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Oscillations of Third Order Half Linear Neutral Differential Equations

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Abstract:

In this paper the oscillation criterion was investigated for all solutions of the third-order half linear neutral differential equations. Some necessary and sufficient conditions are established for every solution of

$$(a(t)[(x(t) \pm p(t)x(\tau(t)))'']^{\gamma})' + q(t)x^{\gamma}(\sigma(t)) = 0, \qquad t \geq t_0,$$

to be oscillatory. Examples are given to illustrate our main results.

Key words: Third order half linear Neutral differential equations, oscillation of solutions.

Introduction:

The study of oscillation theory for solution of half linear neutral differential equations has been recently considered the attention of many researches for the last several years, see [1]-[8]. A few of them have been investigated the case with variable coefficients and delays, see [5], [7-9]. Consider the half linear neutral differential equations.

$$\left.\begin{aligned} &(a(t)[(x(t) + p(t)x(\tau(t)))'']^{\gamma})' + q(t)x^{\gamma}(\sigma(t)) = 0, \quad t \geq t_0 \quad (1.\text{i}) \\ &(a(t)[(x(t) - p(t)x(\tau(t)))'']^{\gamma})' + q(t)x^{\gamma}(\sigma(t)) = 0, \quad t \geq t_0 \quad (1.\text{ii}) \end{aligned}\right\} \quad (1)$$

We define functions $Z(t) = x(t) + p(t)x(\tau(t))$ (2.i)

$z(t) = x(t) - p(t)x(\tau(t))$ (2.ii)

In this paper we will assume that the following conditions are satisfied

H1. $a(t), p(t) \in C([t_0, \infty), R^+), q(t) \in C([t_0, \infty), R)$, $\gamma > 0$ is the quotient of odd positive integers.

H2. $\tau(t), \sigma(t)$ are continuous functions $\sigma(t) < t, \lim_{t\to\infty} \tau(t) = \infty$, $\lim_{t\to\infty} \sigma(t) = \infty$.

H3. $\int_T^{\infty} \left(\frac{1}{a(s)}\right)^{\frac{1}{\gamma}} ds = \infty$.

Where $a(t)$ is continuous positive function. By a solution of eq.(1) we mean a nontrivial function $x(t) \in C([T_x, \infty), R), T_x \geq t_0$ for which $x(t) \pm p(t)x(\tau(t)) \in C^2([T_x, \infty), R)$, $a(t)(z''(t))^{\gamma} \in C^1([T_x, \infty), R)$, and (1.1) is satisfied on some interval $([T_x, \infty), R)$, where $T_x \geq t_0$, A non trivial solution of eq.(1) is said to be oscillatory if it has arbitrarily large zeros, otherwise is said to be nonoscillatory that is eventually positive solution or eventually negative solution. The purpose of this paper is to obtain necessary and sufficient conditions for the oscillation of all solutions of eq.(1).

Some Basic Lemmas

The following lemmas will be useful in the proof of the main results:

Lemma 1. [5]

Suppose that $p, q \in C[R^+, R^+]$, $q(t) < t$ for $t \geq t_0$,$\lim_{t\to\infty} q(t) = \infty$ and

$$\liminf_{t\to\infty} \int_{q(t)}^{t} p(s)ds > \frac{1}{e} \qquad (3)$$

Then the inequality $y'(t) + p(t)y(q(t)) \leq 0$ has no eventually positive solutions, and the inequality $y'(t) + p(t)y(q(t)) \geq 0$ has no eventually negative solutions.

Lemma 2. [4] Assume that $p \in C([t_0, \infty); R^+), \tau \in C([t_0, \infty); R)$, for $t \geq t_0$,

i. suppose that $0 < p(t) \leq 1$ for $t \geq t_0$. let $x(t)$ be a continuous nonoscillatory solution of a functional inequality

$x(t)\left[x(t) - p(t)x\left(\tau(t)\right)\right] < 0$ in a neighborhood of infinity.

Suppose that $\tau(t) < t$ for $t \geq t_0$, then $x(t)$ is bounded. If moreover $0 < p(t) \leq \delta < 1$, $t \geq t_0$, for some positive constant δ, then $\lim_{t\to\infty} x(t) = 0$.

ii. suppose that $1 \leq p(t)$ for $t \geq t_0$. let $x(t)$ be a continuous nonoscillatory solution of a functional inequality

$x(t)\left[x(t) - p(t)x\left(\tau(t)\right)\right] > 0$ in a neighborhood of infinity.

