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On (σ,τ)-Derivations and Commutativity of Prime and Semi prime Γ-rings

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Abstract:

Let R be a Γ-ring, and σ, τ be two automorphisms of R. An additive mapping d from a Γ-ring R into itself is called a (σ,τ)-derivation on R if d(aαb) = d(a)α σ(b) + τ(a)αd(b), holds for all a,b ∈R and α∈Γ. d is called strong commutativity preserving (SCP) on R if $[d(a), d(b)]_{\alpha} = [a, b]_{\alpha}^{(\sigma,\tau)}$ holds for all a,b∈R and α∈Γ. In this paper, we investigate the commutativity of R by the strong commutativity preserving (σ,τ)-derivation d satisfied some properties, when R is prime and semi prime Γ-ring.

Key words: Prime Γ-ring, Semi prime Γ-ring, (σ,τ)-derivation, Strong commutativity preserving (σ,τ)-derivation, Commutativity.

Introduction

Let R and Γ be two additive abelian groups. If for any a, b, c ∈R and α,β ∈Γ, the following conditions are satisfied,

(i) a α b ∈R

(ii) (a+b)αc = aαc + bαc, a(α +β)b =aαb + aβb, aα(b + c) = aαb + aαc

(iii) (aαb)βc = aα(bβc), then R is called a Γ-ring (see [4]).

The set Z(R) = {a ∈R| aαb = bαa, ∀b ∈R, andα∈Γ} is called the center of R.A Γ-ring R is called prime if aΓRΓb = 0 with a, b ∈R implies a = 0 or b = 0, and R is called semi prime if aΓRΓa = 0 with a ∈R implies a = 0. The notion of a (resp. semi-) prime Γ-ring is an extension for the notion of a (resp. semi-) prime ring. In [1] F.J.Jing defined a derivation on Γ-ring as follows, an additive map d from a Γ-ring R into itself is called a derivation on R if d(aαb) = d(a)αb +aαd(b) , holds for all a,b∈R and α∈Γ, and in [2] S. Ali and M.Salahudin Khan defined (σ,τ)-derivation on R ,for two endomorphism σ and τ as follows: an additive map d from R into R is called a (σ,τ)-derivation on R if d(aαb) = d(a)ασ(b) + τ(a)αd(b) , holds for all a,b ∈R and α ∈Γ.

A mapping f from R into itself is commuting if $[f(a), a]_{\alpha} =0$, and centralizing if $[f(a), a]_{\alpha}$∈Z(R) for all a∈R, α∈Γ. And a map f from a Γ -ring R into itself is called strong commutativity preserving (SCP) on R if $[f(a), f(b)]_{\alpha} = [a, b]_{\alpha}$ holds for all a, b ∈R and α∈Γ. The notion of a strong commutativity preserving map was first introduced by Bell and Mason [3], and in [4] X. Jing Ma, and Y. Zhou proved that a semi prime Γ-ring with a strong commutativity preserving derivation on itself must be commutative. In this paper, we obtain that a Γ-ring R with a strong commutativity preserving (σ,τ)-derivation d on itself must be commutative, when R is prime and semi prime Γ-ring. We write $[a, b]_\alpha$ = aαb –bαa. Throughout this paper R will denote a Γ-ring satisfying an assumption (*)…. aαbβc = aβbαc, for all a,b,c ∈R and α,β∈ Γ .

We will often use the identities:

(i) $[a, b]_\alpha^{(\sigma,\tau)}$ = aασ(b) − τ(b)αa.

(ii) $[a\beta b, c]_\alpha^{(\sigma,\tau)}$= aβ$[b, c]_\alpha^{(\sigma,\tau)}$+ $[a, \tau(c)]_\alpha$ βb= aβ $[b, \sigma(c)]_\alpha$ + $[a,c]_\alpha^{(\sigma,\tau)}$βb.

(iii) $[a, b\beta c]_\alpha^{(\sigma,\tau)}$= τ (b)β$[a, c]_\alpha^{(\sigma,\tau)}$+ $[a, b]_\alpha^{(\sigma,\tau)}$β σ(c).

Main Results

First we prove the following lemmas.

Lemma 1:

Let R be a prime Γ-ring, and d be a non zero (σ,τ)-derivation of R. For any a∈R, if d(R)Γa= {0} then a= 0,and if aΓd(R)= {0} then a= 0.

Proof:

Assume that d(R)Γa= {0}, for a∈R.

Let r∈R and α∈Γ, then we have

(1) d(r)αa = 0.

