Comparison of Maximum Likelihood and some Bayes Estimators for Maxwell Distribution based on Non-informative Priors

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Abstract:

In this paper, Bayes estimators of the parameter of Maxwell distribution have been derived along with maximum likelihood estimator. The non-informative priors; Jeffreys and the extension of Jeffreys prior information has been considered under two different loss functions, the squared error loss function and the modified squared error loss function for comparison purpose. A simulation study has been developed in order to gain an insight into the performance on small, moderate and large samples. The performance of these estimators has been explored numerically under different conditions. The efficiency for the estimators was compared according to the mean square error MSE. The results of comparison by MSE show that the efficiency of Bayes estimators of the shape parameter of the Maxwell distribution decreases with the increase of Jeffreys prior constants. The results also show that values of Bayes estimators are almost close to the maximum likelihood estimator when the Jeffreys prior constants are small, yet they are identical in some certain cases. Comparison with respect to loss functions show that Bayes estimators under the modified squared error loss function has greater *MSE* than the squared error loss function especially with the increase of *r*.

Key words: Maxwell distribution, Bayes Estimators, informative and non-informative prior information's, square and modified square error loss functions.

Introduction:

In physics particularly statistical mechanics, the Maxwell–Boltzmann distribution describes particle speeds in gases, where the particles move freely between short collisions, but do not interact with each other, as a function of the temperature of the system, the mass of the particle, and speed of the particle[1]. The Maxwell distribution gives also the distribution of speeds of molecules in thermal equilibrium as given by statistical mechanics[2].The Maxwell distribution was first introduced in the literature as a lifetime model by Tyagi and Bhattacharya (1989)[3]. They obtained Bayes estimates and minimum variance unbiased estimators of the parameter and reliability function. The leading software 'Mathematica' has included Maxwell distribution and its properties in its software library[4]. Bekker and Roux (2005) studied Empirical Bayes estimation for Maxwell distribution[5]. Ali Kasmi*et al* (2012) discussed Baysian estimation for two component mixture of Maxwell distribution, assuming type I censored data[6].This study attempts to deal with the problem of Bayesian estimation in Maxwell distribution. We propose to obtain Bayes estimators based on a class of non-informative priors under the

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assumption of two loss functions; the squared error loss function and the modified squared error loss function. We consider two non-informative priors; Jeffreys prior and the extension of Jeffreys prior. The obtained Baysian estimates of the shape parameter θ are compared to its maximum likelihood counterpart. The performance of these estimates is assessed using Monte Carlo simulation study, considering various sample sizes; several specific values of the parameter θ and Jeffreys prior constants. The results are summarized in tables and followed by the conclusions.

Maxwell Distribution

Let us consider $x_1, x_2, \ldots, x_n$ to be independent and identically distributed random variables from Maxwell distribution having pdf:

$$f(x|\theta) = \frac{4}{\sqrt{\pi}}\frac{1}{\theta^{\frac{3}{2}}}x^2 e^{-\frac{x^2}{\theta}} \quad 0 < x < \infty;\ \theta > 0 \quad (1)$$

where θ is the shape parameter. The cumulative distribution function (cdf) in its simplest form is given by:

$$F(x|\ \theta) = \frac{1}{\Gamma\left(\frac{3}{2}\right)}\Gamma\left(\frac{x^2}{\theta},\frac{3}{2}\right)$$

where $\Gamma(x,\alpha) = \int_0^x u^{\alpha-1}e^{-u}du$

Yet there are other forms of the cdf found in Krishna H. and Malik M. (2009)[4].