Suppose that $\tau(t) > t$ for $t \geq t_0$, then $x(t)$ is bounded. If moreover $1 < \delta \leq p(t)$, $t \geq t_0$, for some positive constant δ, then $\lim_{t\to\infty} x(t) = 0$.

Lemma 3. Suppose that H1-H3 holds, $q(t) \geq 0$ and let $x(t)$ be an eventually positive solution of $(1.\mathrm{i})$ then there are only the following two cases for $(2.\mathrm{i})$

i. $z(t) > 0$, $z'(t) > 0, z''(t) > 0, [a(t)(\, z''(t))^\gamma]' < 0, t \geq t_1 \geq t_0$.

ii. $z(t) > 0$, $z'(t) < 0$, $z''(t) > 0, [a(t)(\, z''(t))^\gamma]' < 0, t \geq t_1 \geq t_0$.

Proof. Let $x(t) > 0, x(\tau(t)) > 0, x(\sigma(t)) > 0$, for $t \geq t_0$ then from eq.$(1.\mathrm{i})$ we get

$[a(t)(\, z''(t))^\gamma]' = -q(t)x^\gamma\left(\sigma(t)\right) \leq 0$, $t \geq t_0$, hence

$a(t)(z''(t))^\gamma$ is non increasing , so either

$a(t)(z''(t))^\gamma > 0$

or $a(t)(z''(t))^\gamma < 0$, $t \geq t_1 \geq t_0$,

therefore $z''(t) > 0$ or $z''(t) < 0$, $t \geq t_1$ respectively.

Suppose that $a(t)(z''(t))^\gamma < 0$, then there exists $d < 0$ such that

$a(t)(z''(t))^\gamma \leq d$, $t \geq t_2 \geq t_1$, then

$$z''(t) \leq \frac{d^{\frac{1}{\gamma}}}{a^{\frac{1}{\gamma}}(t)} .$$

Integrating the last inequality from t_2 to t and using H3 we get

$$z'(t) - z'(t_2) \leq d^{\frac{1}{\gamma}} \int_{t_2}^{t} \frac{1}{a^{\frac{1}{\gamma}}(s)} ds$$

This lead to $\lim_{t\to\infty} z'(t) = -\infty$

Then $z'(t) < 0$ $t \geq t_3$, for t_3 large enough, this implies that $z(t) < 0$ which is

contradiction a. So $a(t)(z''(t))^\gamma > 0$ hence $z''(t) > 0$. □

Lemma 4. Suppose that H1-H3 hold, $1 \leq p(t) \leq p_1, \tau(t) > t, q(t) \leq 0$, let $x(t)$ be an eventually positive solution of eq.$(1.\mathrm{ii})$ then there are only three cases for (2.ii)

i. $z(t) < 0$, $z'(t) > 0, z''(t) < 0, [a(t)(\, z''(t))^\gamma]' \geq 0, t \geq t_1 \geq t_0$. .

ii. $z(t) > 0$, $z'(t) > 0$, $z''(t) < 0, [a(t)(\, z''(t))^\gamma]' \geq 0, t \geq t_1 \geq t_0$.

iii. $z(t) < 0$, $z'(t) < 0$, $z''(t) < 0, [a(t)(\, z''(t))^\gamma]' \geq 0, t \geq t_1 \geq t_0$.

Proof. Let $x(t) > 0, x(\tau(t)) > 0, x(\sigma(t)) > 0$, for $t \geq t_0$ then from eq.$(1.1.\mathrm{ii})$ we get $[a(t)(\, z''(t))^\gamma]' \geq 0$ hence $a(t)(\, z''(t))^\gamma$ is non decreasing then either $a(t)(\, z''(t))^\gamma$ is eventually positive or eventually negative, it follows that either $z''(t)$ is eventually positive or eventually negative, if $z''(t) > 0$ which mean that $a(t)(z''(t))^\gamma > 0$, for $t \geq t_1 \geq t_0$ so there exists $\beta > 0$ such that $a(t)(z''(t))^\gamma \geq \beta > 0,\ t \geq t_2 \geq t_1$ that is

$$z''(t) \geq \frac{\beta^{\frac{1}{\gamma}}}{a^{\frac{1}{\gamma}}(t)}, \qquad t \geq t_2$$