Replacing r by rβb, b∈R in (1) , we get,

(2)d(r)βσ(b)αa + τ(r)βd(b)αa = 0. for all a,r,b∈R and α,β∈Γ.

By using (1) in (2), and since σ is automorphism, we get

d(r)β b αa = 0. for all a,b,r∈R and α,β∈Γ. Since R is a prime and d ≠ 0, then we have a=0.

A similar argument works if aΓd(R)= {0}.

Lemma 2:

Let R be 2-torsion free prime and d be a(σ,τ)-derivation, and d can be commuted with σ and τ. If d^2= 0 then d = 0.

Proof:

Let, for all a, b∈R, α∈Γ. From the hypothesis we have

(1) 0 = d^2(aαb)

= d(d(a)ασ(b) + τ (a)αd(b))

=d^2(a)ασ^2(b)+τ(d(a))αd(σ(b)) +d(τ(a)) ασ(d(b))+ τ^2(a) αd^2 (b)

Then, we get

0 = τ(d(a))αd(σ(b)) + d(τ(a))ασ(d(b))

= 2τ(d(a))ασ(d(b))

(2) = τ (d(a))ασ(d(b))

Taking bβc, c∈R instead of b in (2) ,we get 0 = τ (d(a))ασ(d(bβc))

= τ (d(a))ασ(d(b)βσ(c) + τ (b)βd(c))

= τ (d(a)ασ(d(b)βσ(c)) + τ(d(a))ασ(τ(b) βd(c)), by equation (2), we get

0 = τ (d(a))ασ(τ(b)βσ(d(c))

= τ (d(a))ΓRΓσ(d(c))

Since σ and τ are automorphisms, and R is prime then d(a) = 0 or d(c) = 0 and as a result we get d = 0.

Theorem 3:

Let R be a prime Γ-ring with a non zero (σ,τ)-derivation d. If d is a strong commutativity preserving then R is commutative ring.

Proof:

For all a,b∈R and α∈Γ, we have

(1) $[d(a),d(b)]_\alpha$= $[a, b]_\alpha^{(\sigma,\tau)}$.

Replacing a by aβc in (1) ,we get

$[d(a\beta c),d(b)]_\alpha$= $[a\beta c, b]_\alpha^{(\sigma,\tau)}$,then,

(2) d(a) β$[\sigma(c),d(b)]_\alpha$+$[d(a),d(b)]_\alpha$βσ(c) + τ(a)β$[d(c),d(b)]_\alpha$ + $[\tau(a),d(b)]_\alpha$βd(c)= aβ$[c,b]_\alpha$+$[a,\tau(b)]_\alpha$βc, for all a,b,c∈R and α,β∈Γ

Replacing τ(b) by b and σ(c) by c in (2) and using (1),we get

(3)d(a)β$[c,d(b)]_\alpha$+$[\tau(a),d(b)]_\alpha$βd(c)= 0 ,

Replacing τ(a) by ain (3) ,we get

(4) d(a)β$[c,d(b)]_\alpha$+$[a,d(b)]_\alpha$βd(c)= 0 ,for all a,b,c∈R and α,β∈Γ.

Multiplying (4) by λr on the right, we get

(5) d(a)β$[c,d(b)]_\alpha$λr +$[a,d(b)]_\alpha$βd(c)λr = 0 ,for all a,b,c,r∈R and α,β,λ∈Γ.

Again replacing c by cλr in (4), we get

(6) $d(a)\beta c\lambda[r,d(b)]_\alpha + d(a)\beta[c,d(b)]_\alpha\lambda r + [a,d(b)]_\alpha\beta d(c)\lambda\sigma(r) + [a,d(b)]_\alpha\beta\tau(c)\lambda d(r) = 0$

Replacing σ(r) by r in (6) ,we get

(7) $d(a)\beta c\lambda[r,d(b)]_\alpha + d(a)\beta[c,d(b)]_\alpha\lambda r + [a,d(b)]_\alpha\beta d(c)\lambda r + [a,d(b)]_\alpha\beta\tau(c)\lambda d(r) = 0$, for all a,b,c,r∈R and α,β,λ∈Γ.

Comparing (5) and (7),we get

(8) $d(a)\beta c\lambda[r,d(b)]_\alpha + [a,d(b)]_\alpha\beta\tau(c)\lambda d(r) = 0$,

Replacing a by d(b) in (8) ,we get

(9) $d^2(a)\beta\ c\ \lambda\ [r,d(b)]_\alpha = 0$, for all a,b,c,r∈R and α,β,λ∈Γ.

Since R is a prime, we have $d^2(a)=0$ or $[r,d(b)]_\alpha=0$. If $d^2(a) = 0$ for all a∈R. Then from lemma 2, d = 0 is obtained. But this is a contradiction. That is

(10) $[r,d(b)]_\alpha=0$, for all b,r ∈R and α∈Γ.

Replacing d(b) by tβd(b), t∈R in (10), and using (10) again, we get

(11) $[r, t]_\alpha\ \beta d(b) = 0$, for all r,t,b ∈R and α,β ∈Γ.

By using lemma 1 in (11), we get

$[r, t]_\alpha = 0$, for all r ,t ∈R and α ∈Γ. Hence R is commutative ring.

Theorem 4:

Let R be a prime Γ-ring, and d is a strong commutativity preserving (σ,τ)-derivation. If σ = τ then either R is commutative ring or d =0.

Proof:

By hypothesis, we have

(1) $[d(a),d(b)]_\alpha = [a,b]_\alpha^{(\sigma,\tau)}$,for all a,b∈R and α∈Γ

Replacing b by bβc in (1) ,we get

$[d(a),d(b\beta c)]_\alpha = [a, b\beta c]_\alpha^{(\sigma,\tau)}$,then,

(2) $d(b)\beta[d(a),\sigma(c)]_\alpha + [d(a),d(b)]_\alpha\beta\sigma(c) + \sigma(b)\beta[d(a),d(c)]_\alpha + [d(a), \sigma(b)]_\alpha\beta d(c) = \sigma(b)\beta[a, c]_\alpha^{(\sigma,\tau)} + [a, b]_\alpha^{(\sigma,\tau)}\beta\ \sigma(c)$, for all a,b,c∈R and α,β∈Γ

By using (1) in (2) ,we get

(3) $d(b)\beta\ [d(a),\sigma(c)]_\alpha + [d(a), \sigma(b)]_\alpha\,\beta d(c) = 0$.

Replacing σ(b) by d(a) in (3) ,we get

(4) $d(b)\beta\ [d(a),\sigma(c)]_\alpha = 0$,for all a,b,c∈R and α,β∈Γ.

Replacing σ(c) by σ(c)δr in (4) ,we get

(5) $d(b)\beta\ \sigma(c)\delta\ [d(a),\ r]_\alpha + d(b)\beta [d(a),\sigma(c)]_\alpha\delta r = 0$, for all a,b,c,r∈R and α,β,δ∈Γ.

Using (4) in (5), and replacing r by d(b) in (5) ,we get

(6) $d(b)\beta\ \sigma(c)\delta\ [d(a), d(b)]_\alpha=0$.

By using (1) in (6), and since σ is automorphism, we have

$d(b)\ \beta\ c\ \delta\ [a, b]_\alpha=0$, for all a,b,c∈R and α,β,δ∈Γ.

Since R is a prime, we have d(b) =0 or $[a, b]_\alpha=0$, for all a,b∈R and α∈Γ

That's mean d=o or R is commutative.

Theorem 5:

Let R be a prime Γ-ring with a (σ,τ)-derivation d such that $[d(a),d(b)]_\alpha = [a^2, b^2]_\alpha^{(\sigma,\tau)}$, then d= 0 or d is commuting.

Proof:

For all a,b ∈R and α ∈Γ ,we have

(1) $[d(a),d(b)]_\alpha = [a^2, b^2]_\alpha^{(\sigma,\tau)}$.

Replacing a by a+b in (1) ,we have

$[d(a),d(b)]_\alpha + [d(b),d(b)]_\alpha = [(a^2 + a\beta b + b\beta a + b^2), b^2]_\alpha^{(\sigma,\tau)}$.

By using (1),we have

(2) $[a\beta b, b^2]_\alpha^{(\sigma,\tau)} + [b\beta a, b^2]_\alpha^{(\sigma,\tau)} = 0$, for all a,b∈R and α,β∈Γ .

Replacing aby d(b)δb in (2) ,we have

$[d(b)\delta b\beta b, b^2]_\alpha^{(\sigma,\tau)} + [b\beta d(b)\delta b, b^2]_\alpha^{(\sigma,\tau)} = 0$, then

$[d(b)\delta b\beta b + b\beta d(b)\delta b,\ b^2]_\alpha^{(\sigma,\tau)} = 0$.

This implies

$[d(b^2)\beta b\ ,\ b^2]_\alpha^{(\sigma,\tau)} = 0$, then

$d(b^2)\beta[b,\ b^2]_\alpha + [d(b^2), \tau(b^2)]_\alpha\,\beta b = 0$,then we have

(3) $[d(b^2), \tau(b^2)]_\alpha\ \beta b = 0$, for all b∈R and α∈Γ.

Replacing $b^2$by c in (3) ,we have

(4) $[d(c), \tau(c)]_\alpha\ \beta b = 0$, for all c∈R and α∈Γ

Replacing τ(c) by c and b by bλd(c) in (4) and using (1),we have

$[d(c), c]_\alpha \beta b \lambda d(c)=0$, for all c,b∈R and α,β,λ∈Γ

Since R is a prime, we have d(c) =0 or $[d(c), c]_\alpha$, for all c ∈R and α∈Γ.

Thus d=o or d is commuting.

Theorem 6:

Let R be a semi prime Γ-ring. If d is a strong commutativity preserving (σ,τ)-derivation, then d is commuting.

Proof:

If d=0 , then $[a,b]_\alpha^{(\sigma,\tau)}=0$, for all a,b∈R and α∈Γ .

Replacing b by d(a) in (1) we get $[a,d(a)]_\alpha^{(\sigma,\tau)}=0$, for all a∈R and α∈Γ.Thus d is commuting.

Now, If d≠0, then by hypothesis, we get

(1) $d(a),d(b)]_\alpha = [a,b]_\alpha^{(\sigma,\tau)}$,for all a,b∈R and α∈Γ

Replacing a by aβc in (1) ,we get

$[d(a\beta c),d(b)]_\alpha = [a\beta c,b]_\alpha^{(\sigma,\tau)}$,then,

(2)$d(a)\beta[\sigma(c),d(b)]_\alpha+[d(a),d(b)]_\alpha\beta\sigma(c) + \tau(a)\beta[d(c),d(b)]_\alpha+[\tau(a),d(b)]_\alpha\beta d(c)= a\beta[c, \sigma(b)]_\alpha+[a, b]_\alpha^{(\sigma,\tau)}\beta c$, for all a,b,c∈R and α,β∈Γ

Replacing σ(c) by c in (2) and using (1),we get

(3) $d(a)\beta[\sigma(c),d(b)]_\alpha+\tau(a)\beta[d(c),d(b)]_\alpha + [\tau(a),d(b)]_\alpha\beta d(c) = a\beta[c, \sigma(b)]_\alpha$.

Replacing τ(a) by a and σ(b) by b in (3) and using (1),we get

(4) $d(a)\beta[\sigma(c),d(b)]_\alpha+ [a ,d(b)]_\alpha\beta d(c) = 0$, for all a,b,c∈R and α,β∈Γ.

Replacing σ(c) by d(b) in (4) ,we get

(5) $[a,d(b)]_\alpha\beta d(c)= 0$, for all a,b,c∈R and α,β∈Γ.

Replacing a by bλa in (5),and using(5), we get

(6)$[b,d(b)]_\alpha\lambda a \beta d(c)= 0$, for all a,b,c∈R and α,β,λ ∈Γ.

Multiply (6) from left by d(c),and from right by $[b ,d(b)]_\alpha$,then we get

$d(c)\delta[b ,d(b)]_\alpha\lambda a \beta d(c)\delta[b ,d(b)]_\alpha= 0$, for all a,b,c∈R and α,β,λ,δ∈Γ.

Since R is a semi prime, then:

$d(c)\delta [b ,d(b)]_\alpha= 0$, for all b,c∈R and α,δ ∈Γ.

By using lemma 1 ,we get

$[b,d(b)]_\alpha= 0$, for all b ∈R and α∈Γ. That's complete the proof.

Theorem 7:

Let R be a semi prime Γ-ring with a non zero (σ,τ)-derivation d. If d is a strong commutativity preserving such that $[d(a), \tau(a)]_\alpha= 0$, then d⊆Z(R).

Proof:

For all a,b∈R and α∈Γ, we have

(1) $[d(a),d(b)]_\alpha= [a,b]_\alpha^{(\sigma,\tau)}$.

Replacing b by aβb in (1) ,we get

$[d(a),d(a\beta b)]_\alpha= [a,a\beta b]_\alpha^{(\sigma,\tau)}$,then:

(2) $d(a)\beta [d(a),\sigma(b)]_\alpha+ [d(a),d(a)]_\alpha \sigma(b) + \tau(a)\beta[d(a), d(b)]_\alpha + [d(a), \tau(a)]_\alpha\beta d(b) = \tau(a)\beta[a, b]_\alpha^{(\sigma,\tau)}+ [a, a]_\alpha^{(\sigma,\tau)}\beta \sigma(b)$, for all a,b∈R and α,β∈Γ

By using (1) in (2) ,we get

(3) $d(a)\beta [d(a),\sigma(b)]_\alpha+[d(a), \tau(a)]_\alpha\beta d(b) =0$.

Replacing σ(b) by r,and using the hypothesis in (3) ,we get

(4) $d(a)\beta [d(a), r]_\alpha = 0$,for all a,b,r∈R and α,β∈Γ.

Replacing rby rδb in (4) ,and using (4), we get

(5) $d(a)\beta r \delta [d(a), b]_\alpha = 0$, for all a,b,c,r∈R and α,β,δ∈Γ.

Multiply (5) from left by $[d(a) ,b]_\alpha$,and from right by d(a),then we get

$[d(a), b]_\alpha\lambda d(a) \beta r \delta [d(a), b]_\alpha\lambda d(a) = 0$, for all a,b∈R and α,β,δ,λ∈Γ.

Since R is a semi prime, we have

$[d(a), b]_\alpha\lambda d(a) =0$,for all a,b∈R and α,λ∈Γ.

By using lemma 1, we get

$[d(a), b]_\alpha = 0$, for all a,b∈R and α∈Γ. That's mean d⊆Z(R).

Theorem 8:

Let R be a semi prime Γ-ring with a (σ,τ)-derivation d such that $[d^2(a),d^2(b)]_\alpha= [a ,b]_\alpha^{(\sigma,\tau)}$, then R is a commutative.

Proof:

For all a,b∈R and α∈Γ ,we have :

(1) $[d^2(a),d^2(b)]_\alpha= [a ,b]_\alpha^{(\sigma,\tau)}$

Replacing $[d^2(a),d^2(b)]_\alpha$ by $[[d^2(a),d^2(b)]_\alpha, c]_\beta$ in (1) ,c∈R, we get $[[d^2(a),d^2(b)]_\alpha,c]_\beta = [a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$,then we have

(2) $[d^2(a)\alpha d^2(b),c]_\beta - [d^2(b)\alpha d^2(a),c]_\beta = [a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$,then we have

(3) $d^2(a)\alpha[d^2(b),c]_\beta +[d^2(a),c]_\beta\alpha d^2(b) - d^2(b)\alpha[d^2(a),c]_\beta -[d^2(b),c]_\beta\ \alpha d^2(a) = [a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$,for all a,b,c∈R and α,β∈Γ.

Replacing c by $d^2(b)\delta d^2(a)$ in (3), we get:

$d^2(a)\alpha[d^2(b),d^2(b)\delta d^2(a)]_\beta+[d^2(a),d^2(b)\delta\ d^2(a)]_\beta\alpha d^2(b)\ -^2(b)\alpha[d^2(a),d^2(b)\delta d^2(a)]_\beta - [d^2(b),d^2(b)\delta d^2(a)]_\beta\ \alpha d^2(a)= [a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$, then we have

(4) $d^2(a)\alpha d^2(b)\delta[d^2(b),d^2(a)]_\beta + [d^2(a),d^2(b)]_\beta\delta d^2(a)\alpha d^2(b) - d^2(b)\alpha[d^2(a),d^2(b)]_\beta\delta d^2(a) - d^2(b)\delta[d^2(b),d^2(a)]_\beta\ \alpha d^2(a) = [a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$,for all a,b∈R and α,β,δ∈Γ

Using (1) in (4), and since σ and τ are automorphism, we get

(5) $d^2(a)\alpha d^2(b)\delta\ [b, a]_\beta^{(\sigma,\tau)} + [a,b]_\beta^{(\sigma,\tau)}\delta d^2(a)\alpha d^2(b) =[a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$,for all a,b∈R and α,β,δ∈Γ.

Replacing c by $[a, b]_\beta^{(\sigma,\tau)}$ in (2),we get

$[d^2(a)\alpha d^2(b),\ [a, b]_\beta^{(\sigma,\tau)}]_\beta - [d^2(b)\alpha d^2(a),[a, b]_\beta^{(\sigma,\tau)}]_\beta =[a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$,for all a,b∈R and α,β∈Γ, then

(6) $d^2(a)\alpha d^2(b)\beta\ [a,b]_\beta^{(\sigma,\tau)} - [a, b]_\beta^{(\sigma,\tau)}\beta d^2(a)\alpha d^2(b) - d^2(b)\alpha d^2(a)\beta\ [a, b]_\beta^{(\sigma,\tau)}+[a, b]_\beta^{(\sigma,\tau)}\beta d^2(b)\alpha d^2(a) = [a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}$,for all a,b∈R and α,β∈Γ.

Putting β=δ , and comparing (6) with (5), we get

(7) $d^2(a)\alpha d^2(b)\delta[a,b]_\beta^{(\sigma,\tau)} - [a, b]_\beta^{(\sigma,\tau)}\delta d^2(a)\alpha d^2(b) - d^2(b)\alpha d^2(a)\delta\ [a, b]_\beta^{(\sigma,\tau)}+ [a,b]_\beta^{(\sigma,\tau)}\delta d^2(b)\alpha d^2(a) - d^2(a)\alpha d^2(b)\delta\ [b, a]_\beta^{(\sigma,\tau)} - [a, b]_\beta^{(\sigma,\tau)}\delta d^2(a)\alpha d^2(b) =0$, for all a,b∈R and α,β,δ∈Γ.

Now, from (1) we have

(8) $d^2(a)\alpha d^2(b) - [a\ \ ,b\ \ ]_\alpha^{(\sigma,\tau)}= d^2(b)\alpha d^2(a)$,for all a,b∈R and α∈Γ.

Substitute (8) in (7), and since σ and τ are automorphism, we get

(9) $d^2(a)\alpha d^2(b)\delta\ [a, b]_\beta^{(\sigma,\tau)} + [b, a]_\beta^{(\sigma,\tau)}\delta d^2(a)\alpha d^2(b) = 0$,for all a,b∈R and α,β,δ∈Γ.

Replacing $d^2(a)\alpha d^2(b)$ by r in (9), we have

(10) $r\ \delta\ [a, b]_\beta^{(\sigma,\tau)} +[b, a]_\beta^{(\sigma,\tau)}\delta\ r = 0$, for all a,b,r∈R and β,δ∈Γ.

Replacing a by aλb in (10), we get

(11) $r\delta[a, \tau(b)]_\beta\ \lambda b +[b, a]_\beta^{(\sigma,\tau)}\lambda\ \sigma(b)\delta\ r = 0$,for all a,b,r∈R and β,δ,λ∈Γ.

Since σ and τ are automorphism, and by using (10) in (11), we get

(12) $[b, a]_\beta^{(\sigma,\tau)}\lambda\ [b, r]_\delta^{(\sigma,\tau)}= 0$, for all a,b,r∈R and β,λ,δ ∈Γ.

Replacing r by rγa in (12), and using (12), we get

(13) $[b, a]_\beta^{(\sigma,\tau)}\lambda\ \tau(r)\ \gamma[b, a]_\delta^{(\sigma,\tau)}= 0$, for all a,b,r ∈R and β,λ,δ,γ ∈Γ.

Putting β=δ in (13). Since τ is automorphism and R is semi prime, then we get:

$[b,\ a]_\beta^{(\sigma,\tau)} = 0$, for all a,b∈R and β∈Γ. Hence R is a commutative.

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حول الاشتقاقات -(σ,τ) والابدالية على الحلقات الأولية وشبه الاولية من النمط -Γ

افراح محمد ابراهيم

قسم الرياضيات / كلية التربية / الجامعة المستنصرية

الخلاصة :

لتكن R حلقة من النمط- Γ و σ,τ دوال متشاكلة خارجياً على R. التطبيق الجمعي d من R الى نفسها يسمى إشتقاق(σ,τ)على R اذا كان $d(a\alpha b)= d(a)\alpha\sigma(b) + \tau(a)\alpha d(b)$ يتحقق لكل a,b∈R و α∈Γ . d تسمى إشتقاق(σ,τ) محافظ على الإبدالية القوية على R اذا كان $[d(a), d(b)]_\alpha = [a,b]_\alpha^{(\sigma,\tau)}$ يتحقق لكل a,b∈R وα∈Γ.

في هذا البحث نحقق الابدالية على R عن طريق الإشتقاق(σ,τ) المحافظ على الابدالية القوية محققين بعض الخواص عندما تكون Rحلقة اولية وشبه اولية من النمط - Γ .

الكلمات المفتاحية :الحلقة الاولية من النمط - Γ ،الحلقة شبه الاولية من النمط- Γ ،الاشتقاق(σ,τ)، الاشتقاق(σ,τ) المحافظ على الابدالية القوية ،الابدالية.