1, τ_2) be abitopological space and let $(f, X, A, \geq)$ be a net in X. A point x_o in X is called:

1- "L-pre-cluster point" of f if and only if for each $a \in A$ and for each L-pre-nhd. M of x_o there exists an element $b \geq a$ in A such that $f_b \in M$.

2- "L-semi-p-cluster point" of f if and only if for each $a \in A$ and for each L-semi-p-nhd. M of x_o there exists an element $b \geq a$ in A such that $f_b \in M$.

Theorem (2.27):

Let (X, τ_1, τ_2) be abitopological space and let $(f, X, A, \geq)$ be a net in X. for each $a \in A$,let $K_a = \{f_x : x \geq a \, in \, A\}$,then a point p of X is an L-pre-cluster(L-semi-p-cluster) point of f if and only if $p \in L-Pcl(K_a)(L-SPcl(K_a))$.

Proof:

Assum that p is an L-pre-(L-semi-p-) cluster point of f and let M be an L-pre-(L-semi-p-) nhd.of p , then for each $a \in A$, there exists an element $x \geq a$ in A such that $f_x \in M$ .hence $K_a \cap M \neq \phi$ for each $a \in A$. So by theorem (2.23) $p \in L-Pcl(K_a)(L-SPcl(K_a))$ for each $a \in A$.

Conversely

Assum that $p \in L-Pcl(K_a)(L-SPcl(K_a))$ for each $a \in A$,and suppose, if possible, pis not an L-pre-(L-semi-p-) cluster point of f, then there exists an L-pre-(L-semi-p-) nhd.Mof p and an element $a \in A$ such that $f_x \notin M$ for every $x \geq a$ in A. this implies that $K_a \cap M = \phi$, it follows that $p \notin L-Pcl(K_a)(L-SPcl(K_a))$for this a which is a contradiction. Hence p must be an an L-pre-(L-semi-p-) cluster point of the net f**.**

Definition (2.28):

Let (X, τ_1, τ_2) be a bitopological space and let $\mathcal{F}$ be a filter on X . Apoint x inX is called

1- An" L-pre-cluster" point of $\mathcal{F}$ if and only if each L-pre-nhd. Of x intersects every member of $\mathcal{F}$.

2- An" L-semi-p-cluster" point of $\mathcal{F}$ if and only if each L-semi-p-nhd. Of x intersects every member of $\mathcal{F}$.

Theorem (2.29):

Let (X, τ_1, τ_2) be a bitopological space and let $\mathcal{F}$ be a filter on X . A point p inX is an L-pre-(L-semi-p-) cluster point of $\mathcal{F}$ if and only if $p \in L-Pcl(F)(L-SPcl(F))$, for each $F \in \mathcal{F}$.

Proof:

Suppose that p is an L-pre-(L-semi-p-) cluster point of $\mathcal{F}$, then each L-pre-(L-semi-p-) nhd.M of p , $F \cap M \neq \phi$for every $F \in \mathcal{F}$.It follows by theorem (2.23) that $p \in L-Pcl(F)(L-SPcl(F))$ for each $F \in \mathcal{F}$.

Conversly

Assum that $p \in L-Pcl(K_a)(L-SPcl(K_a))$ for each $F \in \mathcal{F}$, then by theorem (2.23) every L-pre-(L-semi-p-) nhd.of p intersects F for each $F \in \mathcal{F}$.Hence p is an L-pre-(L-semi-p-) cluster point of $\mathcal{F}$.

Theorem (2.30):[6]

Let A be anon empty collection of subsets of a set X such that A has the FIP. Then there exists an ultrafilter $\mathcal{F}$ containing A .

Remark (2.31): [6]

Every filter in anon- empty set X has the FIP.

Theorem (2.32):

Let (X, τ_1, τ_2) be a bitopological space. Then the following statements are equivalent

1- X an L-pre-(L-semi-p-) compact space , 2- Every collection of an L-pre-(L-semi-p-) closed subsets of Xwith FIP has a non empty intersection ,and

3- Every filter on X has an L-pre-(L-semi-p-) cluster point

Proof:

1→2

Let $\{F_\alpha : \alpha \in \Lambda\}$ be a collection of L-pre-(L-semi-p-) closed subset of X with the FIP . suppose that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$,it follows by De-Morgen Laws that $\bigcup_{\alpha\in\Lambda} F_\alpha^c = X$ where F_α^c is an L-pre-(L-semi-p-) open set for each $\alpha \in \Lambda$ .therefore $\left\{F_\alpha^c : \alpha \in \Lambda\right\}$ forms an L-pre-(L-semi-p-) open cover for X which is an L-pre-(L-semi-p-) compact space, then there exist fintiely many elements $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that $\bigcup_{i=1}^{n} F_{\alpha_i}^c = X$. Again by De-Morgen Laws we have that $\bigcap_{i=1}^{n} F_{\alpha_i} = \phi$ which is a contradiction since $\{F_\alpha : \alpha \in \Lambda\}$ has the FIP. Hence $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$

2→3

Let $\mathcal{F}$ be a filter on X, then by remark(2.31) $\mathcal{F}$ has the FIP, it follows that the collection $\{L-Pcl(F)(L-SPcl(F)) : F \in \mathcal{F}\}$ of L-pre-(L-semi-p-) closed subsets of X also has the FIP, so by (2) there exists at least one point $x \in \bigcap\{L-Pcl(F)(L-SPcl(F)) : F \in \mathcal{F}\}$ then by theorem (2.29) x is an L-pre-(L-semi-p-) cluster point of $\mathcal{F}$. thus every filter on X has an L-pre-(L-semi-p-) cluster point.

3→1

Assume that every filter on X has an L-pre-(L-semi-p-) -cluster point. To show that X is an L-pre-(L-semi-p-) compact space and let $\Im$ be an L-pre-(L-semi-p-) open cover of X and suppose ,if possible, $\Im$ has no finite sub cover the collection A $=\{X - G : G \in \Upsilon\}$ has the FIP, for if there is afinite sub collection $\{X - G_i : 1 \le i \le n\}$of A such that $\bigcap\{X - G_i : 1 \le i \le n\} = \phi$ this implies that $\bigcup\{G_i : 1 \le i \le n\} = X$ which is contradicts our supposition that $\Im$ has no finite sub cover, thus A must have the FIP, it follows by theorem (2.30)that there exists an ultrafilter $\mathcal{F}$ on X containing A .by (3) $\mathcal{F}$ has an L-pre-(L-semi-p-) cluster point $x \in X$, then by theorem (3.39) $x \in L-Pcl(F)(L-SPcl(F))$ for each $F \in \mathcal{F}$, in particular $x \in L-Pcl(X-G)(L-SPcl(X-G))$ for each $G \in \Im$. But X-G is an L-pre-(L-semi-p-) closed subset of X for each $G \in \Im$, therefore by propostion (2.24) $L-Pcl(X-G)(L-SPcl(X-G)) = X - G$ for every $G \in \Im$. This implies $x \in \bigcap\{X - G : G \in \Im\}$, so by De-Morgen Laws $x \in X - \bigcup\{G : G \in \Im\}$,that is, $x \notin \bigcup\{G : G \in \Im\}$,which is acontradiction with the fact that $\Im$ is an L-pre-(L-semi-p-) open cover of X ,hence $\Im$ must have a finite sub cover and consequently X is an L-pre-(L-semi-p-) compact space.

Theorem (2.33):

Let (X, τ_1, τ_2) be a bitopological space, if X is an L-pre-(L-semi-p-) compact space, then every net in X has an L-pre-(L-semi-p-) cluster point

Proof:

let $(f, X, A, \ge)$ be a net in X. for each $a \in A$ let $K_a = \{f_x : x \ge a \, in \, A\}$. Since A is directed by $\ge$, so the collection $\{K_a : a \in A\}$ has the FIP. Hence $\{L-Pcl(K_a)(L-SPcl(K_a)) : a \in A\}$ also has FIP , it follows by theorem (2.32)

that
$\bigcap\{L-Pcl(K_a)(L-SPcl(K_a)): a\in A\}\neq\phi$.
Let
$p\in\bigcap\{L-Pcl(K_a)(L-SPcl(K_a)): a\in A\}\neq\phi$
, then $p\in L-Pcl(K_a)(L-SPcl(K_a))$
for each $a\in A$,so by theorem (3.37) p is an L-pre-(L-semi-p-) cluster point of f.

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فضاءات الرص من النوع (L-pre- and L-semi-P-)

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الخلاصة:

الغرض من هذا البحث هو دراسة انواع جديدة من التراص في الفضاءات التبلوجية الثنائية . اذ سنقدم التراص من النوع (**L-pre- and L-semi-P-**)