Integrating the last inequality from t_2 to t and using H3 we get

$$z'(t) - z'(t_2) \geq \beta^{\frac{1}{\gamma}} \int_{t_2}^{t} \frac{1}{a^{\frac{1}{\gamma}}(s)} ds$$

then $\lim_{t\to\infty} z'(t) = \infty$ which implies that $\lim_{t\to\infty} z(t) = \infty$ hence there exist $t_3 \geq t_2$ such that $x(t)z(t) > 0$, for $t \geq t_3$ then by Lemma 2 it follows that $x(t)$ is bounded which is a contradiction. Then $a(t)(z''(t))^\gamma < 0$ eventually which implies that $z''(t) < 0$. □

Main Results:

In this section, we give the main results.

Theorem 1. Suppose that H1-H3 hold, $0 \leq p(t) < 1, \tau(t) < t, q(t) \geq 0$, and

$$\int_{t_1}^{\infty} q(s)\left[1 - p(\sigma(s))\right]^\gamma ds = \infty \quad (4)$$

Then every unbounded solution of eq.(1.1.i) oscillates.

Proof. Suppose the contrary that eq.(1.1.i) has eventually positive solution $x(t)$ then we have $[a(t)(z''(t))^\gamma]' \leq 0$, so by Lemma 3 there are only the following two cases for (2.i)

i. $z(t) > 0, z'(t) > 0, z''(t) > 0, (a(t)(z''(t))^\gamma)' \leq 0, t \geq t_1 \geq t_0$.

ii. $z(t) > 0, z'(t) < 0, z''(t) > 0, (a(t)(z''(t))^\gamma)' \leq 0, t \geq t_1 \geq t_0$.

Case **i.** In this case $a(t)(z''(t))^\gamma$ is positive non increasing,

$z(t) \leq x(t) + p(t)z(\tau(t))$, then

$$x(\sigma(t)) \geq z(\sigma(t)) - p(\sigma(t))z(\tau(\sigma(t))) \geq z(\sigma(t))[1 - p(\sigma(t))]$$

$$x^\gamma(\sigma(t)) \geq z^\gamma(\sigma(t))[1 - p(\sigma(t))]^\gamma, \quad t \geq t_1 \geq t_0 \qquad (5)$$

Integrating eq(1.i) from t_1 to t we get

$$a(t)(z''(t))^\gamma - a(t_1)(z''(t_1))^\gamma = -\int_{t_1}^{t} q(s)x^\gamma(\sigma(s))ds$$

$$\leq -\int_{t_1}^{t} q(s)z^\gamma(\sigma(s))\left[1 - p(\sigma(s))\right]^\gamma ds$$

$$\leq -z^\gamma(\sigma(t_1)) \int_{t_1}^{t} q(s)\left[1 - p(\sigma(s))\right]^\gamma ds$$

Which as $t \to \infty$ leads to a contradiction.

Case **ii.** Since $x(t)$ is unbounded then $z(t)$ is unbounded which is a contradiction in this case. □

Theorem 2. Suppose that H1-H3 hold, $0 \leq p(t) < 1, \tau(t) > t, q(t) \geq 0$, and there exist a continuous functions $\alpha(t), \beta(t)$ such that $\alpha(t) > t, \beta(t) > t$

$$\liminf_{t\to\infty} \int_{F(t)}^{t} \int_{s}^{\beta(s)} \left(\frac{1}{a(s)}\right)^{\frac{1}{\gamma}} \left(\int_{v}^{\alpha(v)} q(w)(1 - p(\sigma(w)))^\gamma dw\right)^{\frac{1}{\gamma}} dv ds > \frac{1}{e} \quad (6)$$

$F(t) = \sigma\left(\alpha\left(\beta(t)\right)\right)$. Then every bounded solution of eq.(1.i) oscillates.

Proof. Suppose that eq.(1.i) has eventually positive solution $x(t)$ then we have $[a(t)(z''(t))^\gamma]' \leq 0$, so by Lemma 3 there are only the following two cases for (2.i)

i. $z(t) > 0, z'(t) > 0, z''(t) > 0, (a(t)(z''(t))^\gamma)' \leq 0, t \geq t_1 \geq t_0$.

ii. $z(t) > 0, z'(t) < 0, z''(t) > 0, (a(t)(z''(t))^\gamma)' \leq 0, t \geq t_1 \geq t_0$..

Case **i.** In this case $\lim_{t\to\infty} z(t) = \infty$, however $x(t)$ and $p(t)$ are bounded leads to $z(t)$ is bounded which is a contradiction.

Case **ii.** It follows from $(2.a)$ $z(t) \le x(t) + p(t)z(\tau(t))$, then

$$x(\sigma(t)) \ge z(\sigma(t)) - p(\sigma(t))z(\tau(\sigma(t))) \ge z(\sigma(t))[1 - p(\sigma(t))]$$

$$x^\gamma(\sigma(t)) \ge z^\gamma(\sigma(t))[1 - p(\sigma(t))]^\gamma$$

Integrating eq.$(1.\mathrm{i})$ from t to $\alpha(t)$ we get

$$-a(t)(z''(t))^\gamma \le -\int_t^{\alpha(t)} q(s)x^\gamma(\sigma(t))ds$$

$$z''(t) \ge \frac{1}{a^{\frac{1}{\gamma}}(t)}[\int_t^{\alpha(t)} q(s)x^\gamma(\sigma(s))ds]^{\frac{1}{\gamma}}$$

Using (5) in the last inequality we get

$$z''(t) \ge \frac{1}{a^{\frac{1}{\gamma}}(t)}[\int_t^{\alpha(t)} q(s)z^\gamma(\sigma(s))(1 - p(\sigma(s)))^\gamma ds]^{\frac{1}{\gamma}}$$

$$\ge \frac{z(\sigma(\alpha(t)))}{a^{\frac{1}{\gamma}}(t)}[\int_t^{\alpha(t)} q(s)(1 - p(\sigma(s)))^\gamma ds]^{\frac{1}{\gamma}}$$

Integrating the last inequality from t to $\beta(t)$ we get

$$-z'(t) \ge \int_t^{\beta(t)} \frac{z(\sigma(\alpha(s)))}{a^{\frac{1}{\gamma}}(s)}[\int_s^{\alpha(s)} q(v)(1 - p(\sigma(v)))^\gamma dv]^{\frac{1}{\gamma}}ds$$

$$z'(t) \le -z(\sigma(\alpha(\beta(t))))\int_t^{\beta(t)} \frac{1}{a^{\frac{1}{\gamma}}(s)}[\int_s^{\alpha(s)} q(v)(1 - p(\sigma(v)))^\gamma dv]^{\frac{1}{\gamma}}ds$$

$$z'(t) + z(F(t))\int_t^{\beta(t)} \frac{1}{a^{\frac{1}{\gamma}}(s)}[\int_s^{\alpha(s)} q(v)(1 - p(\sigma(v)))^\gamma dv]^{\frac{1}{\gamma}}ds \le 0,$$

Where $F(t) = \sigma(\alpha(\beta(t)))$

by Lemma 1 and condition (6) the last inequality cannot has eventually positive solution which is a contradiction. □

Example 1. Consider the third-order nonlinear differential equation

$$\left(\frac{1}{4}\left[\left(x(t) + \frac{1}{3}x(t+\pi)\right)''\right]\right)' + \frac{1}{6}x\left(t - \frac{3\pi}{2}\right) = 0, \qquad (E1)$$

In equation $(E1)$ we find $\gamma = 1, a(t) = \frac{1}{4}, \tau(t) = t + \pi, \sigma(t) = t - \frac{3\pi}{2}$,

If we set $\alpha(t) = \beta(t) = t + \frac{\pi}{2}$, and using the condition (3.3) we get

$$\frac{4}{9}\lim_{t\to\infty} inf \int_{t-\frac{\pi}{2}}^{t}\int_s^{s+\frac{\pi}{2}}\int_v^{v+\frac{\pi}{2}} dwdvds = \frac{\pi^3}{18} > \frac{1}{e}$$

Then according to theorem 2 every solution of equation $(E1)$ is oscillatory, for instance $x(t) = \sin t$ is such oscillatory solution.

Theorem 3. Suppose that H1-H3 hold, $1 \le p(t) \le p_1, \tau(t) > t, q(t) \le 0$, and there exists continuous functions $\alpha(t), \beta(t)$ such that $\alpha(t) > t, \beta(t) > t$

$$\lim_{t\to\infty}\inf \int_{F(t)}^{t}\int_s^{\beta(s)} \left(\frac{1}{a(v)}\right)^{\frac{1}{\gamma}}\left[\int_v^{\alpha(v)} \frac{|q(w)|}{p^\gamma\left(\tau^{-1}(\sigma(w))\right)}dw\right]^{\frac{1}{\gamma}} dvds > \frac{1}{e}, \quad (7)$$

$$H(t) = \tau^{-1}(\sigma(\alpha(\beta(t)))) < t.$$

$$\lim_{t\to\infty}\inf \int_t^{\alpha(t)} |q(s)|\, ds > 0, \qquad (8)$$

$$\int_{t_1}^{\infty} \frac{|q(s)|}{p^\gamma\left(\tau^{-1}(\sigma(s))\right)}ds = \infty, \quad t \ge T \qquad (9)$$

Then every solution of eq $(1.\mathrm{ii})$ is oscillatory.

Proof. Suppose that eq $(1.\text{ii})$ has eventually positive solution $x(t)$ then we have $[a(t)(z''(t))^{\gamma}]' \geq 0$, so by Lemma 4 there are only the following three cases for $(2.b)$

i. $z(t) < 0$, $z'(t) > 0, z''(t) < 0, [a(t)(z''(t))^{\gamma}]' \geq 0, t \geq t_1 \geq t_0$.

ii. $z(t) > 0$, $z'(t) > 0$, $z''(t) < 0, [a(t)(z''(t))^{\gamma}]' \geq 0, t \geq t_1 \geq t_0$.

iii. $z(t) < 0$, $z'(t) < 0$, $z''(t) < 0, [a(t)(z''(t))^{\gamma}]' \geq 0, t \geq t_1 \geq t_0$.

Case **i.** From eq $(2.\text{ii})$ it follows that

$$x(\tau(t)) > \frac{-z(t)}{p(t)}$$

$$x(t) > \frac{-z(\tau^{-1}(t))}{p(\tau^{-1}(t))}$$

$$x(\sigma(t)) > \frac{-z(\tau^{-1}(\sigma(t)))}{p(\tau^{-1}(\sigma(t)))} \quad (10)$$

Integrating eq $(1.\text{ii})$ from t to $\alpha(t)$ and using (10) we get

$$-a(t)(z''(t))^{\gamma} \geq -\int_t^{\alpha(t)} |q(s)| \frac{z^{\gamma}(\tau^{-1}(\sigma(s)))}{p^{\gamma}(\tau^{-1}(\sigma(s)))} ds$$

$$\geq -z^{\gamma}(\tau^{-1}(\sigma(\alpha(t)))) \int_t^{\alpha(t)} \frac{|q(s)|}{p^{\gamma}(\tau^{-1}(\sigma(s)))} ds$$

$$z''(t) \leq \frac{z(\tau^{-1}(\sigma(\alpha(t))))}{a^{\frac{1}{\gamma}}(t)} [\int_t^{\alpha(t)} \frac{|q(s)|}{p^{\gamma}(\tau^{-1}(\sigma(s)))} ds]^{\frac{1}{\gamma}}$$

Integrating the last inequality from t to $\beta(t)$ we get

$$-z'(t) \leq \int_t^{\beta(t)} \frac{z(\tau^{-1}(\sigma(\alpha(s))))}{a^{\frac{1}{\gamma}}(s)} [\int_s^{\alpha(s)} \frac{|q(v)|}{p^{\gamma}(\tau^{-1}(\sigma(v)))} dv]^{\frac{1}{\gamma}} ds$$

$$-z'(t) \leq z\left(\tau^{-1}\left(\sigma\left(\alpha(\beta(t))\right)\right)\right) \int_t^{\beta(t)} \frac{1}{a^{\frac{1}{\gamma}}(s)} [\int_s^{\alpha(s)} \frac{|q(v)|}{p^{\gamma}(\tau^{-1}(\sigma(v)))} dv]^{\frac{1}{\gamma}} ds$$

$$z'(t) + z(H(t)) \int_t^{\beta(t)} \frac{1}{a^{\frac{1}{\gamma}}(s)} [\int_s^{\alpha(s)} \frac{|q(v)|}{p^{\gamma}(\tau^{-1}(\sigma(v)))} dv]^{\frac{1}{\gamma}} ds \geq 0$$

Where $H(t) = \tau^{-1}(\sigma(\alpha(\beta(t))))$

By lemma 1 and condition (7) the last inequality cannot has eventually negative solution which is contradiction.

Case **ii.** From eq $(2.\text{ii})$we get $x(t) > z(t)$, $t \geq t_1 \geq t_0$

$$x(\sigma(t)) > z(\sigma(t)) \quad (11)$$

Integrating eq $(1.1.\text{ii})$ from t to $\alpha(t)$ and using (11) we get

$$-a(t)(z''(t))^{\gamma} \geq -\int_t^{\alpha(t)} q(s) z^{\gamma}(\sigma(s)) ds$$

$$z''(t) \leq \frac{-1}{a^{\frac{1}{\gamma}}(t)} [\int_t^{\alpha(t)} |q(s)| z^{\gamma}(\sigma(s))\, ds]^{\gamma}$$

$$\leq \frac{-z(\sigma(t))}{a^{\frac{1}{\gamma}}(t)} \left[\int_t^{\alpha(t)} |q(s)| ds\right]^{\gamma}$$

Integrating the last inequality from t_1 to t

$$z'(t) - z'(t_1) \leq -\int_{t_1}^{t} \frac{z(\sigma(s))}{a^{\frac{1}{\gamma}}(s)} \left[\int_s^{\alpha(s)} |q(v)| dv\right]^{\gamma} ds$$

$$\leq -z(\sigma(t_1)) \int_{t_1}^{t} \frac{1}{a^{\frac{1}{\gamma}}(s)} \left[\int_s^{\alpha(s)} |q(v)| dv\right]^{\gamma} ds$$

as $t \to \infty$ and in view of condition H3 and (8) the last inequality leads to a contradiction.

Case **iii.** In this case $a(t)(z''(t))^{\gamma} < 0$ and nondecreasing for $t \geq t_1$ hence it is bounded.

Integrating eq $(1.\text{ii})$ from t_1 to t and using (10) we get

$$a(t)(z''(t))^{\gamma} - a(t_1)(z''(t_1))^{\gamma} \geq -\int_{t_1}^{t} |q(s)| \frac{z^{\gamma}\left(\tau^{-1}(\sigma(s))\right)}{p^{\gamma}\left(\tau^{-1}(\sigma(s))\right)} ds$$

$$\geq -z^{\gamma}\left(\tau^{-1}(\sigma(t_1))\right)\int_{t_1}^{t}\frac{|q(s)|}{p^{\gamma}\left(\tau^{-1}(\sigma(s))\right)}ds$$

as $t \to \infty$ and in the view of (9) the last inequality leads to a contradiction

Example 2. Consider the third-order nonlinear differential equation

$$\left(\frac{1}{t}\left[\left(x(t) - 2x(4t)\right)''\right]^3\right)' - 8x\left(\frac{t}{2}\right) = 0, \qquad (E2)$$

One can see that $\gamma = 3, a(t) = \frac{1}{t}, \tau(t) = 4t+, \sigma(t) = \frac{t}{2}$,

If we set $\alpha(t) = \beta(t) = 2t$, then $H(t) = \frac{t}{2}$

We can see that all conditions (6) hold as follows

$$\sqrt[3]{4}\lim_{t\to\infty}\int_{\frac{t}{2}}^{t}\int_{s}^{2s}[v\int_{v}^{2v} dw]^{\frac{1}{3}}dvds = \infty > \frac{1}{e}$$

$$8\lim_{t\to\infty}\int_{t}^{2t} ds = \infty > 0$$

$$\int_{t_1}^{\infty} ds = \infty$$

so according to theorem 2 every solution of equation $(E2)$ is oscillatory.

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تذبذب حلول المعادلات التفاضلية المحايدة نصف الخطية من الرتبة الثالثة

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الخلاصة :

في البحث بعض المفاهيم التذبذب لكل حلول المعادلات التفاضلية المحايدة نصف الخطية من الرتبة الثالثة نوقشت و تم استخراج بعض الشروط الضرورية والكافية لحلول المعادلة:
$(a(t)[(x(t) \pm p(t)x(\tau(t)))'']^{\gamma})' + q(t)x^{\gamma}(\sigma(t)) = 0, \quad t \geq t_0,$ كي تكون متذبذبة.كما اعطينا بعض الامثلة لتوضيح النتائج المستخرجة.

الكلمات المفتاحية: المعادلات التفاضلية المحايدة نصف الخطيةمن الرتبة الثالثة، تذبذب الحلول.