Comparative study of creep and fatigue crack growth in Poly (Vinyl chloride) pipe

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Received 17, February, 2011
Accepted 20, November, 2011

Abstract:
The Rate theory of crack growth in PVC pipe has been studied for creep and fatigue crack propagation. Rate theory function parameters, (RTFP), were estimated theoretically from exponential function parameters, (EFP), to experimental data of crack velocity versus stress intensity factor , (V-K) diagram, to creep crack propagation. Also (RTFP) were estimated theoretically from (EFP) to experimental data of (V-ΔK) diagram to fatigue crack propagation. Temperature effect with (RTFP) was discussed. Crack velocity function denoted with stress intensity factor  and temperature degrees has been determined to fatigue and creep crack propagation theoretically and comparative results this function with experimental data of (V-K or ΔK) diagram.

Key words: Poly Vinyl Chloride(PVC) , Rate theory , Creep crack propagation , Fatigue crack propagation.

Introduction:
A.S. Krausz studied the Fracture theories in the brittle material in the different temperature such as the rate theory of creep crack growth to the lead glass 61% specimens [1] and porcelain ceramics specimens [2] and he studied the fatigue crack growth to the brittle materials. Creep-Fatigue crack interaction comparative studied by the F. Djavanroodi [3]. The exponential function was a good fitting to bind relation between crack velocity and stress intensity factor for creep and fatigue crack growth[4] . Many factors control crack velocity are external and internal constraints [1]:

1. External constraints: the load and displacement boundary condition , the component and crack geometry ,the chemical and thermal environment , and their variation in time .

2. Internal constraints: the microstructure of material, chemical composition, the special arrangement of the constituent atoms , the type, configuration and distribution of defects , and their variation in time .

Accordingly, crack velocity behavior is defined explicitly as:

\[ V(\text{m/sec}) = F(\text{ external work }, \text{ geometry, thermal, microstructure, and their variation in time}). \]

The external work, \( W \), depends on the stress intensity factor, \( K \), for creep crack propagation , and it depends on stress intensity factor range , \( ΔK \), for fatigue crack propagation[1].

\[ \frac{W_c}{W_f} = \frac{\delta_c * K}{\delta_f * ΔK} \] \( (1 - 1) \)

Where \( W_c, W_f \) are external works for creep and fatigue crack propagation

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, respectively, and \(\delta_c\), \(\delta_f\) are works constants for creep and fatigue crack propagation, respectively.

### Theory:

In the Rate theory form (RTF), the crack velocity \(V\), can be expressed as [1].

\[
V = \frac{da}{dt} = L \frac{K_B T}{h} \exp\left[\frac{-\Delta G}{K_B T}\right] \quad (2-1)
\]

Where \(L\) is a crack propagation length per one step of activation energy \(\Delta G\). \(K_B\) is Boltzmann's constant, \(h\) is Plank's constants \((K_B=1.38E-23 JK^{-1}, \ h=6.62E-34 JS\)\), \(T\) is absolute temperature. The activation energy \(\Delta G\) can be expressed as function of external work, \(W\), for creep and fatigue crack propagation in equation (2-2), (2-3), respectively [1].

\[
\Delta \Gamma_c(W) = \Gamma_c - \delta_c * K \quad (2-2)
\]

\[
\Delta \Gamma_f(W) = \Gamma_f - \delta_f * \Delta K \quad (2-3)
\]

Where \(\Gamma_c\), \(\Gamma_f\) are surface activation energy for creep and fatigue crack propagation, respectively, and \(\Delta \Gamma_c\), \(\Delta \Gamma_f\) are activation energy for creep and fatigue crack propagation, respectively. The activation energy \((\Delta G)\) closed to zero where the stress intensity factor equal to threshold stress intensity factor for creep crack propagation \((K = K_{th})\), all one for fatigue crack propagation at \((\Delta K = \Delta K_{th})\) [2]. Therefore, the surface activation energy for creep and fatigue crack propagation is given by function of threshold stress intensity factor, \(K_{th}\) or \(\Delta K_{th}\), [1-3]:

\[
\delta_c = \alpha_c K_B T \quad (2-4)
\]

\[
L_c = (\beta_c h/K_B T) \exp(\Gamma_c/K_B T) \quad (2-14)
\]

\[
\delta_f = \alpha_f K_B T \quad (2-5)
\]

\[
L_f = (\beta_f h/K_B T) \exp(\Gamma_f/K_B T) \quad (2-15)
\]

RTF of crack velocity is given in the equations below for creep and fatigue crack propagation, when equation(2-2) and equation(2-3) were substituted in the equation(2-1):

\[
V = \frac{da}{dt} = L_c \frac{K_B T}{h} \exp\left[\frac{-G_c - \delta_c}{K_B T}\right] \quad (2-6)
\]

\[
V = \frac{da}{dt} = L_f \frac{K_B T}{h} \exp\left[\frac{-G_f - \delta_f}{K_B T}\right] \quad (2-7)
\]

The exponential function \(EF\) is represented one of the fitting models in \((V-K, \Delta K)\) diagrams, its given as the equation below [3] :

\[
V = \frac{da}{dt} = \beta e^{\alpha (K, \Delta K)} \quad (2-8)
\]

