Semi – Bounded Modules

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Abstract*:*

Let R be a commutative ring with identity, and let M be a unity R-module. M is called a bounded R-module provided that there exists an element $x \in M$ such that $ann_R(M) = ann_R(x)$. As a generalization of this concept, a concept of semi-bounded module has been introduced as follows: M is called a semi-bounded if there exists an element $x \in M$ such that $\sqrt{ann_R M} = \sqrt{ann_R(x)}$. In this paper, some properties and characterizations of semi-bounded modules are given. Also, various basic results about semi-bounded modules are considered. Moreover, some relations between semi-bounded modules and other types of modules are considered.

Key words : Commutative ring, R-module, semi-bounded modules.

Introduction:

Let R be a commutative ring with unity, and let M be an R-module. An R-module M is called a semi-bounded module if there exists an element $x \in M$ such that $\sqrt{ann_R M} = \sqrt{ann_R(x)}$, where $ann_R M = \{r : r \in R \text{ and } r\, m = 0,\ \forall m \in M\}$. Our concern in this paper is to study semi-bounded modules and to look for any relation between semi-bounded modules and certain types of well-known modules especially with bounded modules. This paper consists of two sections. In the first section, the definition of a semi-bounded module is recallded and we illustrate it by some examples, we also give some of the basic properties of semi-bounded modules. We end the section by studying the localization of semi-bounded modules, see (1.13).

In section two, we study the relation between semi-bounded modules and bounded modules. It is clear that every bounded module is semi-bounded module, but the converse is not true in general. We give in (2.1), a conditions under which the two concepts are equivalent. Next we investigate the relationships between semi-bounded, prime, quasi-Dedekined, cyclic and multiplication modules see (2.3), (2.8).1. Semi-Bounded Modules Following [1] an R-module M is said to be a bounded module if there exists an element $x \in M$ such that $ann_R M = ann_R(x)$, where $ann_R M = \{r \in R;\ r\, m=0, \forall m \in M\}$. In this section the concept of semi-bounded module is introduced as a generalization of a bounded module and we give some properties and characterizations for this concept. We end this section by studying the behaviour of semi-bounded modules under localization.

Definition 1.1:

An R-module M is said to be semi-bounded module if there exists an element $x \in M$ such that $\sqrt{ann_R M} = \sqrt{ann_R(x)}$. We give some examples and remarks.

Remarks and Examples 1.2:

