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On Fully (m,n)-stable modules relative to an ideal A of $R^{n\times m}$

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Abstract:

Let R be a commutative ring with non-zero identity element. For two fixed positive integers m and n. A right R-module M is called fully (m,n) -stable relative to ideal A of $R^{n\times m}$, if $\theta(N)\subseteq N+M^nA$ for each n-generated submodule of M^m and R-homomorphism $\theta: N\to M^m$. In this paper we give some characterization theorems and properties of fully (m,n) -stable modules relative to an ideal A of $R^{n\times m}$. which generalize the results of fully stable modules relative to an ideal A of R.

Key words: fully (m,n) -stable modules relative to an ideal A of $R^{n\times m}$,(m,n)-multiplication modules and (m,n)-quasi injective modules.

Introduction:

Throughout, R is a commutative ring with non-zero identity and all modules are unitary right R-module. We use the notation $R^{m\times n}$ for the set of all $m\times n$ matrices over R. For G $\in R^{m\times n}$,G^T will denote the transpose of G. In general, for an R-module N, we write $N^{m\times n}$ for the set of all formal $m\times n$ matrices whose entries are elements of N. Let M be a right R-module and N be a left R-module. For x $\in M^{l\times m}$, s $\in R^{m\times n}$ and y $\in N^{n\times k}$, under the usual multiplication of matrices, xs (resp. sy) is a well defined element in $M^{l\times m}$ (resp. $N^{n\times k}$). If X $\in M^{l\times m}$, S$\in R^{m\times n}$ and Y$\in N^{n\times k}$, define

$$\ell_{M^{l\times m}}(S) = \{ u \in M^{l\times m} : us = 0, \forall s \in S \}$$

$$\gamma_{N^{n\times k}}(S) = \{ v \in N^{n\times k} : vs = 0, \forall s \in S \}$$

$$\ell_{R^{m\times n}}(Y) = \{ s \in R^{m\times n} : sy = 0, \forall y \in Y \}$$

$$\gamma_{R^{m\times n}}(X) = \{ s \in R^{m\times n} : xs = 0, \forall x \in X \}$$

We will write $N^n = N^{1\times n}$, $N_n = N^{n\times 1}$ .Fully stable module relative to an ideal have been discussed in [1], an R-module M is called fully stable relative to an ideal, if $\theta(N)\subseteq N+MA$ for each submodule N of M and R-homomorphism $\theta: N\to M$.It is an easy matter to see that M is fully stable relative to an ideal, if and only if $\theta(xR)\subseteq xR+MA$ for each x in M and R-homomorphism $\theta: xR\to M$. An R-module M for two fixed positive integers m and n is called fully (m,n) -stable relative to an ideal A of R, if $\theta(N)\subseteq N+M^nA$ for each n-generated submodule N of M^m and R-homomorphism $\theta: N\to M^m$ [2]. In this paper, for two fixed positive integers m and n, we introduce the concepts of fully (m,n) -stable modules relative to an ideal A of $R^{n\times m}$ and (m,n)-Baer criterion relative to an ideal A of $R^{n\times m}$ and we prove that an R-module M is fully (m,n) -stable relative to an ideal A of $R^{n\times m}$ if and only if (m,n) -Baer criterion relative to an

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ideal holds for n-generated submodules of M^m.

1. Results:

Definition 1.1 : An R-module M is called fully (m,n) -stable relative to an ideal A of $R^{n\times m}$, if $\theta(N)\subseteq N+M^nA$ for each n-generated submodule N of M^m and R-homomorphism $\theta: N\to M^m$. The ring R is fully (m,n) –stable relative to an ideal, if R is fully (m,n) -stable relative to an ideal as R-module.

It is clear that M is fully (1,1)-stable relative to ideal, if and only if M is fully stable relative to ideal.

It is an easy matter to see that an R-module M is fully (m,n)-stable relative to ideal, if and only if it is fully (m,q)-stable relative to ideal for all $1\le q\le n$, if and only if it is fully (p,n)-stable relative to ideal for all $1\le p\le m$, if and only if it is fully (p,q)-stable relative to ideal for all $1\le p\le m$ and $1\le q\le n$.

In [2], an R-module M is called fully-stable, if $\theta(N)\subseteq N$ for each cyclic submodule N of M and R-homomorphism $\theta: N\to M$. An R-module M is called fully (m,n) -stable, if $\theta(N)\subseteq N$ for each n-generated submodule N of M^m and R-homomorphism $\theta: N\to M^m$ [3]. It is clear that every fully (m,n)-stable module M is a fully (m,n)-stable relative to each non-zero ideal A of R for this follows from the fact $\theta(N)\subseteq N+M^nA$.

An R-module M is fully (m,n)-stable relative to an ideal A of $R^{n\times m}$, if and only if for each $\theta: N(\sum_{i=1}^{n}\alpha_i R)\to M^m$ (where $\alpha_i\in M^m$) and each $w\in N$, there exists $t=(t_1,\dots,t_n)\in R^n$ such that $\theta(w)=\sum_{i=1}^{n}\alpha_i t_i + AM^m = (\alpha_1,\dots,\alpha_n)\,t^T + M^{mA}$, if $r=(r_1,\dots,r_n)\in R^n$, then $\theta((\alpha_1,\dots,\alpha_n)\,r^T) + M^mA = (\alpha_1,\dots,\alpha_n)\,t^T + M^mA$.

Proposition 1.2 : An R-module M is fully (m,n)-stable relative to an ideal A of $R^{n\times m}$, if and only if any two m-element subsets $\{\alpha_1,\dots,\alpha_m\}$ and $\{\beta_1,\dots,\beta_m\}$ of M^n, if $\beta j\notin \sum_{i=1}^{n}\alpha_i R + M^nA$. , for each $j=1,\dots,m$ implies $\gamma_{R_n}\{\alpha_1,\dots,\alpha_m\}\not\subset \gamma_{R_n}\{\beta_1,\dots,\beta m\}$.

Proof : Assume that M is fully (m,n)-stable module relative to an ideal A of R and there exist two m-element subsets $\{\alpha_1,\dots,\alpha_m\}$ and $\{\beta_1,\dots,\beta_m\}$ of M^n such that $\beta j\notin \sum_{i=1}^{n}\alpha_i R + M^nA$, $\forall j=1,\dots,m$ and $\gamma_{R_n}\{\alpha_1,\dots,\alpha_m\}\subseteq \gamma_{R_n}\{\beta_1,\dots,\beta_m\}$. Define $f: \sum_{i=1}^{n}\alpha_i R\to M^m$ by $f(\sum_{i=1}^{n}\alpha_i r_i)=\sum_{i=1}^{n}\beta_i r_i$. Let $\alpha_i=(a_{1i},a_{i2},\dots,a_{in})$. If $\sum_{i=1}^{n}\alpha_i r_i=0$, then $\sum_{i=1}^{n}a_{ij}\,r_i=0$, $j=1,\dots,m$ implies that $\alpha_j r^T=0$ where $r=(r_1,\dots,r_n)$ and hence $r^T\in\gamma_{R_n}\{\alpha_1,\dots,\alpha_m\}$. By assumption $\beta_j\ r^T=0$, $j=1,\dots,m$ so $\sum_{i=1}^{n}\beta_i r_i=0$. This show that f is well defined. It is an easy matter to see that f is R-homomorphism. Fully (m,n)-stability relative to an ideal A of $R^{n\times m}$ implies that there exists $t=(t_1,\dots,t_n)\in R^n$ and $w\in M^nA$ such that $f(\sum_{i=1}^{n}\alpha_i r_i)=\sum_{k=1}^{n}(\sum_{i=1}^{n}\alpha_i r_i)\,t_k + w = \sum_{k=1}^{n}(\sum_{i=1}^{n}\alpha_i r_i\,t_k)+w$ for each $\sum_{i=1}^{n}\alpha_i r_i\in\sum_{i=1}^{n}\alpha_i R$. Let $r_i=(0,\dots,0,1,0\dots,0)\in R^n$ where 1 in the i th position and 0 otherwise. $\beta_i = f(\alpha_i) = \sum_{k=1}^{n} \alpha_i t_k + w$ which is contradiction. Conversely assume that there exists n-generated submodule of M^m and R-homomorphism $\theta : \sum_{i=1}^{n} \alpha_i R \to M^m$ such that $\theta(\sum_{i=1}^{n} \alpha_i R) \notin \sum_{i=1}^{n} \alpha_i R + M^n A$. Then there exists an element $\beta (= \sum_{i=1}^{n} \alpha_i r_i) \in \sum_{i=1}^{n} \alpha_i R$ such that $\theta(\beta) \notin \sum_{i=1}^{n} \alpha_i R + AM^n$. Take $\beta_j = \beta$, j = 1…,m, then we have m-element subset $\{\theta(\beta),\ldots, \theta(\beta)\}$, such that $\theta(\beta) \notin \sum_{i=1}^{n} \alpha_i R + M^n A$, j =1,…,m. Let $\eta = (t_1,\ldots,t_n)^T \in \gamma_{R_n}\{\alpha_1,\ldots,\alpha_m\}$then $\alpha_j \eta = 0$, i.e $\sum_{i=1}^{n} a_{ij} t_i = 0$, $\forall j = 1,\ldots, m$, $\alpha_j = (a_{1j},a_{2j},\ldots,a_{nj})$ and $\{\theta(\beta),\ldots,\theta(\beta)\}\eta = \sum_{k=1}^{n} \theta(\beta) t_k = \sum_{k=1}^{n} \theta(\sum_{i=1}^{n} \alpha_i r_i) t_k = \sum_{k=1}^{n} (\theta(\sum_{i=1}^{n} \alpha_i r_i t_k) = 0$, hence $\gamma_{R_n}\{\alpha_1,\ldots,\alpha_m\} \subseteq \gamma_{R_n}\{\theta(\beta),\ldots,\theta(\beta)\}$, thus $\gamma_{R_n}\{\alpha_1,\ldots,\alpha_m\} \subseteq \gamma_{R_n}\{\theta(\beta_1),\ldots, \theta(\beta m)\}$which is a contradiction. Thus M is fully (m,n)-stable module relative to ideal .

Corollary 1.3 : Let M be fully (m,n)-stable module relative to an ideal A of $R^{n\times m}$, then for any two m-element subsets $\{\alpha_1, \ldots, \alpha_m\}$ and $\{\beta_1, \ldots, \beta_m\}$ of M^n, $\gamma_{R_n}\{\alpha_1,\ldots,\alpha_m\} \subseteq \gamma_{R_n}\{\beta_1,\ldots,\beta_m\}$ implies $\alpha_1 R + \ldots + \alpha_m R + AM^n = \beta_1 R + \ldots + \beta_m R + M^n A$.

Corollary 1.4: [1] Let M be a fully stable module relative to an ideal A of , then for each x,y in M, $y \notin (x)$, $\gamma_R(x) = \gamma_R(y)$ implies (x) + AM = (y) + AM.

A submodule N of an R-module M satisfies Baer criterion relative to an ideal A of R, if for every R-homomorphism $f : N \to M$, there exists an element $r \in R$ such that f(n) - rn $\in AM$ for each $n \in N$. An R-module M is said to satisfy Baer criterion relative to A, if each submodule of M satisfies Baer criterion relative to A and it is proved that an R-module M satisfies Baer criterion relative to A for cyclic submodules, if and only if M is fully stable relative to A [1].

Definition 1.5 : For a fixed positive integers n and m, we say that an R-module M satisfies (m,n)-Bear criterion relative to an ideal A of R, if for any n-generated submodule N of M^m and any R-homomorphism $\theta : N \to M^m$ there exists t $\in$ R such that θ (x) - xt $\in M^m A$ for each x in N.

It is clear that if M satisfies (m,n) -Baer criterion relative to an ideal A then M
satisfies (p,q) -Baer criterion relative to an ideal A, $\forall\ 1 \le p \le m$ and $1 \le q \le n$.

Proposition 1.6 : Let A be an ideal of $R^{n\times m}$ and M be an R-module such that $\gamma_R(N \cap K) = \gamma_R(N) + \gamma_R(K)$ for each two n-generated submodule of M^m.If M satisfies (m,1)-Bear criterion relative to A . Then M satisfies (m,n)-Bear criterion relative to A for each n $\ge$ 1.

Proof : Let $L = x_1R + x_2R + \ldots + x_nR$ be n-generated submodule of M^m and f : $L \to M^m$ an R-homomorphism. We use induction on n. It is clear that M satisfies (m,n) -Bear criterion, if n =1. Suppose that M satisfies (m,n) -Bear criterion for all k-generated submodule of M^m, for $k \le n-1$. Write $N = x_1R$, $K = x_2R + \ldots + x_nR$, then for each $w_1 \in N$ and $w_2 \in K$, $f|_N(w_1) = w_1 r$, $f|_K(w_2) = w_2 s$ for some r, s $\in$ R. It is clear

$r - s \in \gamma_R(N \cap K) = \gamma_R(N) + \gamma_R(K)$. Suppose that $r\text{-}s = u+v$ with $u \in \gamma_R(N)$, $v \in \gamma_R(K)$ and let $t = r - u = s + v$. Then for any $w = w_1 + w_2 \in L$ with $w_1 \in N$ and $w_2 \in K$, $f(w) - wt = f(w_1) + f(w_2) - (w_1 + w_2)t = f(w_1) - w_1t + f(w_2) - w_2t = f(w_1) - w_1(r - u) + f(w_2) - w_2(s+v) = f(w_1) - w_1r + w_1u + f(w_2) - w_2s - w_2v = f(w_1) - w_1r + f(w_2) - w_2s \in M^mA$.

Proposition 1.7 : Let M be an R-module and A be an ideal of R. Then M satisfies (m,n)-Baer criterion relative to an ideal A, if and only if $\ell_{M^n}\,\gamma_{R_n}(\alpha_1R, \dots, \alpha_nR) \subseteq \alpha_1R + \dots + \alpha_nR + M^nA$ for any n-element subset $\{\alpha_1, \dots, \alpha_n\}$ of M^n.

Proof : First assume that (m,n)-Baer criterion relative to an ideal A holds for n-generated submodule of M^m, let $\alpha_i = (a_{i1}, a_{i2}, \dots, a_{im})$, for each $i = 1, \dots, n$ and $\beta = \{\beta_1, \dots, \beta_n\} \in \ell_{M^n}\,\gamma_{R_n}(\alpha_1R + \dots + \alpha_nR)$, $\alpha_i = (a_{1j}, a_{2j}, \dots, a_{nj})$. Define $\theta: \alpha_1R, \dots, \alpha_nR \to M^m$ by $\theta(\sum_{i=1}^{n}\alpha_ir_i) = \sum_{i=1}^{n}\beta_ir_i$. If $\sum_{i=1}^{n}\alpha_ir_i = 0$, then $\sum_{i=1}^{n} a_{ij}\, r_i = 0$, $j = 1, \dots, m$, this implies that $\alpha_i r^T = 0$ where $r = (r_1, \dots, r_n)$ and hence $r^T \in \gamma_{R_n}(\alpha_1R, \dots, \alpha_nR)$. By assumption $\beta_i\, r^T = 0$, $\forall\, i = 1, \dots, n$ so $\sum_{i=1}^{n}\beta_ir_i = 0$. This show that f is well defined. It is an easy matter to see that θ is R-homomorphism. By assumption there exists $t \in R$ such that $\theta(\sum_{i=1}^{n}\alpha_ir_i) - (\sum_{i=1}^{n}\alpha_ir_i)t \in M^nA$ for each $\sum_{i=1}^{n}\alpha_ir_i \in \sum_{i=1}^{n}\alpha_iR$. Let $r_i = (0, \dots, 0, 1, 0, \dots, 0) \in R^n$ where 1 in the ith position and 0 otherwise. $\beta_i - \alpha_i t = \theta(\alpha_i) - \alpha_i t \in AM^n$ thus $\beta_i \in \sum_{i=1}^{n}\alpha_iR + AM^n$ which is contradiction. This implies that $\ell_{M^n}\,\gamma_{R_n}(\alpha_1R + \dots + \alpha_nR) \subseteq \alpha_1R + \dots + \alpha_nR + M^nA$. Conversely, assume that $\ell_{M^n}\,\gamma_{R_n}(\alpha_1R + \dots + \alpha_nR) \subseteq \alpha_1R + \dots + \alpha_nR + M^nA$, for each $\{\alpha_1, \dots, \alpha_n\}$ of M^n. Then for each R-homomorphism $f: \alpha_1R + \dots + \alpha_nR \to M^m$ and $s = (s_1, \dots, s_n) \in \gamma_{R_n}(\alpha_1R + \dots + \alpha_nR)$, $\sum_{k=1}^{n}(\sum_{i=1}^{n}\alpha_ir_i)\, s_k = 0$ for each $\sum_{i=1}^{n}\alpha_ir_i \in \sum_{i=1}^{n}\alpha_iR$, hence $\sum_{k=1}^{n} f(\sum_{i=1}^{n}\alpha_ir_i)\, s_k = \sum_{k=1}^{n} f(\sum_{i=1}^{n}\alpha_ir_i\, s_k) = 0$, thus $f(\sum_{i=1}^{n}\alpha_ir_i) \in \ell_{M^n}\,\gamma_{R_n}(\alpha_1R + \dots + \alpha_nR) = \alpha_1R + \dots + \alpha_nR + M^nA$, then $f(\sum_{i=1}^{n}\alpha_ir_i) = f(\alpha_i r^T) = f(\alpha_i)r^T \in \alpha_1R + \dots + \alpha_nR + M^nA$, for some $r \in R$. Then M satisfies (m,n) -Baer criterion.

Corollary 1.8: An R-module M is fully (m,n)-stable relative to an ideal A of $R^{n\times m}$, if and only if $\ell_{M^n}\,\gamma_{R_n}(\alpha_1R + \dots + \alpha_nR) \subseteq \alpha_1R + \dots + \alpha_nR + M^nA$ for any n-element subset $\{\alpha_1, \dots, \alpha_n\}$ of M^n.

We can summarize the above results in the following theorem.

Theorem 1.9: The following statements are equivalent for an R-module M and an ideal A of R.

1. M is fully (m,n)-stable relative to A
2. For any two m-element subsets $\{\alpha_1, \dots, \alpha_m\}$ and $\{\beta_1, \dots, \beta_m\}$ of M^n, if $\beta j \notin \sum_{i=1}^{n}\alpha_iR + M^nA$. , for each $j = 1, \dots, m$ implies $\gamma_{R_n}\{\alpha_1, \dots, \alpha_m\} \not\subset \gamma_{R_n}\{\beta_1, \dots, \beta m\}$.

3.(m,n)-Baer criterion relative to A for n-generated submodules of M^m.

4. $\ell_{M^n}\ \gamma_{R_n}\ (\alpha_1 R + \dots + \alpha_n R) \subseteq \alpha_1 R + \dots + \alpha_n R + M^n A$ for any n-element subset $\{\alpha_1, \dots, \alpha_n\}$ of M^n.

Corollary 1.10: [1] The following statements are equivalent for an R-module M and an ideal A of R.

1. M is fully-stable relative to A
2. For each x, y in M , y $\notin$(x), γ_R (x) = γ_R (y) implies (x) + MA = (y) + MA..
3. M satisfies Baer criterion to A for for each cyclic submodule.
4. For each x in M, $l_M(\gamma_R(x)) \subseteq$ (x)+ AM.

Recall that an R-module M is (m,n)-multiplication module if each n-generated submodule of M^m is of the form $M_n I$ for some ideal I of $R^{n\times m}$ [3].

Proposition 1.11 : Let M be an (m,n)-multiplication R-module. Then M is fully (m,n)-stable module if and only if M is fully (m,n)-stable relative to each non-zero ideal of $R^{n\times m}$.

Proof : $\Rightarrow$It is clear

$\Leftarrow$Let N be any n-generated submodule of M^m and $f : N \to M^m$ be any R-homomorphism. If N = (0), then it is clear that M is fully (m,n)-stable relative to ideal. Let N $\neq$ (0), and since M is an (m,n)-multiplication module, then $N = M_n I$, for some non-zero ideal I of $R^{n\times m}$.By hypothesis $f(N) \subseteq N + IM_n = N + N = N$. Hence, M is fully (m,n)-stable module.

Corollary 1.12: [1] Let M be multiplication R-module. Then M is fully stable module if and only if M is fully stable relative to each non-zero ideal of R.

Recall that an R-module M is (m,n)-quasi-injective in each R-homomorphism from an n-generated submodule of M^m to M extends to one from M^m to M [4].

The following theorem follows from Theorem(2.14) in [5] and Proposition(1.11).

Theorem 1.13: Let M be an (m,n)-multiplication R-module. Then M is (m,n)-quasi injective if and only if M is fully (m,n)-stable relative to each non-zero ideal of $R^{n\times m}$.

Now we introduce the concept of (m,n)-quasi injective module relative to an ideal A of $R^{n\times m}$

Definition 1.14: An R-module M is called (m,n)-quasi injective relative to an ideal A of $R^{n\times m}$ if for every R-homomorphism $g : N\to M^m$ where N is n-generated submodule of M^m and R-homomorphis $f : N \to M$ there exists R-homomorphim $h : M^m \to M$ such that $fg(x) - h(x) \in M^n A$ for each x in N.

Proposition 1.15: If M is a fully (m,n)-stable R-module relative to an ideal A of $R^{n\times m}$, then M is (m,n)-quasi injective relative to A.

Proof:

Let $N = \alpha_1 R + \dots + \alpha_n R$ be n-generated submodule of M^m where $\alpha_i, \dots, \alpha_i \in M^m$ and $f: N \to M^m$ be any R-homomrphism. Since M is a fully (m,n)-stable module relative to A, then $f(\alpha_1 R + \dots + \alpha_n R) \subseteq \alpha_1 R + \dots + \alpha_n R + M^n A$, thus there exist $s = (s_1, \dots, s_n) \in R^m$ and $w \in M^n A$. Let r_i = (0, … , 1 . 0, … , 0) such that $f(\sum_{i=1}^{n}\alpha_i) = (\sum_{i=1}^{n}\alpha_i)s + w$. Define $g : M^m \to M$ by $g(\alpha_i) = \alpha_i s^T$, it is clear that g is a well defined R-homomorphism. Now $f(\sum_{i=1}^{n}\alpha_i) - g(\sum_{i=1}^{n}\alpha_i) = (\sum_{i=1}^{n}\alpha_i)s + w - (\sum_{i=1}^{n}\alpha_i)s = w \in M^n A$ and since for each y $\in$ $\alpha_1 R + \ldots + \alpha_n R$, y = $\sum_{i=1}^{n} \alpha_i t_i$ for some t= (t_1, …, t_n) $\in$ R, f(y) – g(y) = f($\sum_{i=1}^{n} \alpha_i t_i$) – g($\sum_{i=1}^{n} \alpha_i t_i$) = f(($\sum_{i=1}^{n} \alpha_i$)t) – g(($\sum_{i=1}^{n} \alpha_i$)t) = (f($\sum_{i=1}^{n} \alpha_i$) – g($\sum_{i=1}^{n} \alpha_i$))t $\in$ $M^m A$. Therefore M is (m,n) – quasi injective module.

The following theorem follows from Theorem(1.13) and Proposition (1.115).

Theorem 1.16: If M is (m,n)-quasi injective R-module then M is (m,n)-quasi injective relative to an ideal A of $R^{n\times m}$.

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حول المقاسات تامة الاستقرارية من النمط - (m,n) بالنسبة الى مثالي A في $R^{n\times m}$

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الخلاصة:

لتكن R حتقة ابدالية ذات عنصر محايد M مقاساً أيسراً أحادياً على R و A مثالي في الحلقة $R^{n\times m}$. كتعميم لمفهوم المقاسات تامة الاستقرارية من النمط - (m,n) عرفنا المقاسات تامة الاستقرارية من النمط - (m,n) بالنسبة الى مثالي. نقول ان المقاس M تام الاستقرارية من النمط (m,n) بالنسبة الى A اذا كان $\theta(N) \subseteq N + M^m A$ لكل تشاكل مقاسي θ من N الى M حيث N مقاس جزني متولد من النمط- n . في هذ البحث يتم دراسة علاقة صنف المقاسات تامة الاستقرارية من النمط –(m,n) بالنسبة الى مثالى باصناف اخرى مثل المقاسات الجدائية من النمط – (m,n) ، المقاسات شبه الغامرة من النمط – (m,n) .

الكلمات المفتاحية : المقاسات تامة الاستقرارية من النمط –(m,n) بالنسبة الى مثالى A في $R^{n\times m}$ و المقاسات الجدائية من النمط – (m,n) و المقاسات شبه الغامرة من النمط – (m,n) .