Comparing RTF with \(EF\) for creep and fatigue crack propagation, we find that our formulation of exponential function parameters EFP for creep and fatigue crack propagation is given as the equations below:

\[
\beta_c = \frac{L_c K_B T}{h} \exp\left[\frac{-\Gamma_c}{K_B T}\right] \quad (2-9)
\]

\[
\alpha_c = \frac{\delta_c}{K_B T} \quad (2-10)
\]

\[
\beta_f = \frac{L_f K_B T}{h} \exp\left[\frac{-\Gamma_f}{K_B T}\right] \quad (2-11)
\]

\[
\alpha_f = \frac{\delta_f}{K_B T} \quad (2-12)
\]

From Eq(2-9) ,EQ(2-10) for creep crack propagation, and Eq(2-11) , Eq(2-12) for fatigue crack propagation , we find the formulas of parameters \((L, \delta)\), they are given as the equations below for creep and fatigue crack propagation, respectively [4]:

\[
\delta_c = \alpha_c K_B T \quad (2-13)
\]

\[
L_c = (\beta_c h/K_B T) \exp(\Gamma_c/K_B T) \quad (2-14)
\]

\[
\delta_f = \alpha_f K_B T \quad (2-15)
\]

\[
L_f = (\beta_f h/K_B T) \exp(\Gamma_f/K_B T) \quad (2-16)
\]
Methods of calculations and analysis:

![Fig(1): shows the effect of temperature on the (V-K, ΔK) plots for creep and fatigue crack propagation in PVC pipes][5]

FIG. 1 shows V–K,ΔK diagram with different temperature for creep and fatigue crack propagation in PVC tubes [5], we get the experimental data of crack velocity and stress intensity factor via Get Data Graph Digitizer Program(GDGDP), as shown in fig (2), its a program for digitizing graphs, plots and maps. We inserted the experimental data in table (1) and redraw this data in the Graph Program(GP) to calculate (EFP) as shown in figures (3 & 4) for creep and fatigue crack propagation with different temperature degrees, respectively. (EFP) are inserted in table (2). We calculate RTFP, firstly, the work constant (δ) for creep and fatigue crack propagation from Eq(2-13) & Eq(2-15), respectively, dependent on α_c & α_f values in table (2). We calculate surface energy (G) for creep and fatigue crack propagation from Eq(2-4) & Eq(2-5), respectively, from δ_c, δ_f which were estimated previously and K_{th}, ΔK_{th} which were estimated via (GDGD) , as shown as in the table (1). Finally, we calculate a crack propagation length per one step of activation energy (L) for creep and fatigue crack propagation from Eq(2-14) & Eq(2-16), respectively, dependent on β_c & β_f values in the table (2) and G_c & G_f which were estimated previously. RTFP are inserted in table (3). Fig (5) shows the work constants (δ_c, δ_f) as linear function of temperature degrees for creep and fatigue crack propagation. There are given with the equations (3-1) and (3-2).
Fig(2): GetData Graph Digitizer program

Table (1): shows the experimental data for creep and fatigue crack propagation

<table>
<thead>
<tr>
<th>K(MPa√m)</th>
<th>da/dt(m/sec)</th>
<th>da/dt(MPa/√m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T=50°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.870042</td>
<td>1.709065E-8</td>
<td>0.6800402</td>
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<td>2.037697</td>
<td>1.725378E-8</td>
<td>0.7256337</td>
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<td>2.23778</td>
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<td>2.40654</td>
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<td>0.8074247</td>
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<tr>
<td>2.545459</td>
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<td>0.81731</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>1.624929</td>
<td>1.649618E-8</td>
<td>1.180990E-8</td>
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<td>1.498718E-7</td>
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<tr>
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<td>3.331953</td>
<td>2.041996E-8</td>
<td>1.629399E-7</td>
</tr>
</tbody>
</table>

Table (1): shows the experimental data for creep and fatigue crack propagation.
δc(T)=2.20329E-22*T–6.70902E-20 (3-1)
δf(T)=2.42572E-23*T–1.29104E-21 (3-2)
Linear regression coefficients (LRC)[4,6,7] have been calculated to the eq(3-1) and eq(3-2) in fig(5),ther are given with values below, respectively:
\[ \text{LRC}_c = 0.84343 \quad \text{LRC}_f = 0.772188 \]

Fig (6) shows the surface energy \((G_c, G_f)\) as linear function of temperature degrees for creep and fatigue crack propagation. Ther are given with the following equations.
\[ G_c(T) = -8.19138E-23*T + 3.53987E-20 \]  (3-3)
\[ G_f(T) = 5.37194E-24*T - 5.21007E-21 \]  (3-4)
\[ \text{LRC}_c = 0.822 \quad \text{LRC}_f = 0.12638 \]

Fig (7) shows the a crack propagation length per one step of activation energy \((L_c, L_f)\) as linear function of temperature degrees for creep and fatigue crack propagation. Ther are given with the following equations.
\[ L_c(T) = 1.96618E-22 T - 5.96698E-20 \]  (3-5)
\[ L_f(T) = 1.88768E-21*T - 5.5841E-19 \]  (3-6)
\[ \text{LRC}_c = 0.53973 \quad \text{LRC}_f = 0.6364 \]