1. Every bounded R-module is a semi-bounded module. But the converse is not true in general. However we have no example.

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2. Every simple R-module is a semi-bounded module. But the converse is not true in general, for example: The Z-module Z is a semi-bounded module but not simple.
3. Q as a Z-module is a semi-bounded module.
4. Consider, the Z-module M = $Z\oplus Z_6$. Then $\sqrt{ann_Z M}=\sqrt{ann_Z(1,0)}$. Therefore, M is a semi-bounded module.
5. Z_{p^∞} is not a semi-bounded Z-module.
Proof: We know that every submodule of Z_{p^∞} is of the form $<\frac{1}{p^n}+Z>$, where n be a non-negative integer, so $\sqrt{ann_Z<\frac{1}{p^n}+Z>}=\sqrt{p^nZ}=pZ$. But $\sqrt{ann_Z Z_{p^\infty}}\neq\sqrt{ann_Z<\frac{1}{p^n}+Z>}$, so Z_{p^∞} is not a semi-bounded module.
6. Every cyclic R-module is a semi-bounded.
Proof: It follows directly by [2, Corollary 1.1.3, ch.1] and (1.2,1).
However, the converse is not true in general for example: The Z-module Q is a semi-bounded by (1.2,3), but not cyclic.
7. For each positive integer n, Z_n as a Z-module is a semi-bounded.
It is known that, if M is an *R*-module and I is an ideal of R which is contained in $ann_R M$ then M is an R\I-module, by taking (r+I) x = *r* x, $\forall$ x$\in$M, r$\in$R Now, we can give the following result.
Theorem 1.3 Let M be an R-module and let I be an ideal of R, which is contained in $ann_R M$. Then M is a semi-bounded R-module if and only if M is a semi-bounded R\ I- module.
Proof: If M is a semi-bounded R-module. To prove M is a semi-bounded R\I-module , we must prove $\sqrt{ann_{R\backslash I}M}=\sqrt{ann_{R\backslash I}(x)}$, for some x$\in$M .
It is clear that $\sqrt{ann_{R\backslash I}M}\subseteq\sqrt{ann_{R\backslash I}(X)}$ .Let (r + I) $\in\sqrt{ann_{R\backslash I}(X)}$,then $(r+I)^n x=0$ for some n$\in Z_+$, and so $(r^n+I)x=0$.Hence $r^n x=0$, then r$\in\sqrt{ann_R(X)}$. But M is a semi-bounded R-module ,so r$\in\sqrt{ann_R M}$ and $r^n\in ann_R M$. Then
$r^n m = 0$ for some n$\in Z_+$ and for all m$\in$M and hence $r^n m=(r^n+I)m=(r+I)^n m = 0$ for all m$\in$M. This implies $(r+I)^n\in ann_{R\backslash I}M$ and so (r+I)$\in\sqrt{ann_{R\backslash I}M}$.
Therefore, $\sqrt{ann_{R\backslash I}M}=\sqrt{ann_{R\backslash I}(x)}$ and hence M is a semi-bounded R\I-module.
Conversely , if M is a semi-bounded R\I-module, to prove M is a semi-bounded R-module,we must prove $\sqrt{ann_R M}=\sqrt{ann_R(X)}$, for some x $\in$ M. It is clear that $\sqrt{ann_R M}\subseteq\sqrt{ann_R(X)}$. Let r $\in\sqrt{ann_R(X)}$, then $r^n x=0$, for some n$\in Z_+$, but $r^n x = (r^n+I)x=(r+I)^n x = 0$, then (r+I) $\in\sqrt{ann_{R\backslash I}(X)}$. But M is a semi-bounded R\I-module, so (r+I)$\in\sqrt{ann_{R\backslash I}M}$ and $(r+I)^n\in ann_{R\backslash I}M$. Then, $(r+I)^n m = 0$ for some n$\in Z_+$ and for all m $\in$M and hence $(r^n+I) m = r^n m = 0$. This implies $r^n\in ann_R M$ and so are r$\in\sqrt{ann_R M}$ .Therefore, $\sqrt{ann_R M}=\sqrt{ann_R(X)}$ and hence M is a semi- bounded R-module .
The following result is an immediate consequence of theorem (1.3).

Corollary 1.4:
Let M be an R-module, then M is a semi-bounded R-module if and only if M is a semi-bounded $R\backslash ann_R M$-module.

Recall that an R-module M is called the internal direct sum of two R-modules M_1 and M_2 of M, written as $M=M_1\oplus M_2$ if and only if $M=M_1+M_2$ and $M_1\cap M_2=\{0\}$.

Now, the following result has been stated and proved.

Proposition 1.5:
Let M_1 and M_2 be two semi-bounded R-modules, then $M_1\oplus M_2$ is a semi-bounded R-module.

Proof: There exists $x\in M_1$ such that $\sqrt{ann_R M_1}=\sqrt{ann_R(x)}$. Also there exists $y\in M_2$ such that $\sqrt{ann_R M_2}=\sqrt{ann_R(y)}$. So $(x,y)\in M_1\oplus M_2$. We claim that $\sqrt{ann_R(M_1\oplus M_2)}=\sqrt{ann_R((x,y))}$. Let $r\in\sqrt{ann_R((x,y))}$, then $r^n(x,y)=(0,0)$ for some $n\in Z_+$, and so $(r^n x,r^n y)=(0,0)$. It follows that $r^n x=0$ and $r^n y=0$, that is $r^n\in ann_R(x)$ and $r^n\in ann_R(y)$ and so $r\in\sqrt{ann_R(x)}=\sqrt{ann_R M_1}$ and $r\in\sqrt{ann_R(y)}=\sqrt{ann_R M_2}$. Now, if $(m,m')\in M_1\oplus M_2$ then $r^n(m,m')=(r^n m,r^n m')=(0,0)$ for some $n\in Z_+$, implies that $r^n\in ann_R(M_1\oplus M_2)$ and so $r\in\sqrt{ann_R(M_1\oplus M_2)}$. Therefore $\sqrt{ann_R(M_1\oplus M_2)}=\sqrt{ann_R((x,y))}$.

Recall that a submodule B of an R-module M is called a direct summand of M if and only if there exists a submodule C of M such that $M=B\oplus C$, [3,p.31].

Note that a direct summand of a semi-bounded module need not be semi-bounded in general for example:

Let $M=Z\oplus Z_{p^\infty}$ as a Z-module, M is semi-bounded since $\sqrt{ann_Z M}=\sqrt{ann_Z((1,0))}$, but Z_{p^∞} is not semi-bounded Z-module, by (1.2,(5)).

By proposition (1.5) and by mathematical induction we have the following.

Corollary 1.6:
A finite direct sum of semi-bounded R-modules is semi-bounded.

Remark 1.7:
If M/N is a semi-bounded R-module, then it is not necessary that M is a semi-bunded R-module as the following example shows.

Let $M=Z_2\oplus \underset{p>2}{Z_p}$ as a Z-module and $N=\underset{p>2}{\oplus} Z_p$ is a submodule of M, where p is a prime number, so $M/N\approx Z_2$ is a semi-bounded Z-module. But M is not a semi-bounded Z-module.

Recall that a submodule N of an R-module M is said to be pure if $I\,M\cap N=I\,N$ for every ideal I of R. In case R is a principal ideal domain (PID) or M is cyclic, then N is pure if and only if $rM\cap N=rN$, $\forall r\in R$, [4].

By using this concept, we have the following:

Proposition 1.8:
Let N be a pure submodule of an R-module M such that M/N is a semi-bounded R-module and $ann_R M=[N\underset{R}{:}M]$. Then M is a semi-bounded R-module, where $[N\underset{R}{:}M]=\{r\in R; rM\subseteq N\}$.

Proof: Since M/N is a semi-bounded R-module, then there exists $\bar{x}\in M/N$ such that $\sqrt{ann_R M/N}=\sqrt{ann_R(\bar{x})}$. But $[N\underset{R}{:}M]=ann_R M$ by hypothesis. And on the other hand $ann_R M/N=[N\underset{R}{:}M]$, hence

$\sqrt{\text{ann}_R M} = \sqrt{\text{ann}_R(\bar{x})}$ …(1)

No we can show that $\sqrt{\text{ann}_R(\bar{x})} = \sqrt{\text{ann}_R(m)}$ for some $m \in M$. By [2,proposition (1.1.21), ch.1], we have $\text{ann}_R(\bar{x}) = \text{ann}_R(m)$ for some $m \in M$. Therefore

$\sqrt{\text{ann}_R(\bar{x})} = \sqrt{\text{ann}_R(m)}$ …(2)

Thus by (1) and (2), $\sqrt{\text{ann}_R M} = \sqrt{\text{ann}_R(m)}$ for some $m \in M$ and M is a semi-bounded R-module by definition (1.1).

Recall that an R-module M is called F-regular if every submodule of M is pure, [4].

The following result follows immediately from proposition (1.8).

Corollary 1.9:

Let M be F-regular R-module and N be a submodule of M such that M/N is a semi-bounded and $\text{ann}_R M = [N \underset{R}{:} M]$. Then M is a semi-bounded R-module.

Corollary 1.10:

Let N be a submodule of an R-module M, if every finitely generated submodule of N, is pure in M such that M/N is a semi-bounded and $\text{ann}_R M = [N \underset{R}{:} M]$. Then M is a semi-bounded.

Proof: Since every finitely generated submodule of N is pure in M, implies that N is pure submodule by [4,corollary 2]. Therefore M is a semi-bounded by proposition (1.8).

Hence we have another consequence of proposition (1.8).

Corollary1.11:

Let R be a (PID), M is an R-module, N is a divisible submodule of M such that M/N is a semi-bounded R-module and $\text{ann}_R M = [N \underset{R}{:} M]$, then M is a semi-bounded.

Proof: It is enough to show N is pure in M. Since N is a divisible submodule of M, then rN=N for every $0 \neq r \in R$ and so $rM \cap N = rM \cap rN = rN$. Thus N is pure. Therefore M is a semi-bounded R-module by proposition (1.8).Recall that a subset S of a ring R is called multiplicatively closed if $1 \in S$ and $ab \in S$ for every a, $b \in S$, we know that a proper ideal P in R is prime if and only if R\P is multiplicatively closed, [5 ,p.42].

Now, let M be an R-module and S be a multiplicatively closed subset of R and let R_S be the set of all fractional r/s where $r \in R$ and $s \in S$ and M_S be the set of all fractional x/s where $x \in M$ and $s \in S$. For $x_1, x_2 \in M$ and $s_1, s_2 \in S$, $x_1/s_1 = x_2/s_2$ if and only if there exists $t \in S$ such that $t(s_1x_2 - s_2x_1) = 0$. So, we can make M_S in to R_s-module by setting $x/s+y/t=(tx+sy)/st$ and $r/t \cdot x/s = rx/ts$ for every $x,y \in M$ and s, $t \in S$, $r \in R$. and M_S is the module of fractions**.** If S=R\P where P is a prime ideal we write M_p instead of Ms and R_p instead of R_s. R_p is often called the localization of R at P, and M_p is the localization of M at P. In order, we investigate the behaviour of a semi-bounded module under localization. But first we state and prove the following lemma.

Lemma 1.12:

Let M be an R-module, let S be a multiplicatively closed subset of R. If $(\text{ann}_R(x))_S = \text{ann}_{R_S}(x)_S$, then $\left(\sqrt{\text{ann}_R(x)}\right)_S = \sqrt{\text{ann}_{R_S}(x)_S}$.

Proof: Let $m \in \left(\sqrt{\text{ann}_R(x)}\right)_S$, then m=r/s for some $r \in \sqrt{\text{ann}_R(x)}$ and $s \in S$ and hence $m^n = (r/s)^n \in (\text{ann}_R(x))_S = \text{ann}_{R_S}(x)_S$ for some $n \in Z_+$. Thus $m = r/s \in \sqrt{\text{ann}_{R_S}(x)_S}$ and so

$\left(\sqrt{ann_R(x)}\right)_S \subseteq \sqrt{ann_{R_S}(x)_S}$ …(1)

Now, let $m=r/s\in\sqrt{ann_{R_S}(x)_S}$ then $m^n=(r/s)^n=r^n/s^n\in ann_{R_S}(x)_S=(ann_R(x))_S$ for some $n\in Z_+$. Hence, there exists $r_1\in ann_R(x)$ and $t\in S$ such that $r^n/s^n=r_1/t$ and so there exists $t_1\in S$ such that $t_1tr^n=t_1r_1s^n\in ann_R(x)$, which implies that $t_1tr^n\in ann_R(x)$ and so $t_1tr\in\sqrt{ann_R(x)}$ .Therefore, $m=r/s=t_1tr/t_1ts\in\left(\sqrt{ann_R(x)}\right)_S$ and $\sqrt{ann_{R_S}(x)_S}\subseteq\left(\sqrt{ann_R(x)}\right)_S$ …(2)

By (1) and (2) we get $\left(\sqrt{ann_R(x)}\right)_S=\sqrt{ann_{R_S}(x)_S}$.

Now, the following proposition has been stated and proved:

Proposition 1.13:

Let M be a finitely generated semi-bounded R-module and S be a multiplicatively closed subset of R, then M_S is a semi-bounded R_S-module.

Proof: Since M is a semi-bounded R-module, then there exists $x\in M$ such that $\sqrt{ann_R M}=\sqrt{ann_R(x)}$, and so $\left(\sqrt{ann_R M}\right)_S=\left(\sqrt{ann_R(x)}\right)_S$. And since M is a finitely generated, so $(ann_R M)_S=ann_{R_S}M_S$ by [6,proposition 3.14,p.43]. Hence $\left(\sqrt{ann_R M}\right)_S=\sqrt{ann_{R_S}M_S}$ by [7,lemma 2.1.24,ch.2].

On the other hand, $\left(\sqrt{ann_R(x)}\right)_S=\sqrt{ann_{R_S}(x)_S}$ by lemma (1.12). Therefore $\sqrt{ann_{R_S}M_S}=\sqrt{ann_{R_S}(x)_S}$ and M_S is a semi-bounded R_S-module.

The following corollary follows immediately from proposition (1.13).

If P is a prime ideal of R and M is a finitely generated semi-bounded R-module, then M_P is a semi-bounded R_P-module.

2. Some Relations Between Semi-Bounded Modules and Other Modules

In this section, we study the relationships between semi-bounded modules and other modules such as bounded modules, prime, quasi-Dedekined, cyclic and multiplication modules. As we have mentioned in (1.2(1)), that bounded module is a semi-bounded module and the converse need not be true in general. However the following result shows that the converse is true. But first the following definition is needed.Recall that a submodule N of an R-module M is said to be semi-prime if for every $r\in R$, $x\in M$, $k\in Z_+$, such that $r^kx\in N$, then $rx\in N$, see [7].

Proposition 2.1:

If M is a semi-bounded and (0) is a semi-prime submodule of M, then M is a bounded R-module.

Proof: To prove M is a bounded module, we must prove $ann_R(M)=ann_R(x)$ for some $x\in M$.It is clear that $ann_R(M)\subseteq ann_R(x)$. Let $r\in ann_R(x)$, hence $r\in\sqrt{ann_R(x)}$. But M is a semi-bounded module, so $\sqrt{ann_R M}=\sqrt{ann_R(x)}$. Hence, $r\in\sqrt{ann_R M}$, which implies that $r^n\in ann_R(M)$ for some $n\in Z_+$. Thus, $r^nm=0$ for each $m\in M$. But (0) is a semi-prime submodule of M, then $rm=0$ and hence $r\in ann_R(M)$, so that $ann_R(x)\subseteq ann_R(M)$. Therefore, $ann_R(x)=ann_R(M)$, that is M is bounded R-module.

Next, we study the relationship between semi-bounded modules and prime modules. And we give a condition under which the two concepts are equivalent. Recall that an R-module M is said to be prime module if $ann_R M = ann_R N$ for every non-zero submodule N of M, [8]. It is clear that every prime R-module is bounded and hence it is semi-bounded, but the converse need not be true in general, for example:

Let $M=Z_8$ as a Z-module is bounded and so semi-bounded, but not prime module since $ann_Z Z_8=8Z$ but $ann_Z(\bar{2})=4Z$. In order we can give the following result. But first we need the following definition. Recall that a submodule N of an R-module M is called a bounded if there exists $x\in N$ such that $ann_R N=ann_R(x)$, see [2].

Proposition 2.2:

Let M be an R-module and let $0\neq x\in M$ such that:

1. $\sqrt{ann_R M}=\sqrt{ann_R(x)}$

2. (0) is a semi-prime submodule of M.

3. Every non-zero submodule N of M is bounded. Then M is a prime R-module.

Proof: Let N be a non-zero R-submodule of M, to prove $ann_R M=ann_R N$. Since every non-zero submodule N of M is bounded, then $ann_R N=ann_R(x)$ for some $x\in N$. Therefore (by condition 1) $\sqrt{ann_R M}=\sqrt{ann_R N}$ and M is a primary R-module by [7,theorem (2.1.3),ch.2]. But (0) is a semi-prime submodule of M (by condition 2), then M is a prime R-module by [7,corollary (2.3.3),ch.2].

The following corollary, we give a condition under which a semi-bounded module is prime.

Corollary 2.3:

If M is a semi-bounded R-module such that every non-zero submodule N of M is bounded and (0) is a semi-prime submodule of M, then M is a prime R-module Next, we study the relationship between semi-bounded and quasi-Dedekined module.Now, the following definitions are needed. Let M be an R-module. A submodule N of M is called quasi-invertible if $Hom_R(M\backslash N,M)=0$ [9,definition 1.1.1,ch.1]. And M is called quasi-Dedekined R-module if every submodule N of M is quasi-invertible, [9,definition 2.1.1,ch.2].

Remark 2.4:

Every quasi-Dedekind R-module is a semi-bounded R-module.

Proof: By [9,theorem 1.7,ch.2] everyquasi-Dedekind is prime and hence it is semi-bounded. However, the converse is not true in general, for example:Z_8 as Z-module is semi-bounded. But it is not prime (since $ann_Z Z_8=8Z$ and $ann_Z(\bar{2})=4Z$). So it is not quasi-Dedekind. In the following proposition, we give a condition under which the converse of remark (2.4) is true.

proposition 2.5: If M is a uniform semi-bounded R-module such that (0) is a semi-prime submodule of M and every non-zero submodule of M is bounded, then M is a quasi- Dedekind.

Proof: By proposition (2.3), M is a prime R-module. But M is uniform, so by [9,theorem 11,ch.2] we obtain the result. As we mentioned in (1.2,(6)), that cyclic module is a semi-bounded and the converse need not be true in general. However the following result shows that the converse is true. But first the following definition is needed. Recall that an R-module M is said to be fully stable if $ann_M(ann_R(x))=(x)$ for each $x\in M$. [10,corollary 3.5]. In the following proposition, we give a condition under which the converse of (1.2,(6)) is true.

Proposition 2.6:

If M is a fully stable semi-bounded R-module and (0) is a semi-prime submodule, then M is cyclic R-module.

Proof: Since M is a semi-bounded R-module and (0) is a semi-prime submodule, then M is a bounded by proposition (2.1). But M is a fully stable, so by [2,proposition 1.1.4,ch.1] we obtain the result. Now, the relationship between semi-bounded modules and multiplication modules has been studied. And we give a condition under which the two concepts are equivalent. Recall that an R-module M is said to be multiplication module if for every submodule N of M, there exists an ideal I in R such that N=I M, [11]. Note that it is not necessary that every semi-bounded is multiplication for example: Q as a Z-module is semi-bounded, but not multiplication, since Z is a submodule of Q, but $\nexists$ I an ideal of Z such that IQ=Z. In the following corollary, we give a sufficient condition for semi-bounded module is multiplication.

Corollary 2.7:

If M is a fully stable semi-bounded R-module and (0) is a semi-prime submodule, then M is a multiplication R-module.

Proof: By proposition (2.6), we obtain that M is a cyclic R-module. Then it is clear that M is a multiplication R-module. In the following proposition, we give some condition under which the converse of corollary (2.7) is true. But first we need the following definition. Recall that an R-module M is called a quasi-prime R-module if and only if $ann_R N$ is a prime ideal for each non-zero submodule N of M, [6].

Proposition 2.8:

If M is a multiplication quasi-prime R-module, then M is a semi-bounded R-module.

Proof: Since M is a multiplication quasi-prime R-module, so M is a prime module by [6,theorem 1.4.1,ch.1], hence it is a bounded. Therefore M is a semi-bounded R-module by (1.2,(1)).

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الموديولات شبه المقيدة

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الخلاصة :

لتكن R حلقة ابدالية ذات محايد وليكن M مقاساً احادياً على R. اطلق على M مقاساً مقيداً اذا وجد x عنصراً ينتمي الى M بحيث $ann_R(M) = ann_R(x)$ في المصدر [1].كتعميم لهذا المفهوم، قدمنا مفهوم مقاس شبه مقيداً كما يلي:يطلق على M مقاساً شبه مقيداً اذا وجد x عنصراً ينتمي الى M بحيث $\sqrt{ann_R M} = \sqrt{ann_R(x)}$. في هذا البحث، تم تعريف بعض الخواص والتميزات حول المقاسات شبه المقيدة. كذلك دُرست بعض النتائج الاساسية حول المقاسات شبه المقيدة. بالاضافة الى هذا دُرست بعض العلاقات بين المقاسات شبه المقيدة مع انواع اخرى من المقاسات.