Maximum Likelihood Estimator

The likelihood function for Maxwell pdf is given by:

$$L(x_i;\theta) = \left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{\theta^{3n/2}}\prod_{i=1}^{n} x_i^2\, exp\left[-\frac{\sum_{i=1}^n x_i^2}{\theta}\right]$$

By taking the log and differentiating partially with respect to θ, we get:

$$\frac{\partial \ln L(x_i;\theta)}{\partial\theta} = \frac{-3n}{2\theta} + \frac{\sum_{i=1}^n x_i^2}{\theta^2} \quad (2)$$

Then the MLE of θ is the solution of equation (2) after equating the first derivative to zero is given by:

$$\hat{\theta} = \frac{2\sum_{i=1}^n x_i^2}{3n} \quad (3)$$

Bayes' Estimators

To obtain Bayes estimators, we assume that θ is a real valued random variable with probability density function $\pi(\theta)$. The posterior distribution of θ is the conditional probability density function of θ given the data, which is given by:

$$h(\theta|\boldsymbol{x}) = \frac{\prod_{i=1}^n f(x_i|\theta)\,\pi(\theta)}{\int_0^\infty \prod_{i=1}^n f(x_i|\theta)\pi(\theta)d\theta} \quad (4)$$

Once the posterior has been obtained, a loss function is attached to indicate the loss coming up when the estimate $\hat{\theta}$ deviates from the true value. The loss should be zero if and only if $\hat{\theta} = \theta$. We consider two loss functions

1- The squared error loss function:

$$L_1(\hat{\theta},\theta) = (\hat{\theta}-\theta)^2$$

Bayes' estimator will be the estimator that minimizes the posterior risk given by

$$R_1(\hat{\theta}-\ \theta) = E[L_1(\hat{\theta},\theta)] = \int_0^\infty (\hat{\theta}-\theta)^2\, h(\theta|\boldsymbol{x})\, d\theta$$

which is minimized when:

$$\hat{\theta} = E(\theta|\boldsymbol{x}) = \int_0^\infty \theta\ \ h(\theta|\boldsymbol{x})\ d\theta \quad (5)$$

2- The modified squared error loss function: $L_2(\hat{\theta}-\theta) = \theta^r(\hat{\theta}-\theta)^2$

Bayes' estimator will be the estimator that minimizes the posterior risk given by

$$R_2(\hat{\theta}- \theta) = E[L_2(\hat{\theta},\theta)]$$

$$= \int_0^\infty \theta^r (\hat{\theta}-\theta)^2 \, h(\theta|\boldsymbol{x})\, d\theta$$

which is minimized when

$$\hat{\theta} = \frac{E(\theta^{r+1}|\boldsymbol{x})}{E(\theta^r|\boldsymbol{x})} \quad (6)$$

where

$$E(\theta^r|\boldsymbol{x}) = \int_0^\infty \theta^r h(\theta|\boldsymbol{x}) \; d\theta$$

Bayes estimators for the parameter θ, was considered with non-informative priors.

It refers to the case when very little or limited information is available a priori[7]. As a general rule to find a non-informative prior, Jeffreys suggested the following rule for the likelihood $f(\boldsymbol{x}|\theta)$

$\pi(\theta) \propto \sqrt{I(\theta)}$, where $I(\theta)$ is the Fisher information. Then,

$\pi(\theta) = constant\, \sqrt{I(\theta)}$.

Following is the derivation of these estimators

i)Jeffreys prior information, under squared error loss function

When we introduce a power $c > 0$, we arrive at the following generalization of the non- informative Jeffreys prior:

$$\pi_1(\theta) = \frac{1}{\theta^c} \; ; \; c > 0 \quad (7)$$

The posterior distribution for the parameter θ given the data (x_1, x_2... x_n) is:

$$h(\theta \,|\, \mathbf{x}) = \frac{\prod_{i=1}^n f(x_i|\theta)\,\pi(\theta)}{\int_0^\infty \prod_{i=1}^n f(x_i|\theta)\pi(\theta) d\theta} = \frac{e^{-\frac{\sum_{i=1}^n x_i^2}{\theta}}\theta^{\frac{-3n-2c}{2}}}{\int_0^\infty \; e^{-\frac{\sum_{i=1}^n x_i^2}{\theta}}\theta^{\frac{-3n-2c}{2}} d\theta}$$

Let $y = \frac{\sum_{i=1}^n x_i^2}{\theta}$, then

$$h(\theta|\boldsymbol{x}) = \frac{y^{\frac{3n+2c}{2}} e^{-y}}{-\sum_{i=1}^n x_i^2 \int_0^\infty y^{\frac{3n+2c-4}{2}} e^{-y} \; dy}$$

And the posterior distribution become as follows

$$h(\theta|\boldsymbol{x}) = \frac{-(\sum_{i=1}^n x_i^2)^{\frac{3n+2c-2}{2}} e^{-\frac{\sum_{i=1}^n x_i^2}{\theta}}}{\theta^{\frac{3n+2c}{2}}\Gamma\left(\frac{3n+2c-2}{2}\right)} \quad (8)$$

According to the squared error loss function, the corresponding Bayes' estimator for the parameter θ is such that:

$$\theta_1^* = E(\theta|\boldsymbol{x}) \quad (9)$$

Substituting the posterior distribution (8) in (9), we get:

$$E(\theta|\boldsymbol{x}) = \frac{-(\sum_{i=1}^n x_i^2)^{\frac{3n+2c-2}{2}}}{\Gamma\left(\frac{3n+2c-2}{2}\right)} \int_0^\infty \theta^{\frac{-3n-2c+2}{2}} \; e^{\frac{-\sum_{i=1}^n x_i^2}{\theta}} d\theta$$

Let

$$y = \frac{\sum_{i=1}^n x_i^2}{\theta}$$

Then

$$E(\theta|\boldsymbol{x}) = \frac{-(\sum_{i=1}^n x_i^2)^{\frac{3n+2c-2}{2}}}{\Gamma\left(\frac{3n+2c-2}{2}\right)} \int_0^\infty e^{-y}\left(\frac{\sum_{i=1}^n x_i^2}{y}\right)^{\frac{-3n-2c+2}{2}} \frac{-\sum_{i=1}^n x_i^2}{y^2} dy$$

And after few steps

$$E(\theta|\boldsymbol{x}) = \frac{\sum_{i=1}^{n} x_i^2}{\Gamma\left(\frac{3n+2c-2}{2}\right)} \int_0^\infty e^{-y} y^{\frac{3n+2c-6}{2}} dy = \frac{\sum_{i=1}^{n} x_i^2 \quad \Gamma\left(\frac{3n+2c-4}{2}\right)}{\Gamma\left(\frac{3n+2c-2}{2}\right)}$$

Hence,

$$\theta_1^* = \frac{2\sum_{i=1}^{n} x_i^2}{3n+2c-4}$$

ii)Jeffreys' prior information, under modified squared error loss function

Now, according to the modified squared error loss function, the corresponding Bayes' estimator for θ is such that:

$$\theta_2^* = \frac{E(\theta^{r+1}|\boldsymbol{x})}{E(\theta^r|\boldsymbol{x})} \quad (10)$$

Substituting the posterior distribution (8) in (10), we get:

$$E(\theta^r|\boldsymbol{x}) = \frac{-(\sum_{i=1}^{n} x_i^2)^{\frac{3n+2c-2}{2}}}{\Gamma\left(\frac{3n+2c-2}{2}\right)} \int_0^\infty \theta^{\frac{-3n-2c+2r}{2}} e^{\frac{-\sum_{i=1}^{n} x_i^2}{\theta}} d\theta$$

Let

$$y = \frac{\sum_{i=1}^{n} x_i^2}{\theta}$$

Then

$$E(\theta^r|\boldsymbol{x}) = \frac{-(\sum_{i=1}^{n} x_i^2)^{\frac{3n+2c-2}{2}}}{\Gamma\left(\frac{3n+2c-2}{2}\right)} \int_0^\infty e^{-y} \left(\frac{y}{\sum_{i=1}^{n} x_i^2}\right)^{\frac{3n+2c-2r}{2}} \frac{-\sum_{i=1}^{n} x_i^2}{y^2} dy$$

Hence,

$$E(\theta^r|\boldsymbol{x}) = \left(\sum_{i=1}^{n} x_i^2\right)^r \frac{\Gamma\left(\frac{3n+2c-2r-2}{2}\right)}{\Gamma\left(\frac{3n+2c-2}{2}\right)} \quad (11)$$

In the same manner, we find the numerator of θ_1^* which become:

$$E(\theta^{r+1}|\boldsymbol{x}) = \left(\sum_{i=1}^{n} x_i^2\right)^{r+1} \frac{\Gamma\left(\frac{3n+2c-2r-4}{2}\right)}{\Gamma\left(\frac{3n+2c-2}{2}\right)} \quad (12)$$

And from (11) and (12), we get:

$$\theta_2^* = \frac{2\sum_{i=1}^{n} x_i^2}{(3n+2c-2r-4)}$$

iii) Extension of Jeffreys' prior information, under squared error loss function

The extension of Jeffreys' prior is $\pi(\theta) \propto [I(\theta)]^c$, $c \epsilon R^+$

With $\pi(\theta) \propto \left[\frac{n}{\theta^2}\right]^c$, then

$\pi_2(\theta) = k\frac{n^c}{\theta^{2c}}$; $c > 0$, and k is a constant. (13)

The posterior distribution for the parameter θ given the data ($x_1, x_2 \ldots x_n$) is:

$$h(\theta \mid \mathbf{x}) = \frac{\prod_{i=1}^{n} f(x_i|\theta)\,\pi(\theta)}{\int_0^\infty \prod_{i=1}^{n} f(x_i|\theta)\pi(\theta)d\theta} = \frac{e^{-\frac{\sum_{i=1}^{n} x_i^2}{\theta}} \theta^{\frac{-3n-4c}{2}}}{\int_0^\infty e^{-\frac{\sum_{i=1}^{n} x_i^2}{\theta}} \theta^{\frac{-3n-4c}{2}} d\theta}$$

Let

$$y = \frac{\sum_{i=1}^{n} x_i^2}{\theta}$$

, then

$$h(\theta|\boldsymbol{x}) = \frac{y^{\frac{3n+4c}{2}} e^{-y}}{-\sum_{i=1}^{n} x_i^2 \int_0^\infty y^{\frac{3n+4c-4}{2}} e^{-y}\, dy}$$

And the posterior distribution become as follows:

$$h(\theta|\boldsymbol{x}) = \frac{-(\sum_{i=1}^{n} x_i^2)^{\frac{3n+4c-2}{2}} e^{-\frac{\sum_{i=1}^{n} x_i^2}{\theta}}}{\theta^{\frac{3n+4c}{2}} \Gamma\left(\frac{3n+4c-2}{2}\right)}$$
(14)

According to the square error loss function, the corresponding Bayes' estimator for the parameter θ is such that:

$\theta_3^* = E(\theta|\boldsymbol{x})$
(15)

Substituting the posterior distribution (14) in (15), we get:

$$E(\theta|\boldsymbol{x}) = \frac{-(\sum_{i=1}^{n} x_i^2)^{\frac{3n+4c-2}{2}}}{\Gamma\left(\frac{3n+4c-2}{2}\right)} \int_0^\infty \theta^{\frac{-3n-4c+2}{2}} e^{\frac{-\sum_{i=1}^{n} x_i^2}{\theta}} d\theta$$

Let

$$y = \frac{\sum_{i=1}^{n} x_i^2}{\theta}$$

Then

$$E(\theta|\boldsymbol{x}) = \frac{-(\sum_{i=1}^{n} x_i^2)^{\frac{3n+4c-2}{2}}}{\Gamma\left(\frac{3n+4c-2}{2}\right)} \int_0^\infty e^{-y} \left(\frac{y}{\sum_{i=1}^{n} x_i^2}\right)^{\frac{3n+4c-2}{2}} \frac{-\sum_{i=1}^{n} x_i^2}{y^2} dy$$

And after few steps

$$E(\theta|\boldsymbol{x}) = \frac{\sum_{i=1}^{n} x_i^2}{\Gamma\left(\frac{3n+4c-2}{2}\right)} \int_0^\infty e^{-y} y^{\frac{3n+4c-6}{2}} dy = \frac{\sum_{i=1}^{n} x_i^2 \quad \Gamma\left(\frac{3n+4c-4}{2}\right)}{\Gamma\left(\frac{3n+4c-2}{2}\right)}$$

Hence,

$$\theta_3^* = \frac{2\sum_{i=1}^{n} x_i^2}{3n+4c-4}$$

iv) Extension of Jeffreys' prior information, under modified square error loss function

Now, according to the modified squared error loss function, the corresponding Bayes' estimator for θ is such that:

$\theta_4^* = \frac{E(\theta^{r+1}|\boldsymbol{x})}{E(\theta^r|\boldsymbol{x})}$(16)

Substituting with the posterior distribution (14) in (16), we get:

$$E(\theta^r|\boldsymbol{x}) = \frac{-(\sum_{i=1}^{n} x_i^2)^{\frac{3n+4c-2}{2}}}{\Gamma\left(\frac{3n+4c-2}{2}\right)} \int_0^\infty \theta^{\frac{-3n-4c+2r}{2}} e^{\frac{-\sum_{i=1}^{n} x_i^2}{\theta}} d\theta$$

Let

$$y = \frac{\sum_{i=1}^{n} x_i^2}{\theta}$$

Then

$$E(\theta^r|\boldsymbol{x}) = \frac{-(\sum_{i=1}^{n} x_i^2)^{\frac{3n+2c-2}{2}}}{\Gamma\left(\frac{3n+2c-2}{2}\right)}$$

$$\int_0^\infty e^{-y} \left(\frac{y}{\sum_{i=1}^{n} x_i^2}\right)^{\frac{3n+2c-2r}{2}} \frac{-\sum_{i=1}^{n} x_i^2}{y^2} dy$$

Hence,

$$E(\theta^r|\boldsymbol{x}) = \left(\sum_{i=1}^{n} x_i^2\right)^r \frac{\Gamma\left(\frac{3n+4c-2r-2}{2}\right)}{\Gamma\left(\frac{3n+4c-2}{2}\right)}$$
(17)

In the same manner, we find the numerator of θ_4^* which become:

$$E(\theta^{r+1}|\boldsymbol{x}) = \left(\sum_{i=1}^{n} x_i^2\right)^{r+1} \frac{\Gamma\left(\frac{3n+4c-2r-4}{2}\right)}{\Gamma\left(\frac{3n+4c-2}{2}\right)}$$
(18)

And from (17) and (18), we get:

$$\theta_4^* = \frac{2\sum_{i=1}^{n} x_i^2}{(3n+4c-2r-4)}$$

Simulation study

Monte-Carlo simulation study is performed to compare the efficiencies of various estimates developed in previous sections. For the generation of a sample from Maxwell distribution, we followed an algorithm suggested by Krishna and Malik (2009)[4]. For this study I = 2000 samples of size n = 20, 50, and100 were generated from Maxwell distribution with $\theta = 0.5, 1,$ and *2*. Three values of *c (c = 1, 3,* and *5)* and four values of *r (r = -1, 1, 3,* and *5)* are chosen. The averages of the estimated values of θ and the corresponding mean square errors *(MSE)* were computed, to compare the efficiency of each of the five estimators, where

$$MSE(\hat{\theta}) = \frac{\sum_{i=1}^{2000}(\hat{\theta}-\theta)^2}{I}$$

The results are summarized and tabulated in the following tables for each estimator and for all sample sizes. The entries within parenthesis indicate the *MSE.*

Table 1: Estimates ofthe parameter$\boldsymbol{\theta}$ and *MSE* with $\boldsymbol{\theta}$ **=0.5,** ***c*** **= 1**

n	$\hat{\theta}$	θ_1^*	θ_2^*			θ_3^*	θ_4^*		
			r =1	*r= 3*	*r = 5*		*r =1*	*r = 3*	*r = 5*
20	*.4997 (.0083)*	*.5169 (.0091)*	*.5354 (.0107)*	*.5766 (.0169)*	*.6246 (.0284)*	*.4997 (.0083)*	*.5169 (.0091)*	*.5552 (.0132)*	*.5996 (.0218)*
50	*.4997 (.0033)*	*.5064 (.0034)*	*.5133 (.0037)*	*.5278 (.0045)*	*.5431 (.0058)*	*.4997 (.0033)*	*.5064 (.0034)*	*.5205 (.0040)*	*.5353 (0050)*
100	*.4997 (.0016)*	*.5030 (.0017)*	*.5064 (.0018)*	*.5134 (.0019)*	*.5205 (.0022)*	*.4997 (.0017)*	*.5030 (.0017)*	*.5099 (.0018)*	*.5169 (.0021)*

Table 2: Estimates ofthe parameter$\boldsymbol{\theta}$ and *MSE* with $\boldsymbol{\theta}$ **= 0.5,** ***c*** **= 3**

n	$\hat{\theta}$	θ_1^*	θ_2^*			θ_3^*	θ_4^*		
			r =1	*r= 3*	*r = 5*		*r =1*	*r = 3*	*r = 5*
20	*.4997 (.0083)*	*.4836 (.0080)*	*.4997 (.0083)*	*.5354 (.0107)*	*.5766 (.0169)*	*.4409 (.0099)*	*.4543 (.0089)*	*.4836 (.0080)*	*.5169 (.0091)*
50	*.4997 (.0033)*	*.4931 (.0032)*	*.4996 (.0033)*	*.5133 (.0037)*	*.5278 (.0044)*	*.4743 (.0036)*	*.4804 (.0034)*	*.4931 (.0033)*	*.5064 (.0034)*
100	*.4997 (.0017)*	*.4964 (.0017)*	*.4997 (.0017)*	*.5064 (.0018)*	*.5134 (.0019)*	*.4867 (.0018)*	*.4899 (.0017)*	*.4964 (.0017)*	*.5030 (.0018)*

Table 3: Estimates ofthe parameter$\boldsymbol{\theta}$ and *MSE* with $\boldsymbol{\theta}$ **= 0.5,** $\boldsymbol{c}$ **= 5**

n	$\hat{\theta}$	θ_1^*	θ_2^*			θ_3^*	θ_4^*		
			r =1	r= 3	r = 5		r =1	r= 3	r = 5
20	.4997 (.0083)	.4543 (.0089)	.4685 (.0083)	.4997 (.0083)	.5354 (.0107)	.3945 (.0163)	.4543 (.0089)	.4283 (.0112)	.4543 (.0089)
50	.4997 (.0033)	.4804 (.0034)	.4867 (.0033)	.4997 (.0033)	.5133 (.0037)	.4515 (.0051)	.4804 (.0034)	.4684 (.0039)	.4804 (.0034)
100	.4997 (.0017)	.4899 (.0017)	.4931 (.0017)	.4997 (.0017)	.5064 (.0018)	.4774 (.0022)	.4899 (.0017)	.4774 (.0020)	.4899 (.0017)

Table 4: Estimates ofthe parameter$\boldsymbol{\theta}$ and *MSE* with $\boldsymbol{\theta}$ **= 1,** $\boldsymbol{c}$ **= 1**

n	$\hat{\theta}$	θ_1^*	θ_2^*			θ_3^*	θ_4^*		
			r =-1	r= 1	r = 3		r =-1	r= 1	r = 3
20	.9994 (.0330)	1.0339 (.0365)	.9994 (.0330)	1.0708 (.0429)	1.1532 (.0674)	.9994 (.0330)	.9672 (.0320)	1.0339 (.0356)	1.1105 (.0530)
50	.9993 (.0132)	1.0128 (.0138)	.9993 (.0100)	1.0267 (.0147)	1.0556 (.0179)	.9993 (.0132)	.9862 (.0131)	1.0281 (.0138)	1.0409 (.0160)
100	.9994 (.0067)	1.0061 (.0068)	.9994 (.0067)	1.0129 (.0071)	1.0267 (.0078)	.9994 (.0067)	.9927 (.0067)	1.0061 (.0068)	1.0198 (.0074)

Table 5: Estimates ofthe parameter$\boldsymbol{\theta}$ and *MSE* with $\boldsymbol{\theta}$ **= 1,** $\boldsymbol{c}$ **= 2**

n	$\hat{\theta}$	θ_1^*	θ_2^*			θ_3^*	θ_4^*		
			r =-1	r= 1	r = 3		r =-1	r= 1	r = 3
20	.9994 (.0330)	.9994 (.0330)	.9672 (.0320)	1.0339 (.0365)	1.1105 (.0530)	.9369 (.0330)	.9085 (.0356)	.9672 (.0320)	1.0339 (.0365)
50	.9993 (.0132)	.9993 (.0132)	.9862 (.0131)	1.0128 (.0138)	1.0409 (.0160)	.9733 (.0133)	.9609 (.0138)	.9862 (.0131)	1.0128 (.0138)
100	.9994 (.0067)	.9994 (.0067)	.9927 (.0067)	1.0061 (.0068)	1.0198 (.0074)	.9862 (.0067)	.9798 (.0069)	.9927 (.0067)	1.0061 (.0068)

Table 6: Estimates ofthe parameter$\boldsymbol{\theta}$ and *MSE* with $\boldsymbol{\theta}$ **= 2,** $\boldsymbol{c}$ **= 2**

n	$\hat{\theta}$	θ_1^*	θ_2^*			θ_3^*	θ_4^*		
			r =-1	r= 1	r = 3		r =-1	r= 1	r = 3
20	1.9988 (.1321)	1.9988 (.1321)	1.9344 (.1280)	2.0678 (.1459)	2.2209 (.1321)	1.8739 (.1320)	1.8171 (.1426)	1.9344 (.1280)	2.0678 (.1459)
50	1.9986 (.0530)	1.9986 (.0530)	1.9723 (.0523)	2.0265 (.0550)	2.0819 (.0642)	1.9467 (.0531)	1.9217 (.0551)	1.9723 (.0523)	2.0256 (.0550)
100	1.9987 (.0268)	1.9987 (.0268)	1.9855 (.0267)	2.0121 (.0273)	2.0395 (.0295)	1.9724 (.0269)	1.9595 (.0274)	1.9855 (.0267)	2.0121 (.0273)

Discussion:

In general, comparison shows that Bayes' estimator of the parameter θ of Maxwell distribution based on Jeffreys prior with respect to the squared error loss function gives less *MSE* than the extension of Jeffreys prior, only when θ is small.

Comparison also shows that Bayes estimators with Jeffreys prior and under squared error loss function, gave identical results with the maximum likelihood estimator, when $c=2$. While estimators of the extension of Jeffreys prior under the squared error loss function was identical to maximum likelihood estimator when $c=1$.

It is obvious from tables *1 - 6* that all the estimates of the parameter θ and the *MSE* are reduced with the increase in the sample size. Under the modified squared error loss function one can easily observe that the parameters are generally overestimated with the increase of *r*. The extent of overestimation is higher for small *n*. On the other hand the estimates of the parameter based on both Jeffreys and the extension of Jeffreys prior are observed to be underestimated with the increase of *c*. The extent of underestimation is higher in the case of small *n*.

Finally, from the results, we can conclude that though the extension of Jeffreys prior gives the opportunity of covering a wide spectrum of priors, yet at times Jeffreys prior gives better Bayes estimates.

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*قسم الرياضيات /كلية العلوم/ جامعة بغداد

الخلاصة:

في هذا البحث قمنا باشتقاق مقدرات بيز لمعلمة الشكل لتوزيع ماكسويل ومقارنتها مع مقدر الارجحية العظمى. أخذنا بالاعتبار دوال الاسبقية غير المعلوماتية وهي كل من دالة جفريز ودالة جفريز الموسعة كما اخذنا بالاعتبار دالتي الخسارة التربيعية والتربيعية المعدلة. إستخدمنا اسلوب المحاكاة في مقارنة اداء كل مقدر بافتراض حجوم عينة مختلفة وعند حالات مختلفة. وقد جرت مقارنة كفاءة كل مقدر وفقا لمعيار متوسط مربعات الخطأ (MSE).أظهرت نتائج المقارنة ان كفاءة مقدرات بيز لمعلمة توزيع ماكسويل تتناقص بزيادة ثابت جفريز. كما أظهرت تقارب مقدرات بيز مع مقدر الارجحية العظمى عندما يكون ثابت جفريز صغيرا ومتطابقة عند حالات معينة. فيما يتعلق بدوال الخسارة أظهرت المقارنة أن المقدرات الناتجة عن استخدام دالة الخسارة التربيعية المعدلة لها متوسط مربعات خطأ أعلى من نظيرتها الناتجة عن استخدام دالة الخسارة التربيعية وبشكل خاص وواضح عند زيادةr .