We get to the theoretical crack velocity as function of (\(K, \Delta K, T\)) for creep and fatigue of crack propagation where RTFP as function of (T) are substituted in the Eq(2-6) & Eq(2-7), respectively. That functions are given with the equations below:
\[ \frac{dV}{dt} = \frac{dV}{dt} = \frac{dV}{dt} \]

\[ V(K, T) = \frac{T}{h} \quad \text{Eq}(3-7) \]

\[ V(\Delta K, T) = \frac{T}{h} \quad \text{Eq}(3-8) \]

Eq(3-7) is represented theoretical crack velocity as function to the temperature degrees and stress intensity factor for creep crack propagation . we investigated from this equation by comparative results this function with the experimental data of crack velocity for creep crack propagation as shown as in fig(8) .

Eq(3-8) is represented theoretical crack velocity as function to the temperature degrees and stress intensity factor range for fatigue crack propagation . In the same way, we investigated from this equation by comparative results.
this function with the experimental data of crack velocity for fatigue crack propagation as shown as in fig(9). Fig (8) and Fig(9) clear that the rate theory for crack growth was applied on the creep crack propagation with higher accuracy than the fatigue crack propagation. This theory, RT, was applied on the creep crack propagation for the alumina ceramic samples with different a grain size [7].

Table (2) shows (EFP) for creep and fatigue crack propagation

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>1/2 (M.Pa.(m)  )</th>
<th>α (1/Mpa/sec)</th>
<th>β (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>7.0963159E-9</td>
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<td>2.78767E-9</td>
</tr>
<tr>
<td>55</td>
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<td>4.06736E-9</td>
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<td>60</td>
<td>7.5834675E-9</td>
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<td>7.56978E-9</td>
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<td>65</td>
<td>7.7830296E-9</td>
<td>1.35265</td>
<td>1.27972E-8</td>
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<td>70</td>
<td>8.1754268E-9</td>
<td>2.07304</td>
<td>7.5909E-9</td>
</tr>
</tbody>
</table>

Table (3) shows (RTFP) for creep and fatigue crack propagation

<table>
<thead>
<tr>
<th>T (K)</th>
<th>δ (J/Mpa/m)</th>
<th>G(J)</th>
<th>L (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>323</td>
<td>4.6975E-21</td>
<td>8.784522E-21</td>
<td>2.971E-21</td>
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<tr>
<td>328</td>
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<td>4.319930E-21</td>
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<tr>
<td>333</td>
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<td>7.536724E-21</td>
<td>5.621942E-21</td>
</tr>
<tr>
<td>338</td>
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<td>1.048039E-20</td>
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<td>343</td>
<td>9.81529E-21</td>
<td>7.136124E-21</td>
<td>4.79433E-21</td>
</tr>
</tbody>
</table>

Fig (3): shows (EFP) of (V-ΔK) plot for creep crack propagation in PVC tubes with different temperature degrees.

Fig (4): shows (EFP) of (V-ΔK) plot for fatigue crack propagation in PVC tubes with different temperature degrees.
Fig(5) shows the temperature effect in the work constants for a creep and the fatigue crack propagation.

Fig (6) shows the temperature effect in the surface energy for a creep and the fatigue crack propagation.

Fig(7) shows the temperature effect in a crack propagation length per one step of activation energy for a creep and the fatigue crack propagation.

Fig (8) shows the comparative between experimental data of crack velocity and theoretical crack velocity in eq(3-7).

Fig (9) shows the comparative between experimental data of crack velocity and theoretical crack velocity in eq(3-8).

Conclusions:

The rate theory of crack growth has been applied for data of PVC pipe, making to use of the values (EFP), the (RTFP) were estimated. Energy terms ($\delta$ and $G_h$) of RTFP are taken the instability behavior with increase temperature degrees in the fatigue crack propagation but in the creep crack propagation they are stable behavior, $\delta$ increase and $G_h$ decrease, with the increase temperature degrees, as shown in fig5 & fig6, that refer to unagreement RTFP with experimental data of (V-$\Delta K$) diagram for fatigue crack propagation.

To shed light on the results about the (RT) comparative study to the creep
and fatigue crack propagation. The main consequence of this study is that theoretical crack velocity as function of (T,K),eq(3-7), for creep crack growth is fitting with experimental data to (V-K) diagram as shown in fig (8), this figure explain the comparative between experimental data of crack velocity and results theoretical crack velocity in eq(3-7). Theoretically, that refer to a good agreement between the theoretical prediction obtained using the correlations (RTFP) to the temperature as shown as in fig5, fig6 and fig7, with experimental data to the crack velocity for creep crack growth ,as shown in fig8. In the fatigue crack propagation, theoretical crack velocity as function (ΔK,T),eq(3-8), give far results from the experimental data of (V-ΔK) diagram as shown in fig9, due to LRC in fig5, fig6 and fig7 for fatigue crack growth are lesser than LRC in the same figures for creep crack growth.

References: