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F-Compact operator on probabilistic Hilbert space

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Abstract:

This paper deals with the F-compact operator defined on probabilistic Hilbert space and gives some of its main properties.

Key words: Bounded set, F-compact operators, τ-convergent sequences, and probabilistic Hilbert space.

Introduction:

The notion of probabilistic metric space was introduced by Menger [1]. It was represented as a generalization of the notion of metric space. Menger replaced the distance between the two points p and q by a distribution function $F_{pq}(x)$ for real number x means that the distance between p and q is less than x. The notion of probabilistic metric space corresponds to the situations when we do not know exactly the distance between two points and we know only the possible probabilistic values of these distances. An important family of probabilistic spaces are probabilistic inner product spaces which were introduced by C. Sklar and B. Schweizer in 1983 [2]. The definition of probabilistic Hilbert space was also introduced in 2007 by Su, Y.; Wang, X.; and Gao, J. [3]. In the sequel, we shall define the compact operator on probabilistic Hilbert space and examine some important theorems about it.

Preliminaries:

Definition (1)[4] (Distribution function)

A distribution function (d.f.) is a function F defined on the extended real line $\bar{R}: [-\infty, +\infty]$ that is non-decreasing and left-continuous, such that $F(-\infty) = 0$ and $F(+\infty) = 1$. The set of all d.f. will be denoted by Δ; the subset of Δ formed by the proper d.f.s, i.e. by those d.f.s F for which

$\lim_{t\to-\infty} F(t) = 0$ and $\lim_{t\to+\infty} F(t) = 1$

will be denoted by D.

A special element of Δ is the function which is defined by

$$\varepsilon_0(t) = \begin{cases} 0, & t \le 0 \\ 1, & t > 0 \end{cases}$$

and denoted by $H(t)$.

Definition (2) [2]

A function $\mathcal{T}: [0,1] \times [0,1] \longrightarrow [0,1]$ is called triangle norm (t-norm) *simply* t-norm on $I = [0,1]$ if it satisfies the following conditions, for all $a, b, c,$ and $d \in [0,1]$:

1. $\mathcal{T}(a,1) = a$;
2. $\mathcal{T}(a,b) = \mathcal{T}(b,a)$;
3. $\mathcal{T}(c,d) \geq \mathcal{T}(a,b),\ \forall c \geq a, d \geq b$;
4. $\mathcal{T}\big(a, \mathcal{T}(b,c)\big) = \mathcal{T}(\mathcal{T}(a,b),c)$.

The most important t- norms are the functions which are defined as follows.

1. $W(a,b) = max(a+b-1,0)$,
2. $\prod(a,b) = ab$,
3. $M(a,b) = min(a,b)$.

Definition (3) [2]

A triangle function is a binary operation on Δ^+ which is commutative, associative, non-decreasing and has ε_0 as the identity element. In the other words, a function $\tau: \Delta^+ \times \Delta^+ \longrightarrow \Delta^+$ is called triangle function if for any F, G, and S in Δ^+ we have:

1. $\tau(F, \varepsilon_0(t)) = F$;
2. $\tau(F,G) = \tau(G,F)$;
3. $\tau(F,S) \leq \tau(G,S)$ if $F \leq G$;
4. $\tau(\tau(F,G),S) = \tau\big(F, \tau(G,S)\big)$.

Definition(4) [5]

(Modified probabilistic inner product space)

Let E be a real linear space and let $F: E \times E \to D$ be a function, then the modified probabilistic inner product space is the triple $(E,F,*)$ where F is assumed to satisfy the following conditions:

(MPIP-1) $F_{x,x}(0) = 0$;

(MPIP-2) $F_{x,y} = F_{y,x}$;

(MPIP-3) $F(x,x)\ (t) = H(t) \leftrightarrow x = 0$;

(MPIP-4)

$$F_{\lambda x,y}(t) = \begin{cases} F_{x,y}\left(\frac{t}{\lambda}\right) & , \quad \lambda > 0 \\ H(t) & , \quad \lambda = 0 \\ 1 - F_{x,y}\left({}^{t}/_{\lambda} +\right) & , \quad \lambda < 0 \end{cases}$$

where λ is real number, $F_{x,y}\left({}^{t}/_{\lambda} +\right)$is the right hand limit of $F_{x,y}$ at $\frac{t}{\lambda}$.

(MPIP-5)

If x and y are linearly independent then

$$F_{x+y,z}(t) = \big(F_{x,z} * F_{y,z}\big)(t);$$

Where

$$\big(F_{x,z} * F_{y,z}\big)(t) = \int_{-\infty}^{\infty} F_{x,z}(t-u)dF_{z,y}(u)$$

Note that if x and y are linearly dependent, then

$x + y = x + \alpha x = (1+\alpha)x = \lambda x$

Where $\lambda = 1 + \alpha$

$$F_{x+y,z}(t) = F_{\lambda x,z}(t) = \begin{cases} F_{x,z}\left(\frac{t}{\lambda}\right) & , \quad \lambda > 0 \\ H(t) & , \quad \lambda = 0 \\ 1 - F_{x,z}\left({}^{t}/_{\lambda} +\right) & , \quad \lambda < 0 \end{cases}$$

which is (MPIP-4).

Then $(E,F,*)$ is the modified probabilistic inner product space.

Definition (5) [3]

Let $(E,F,*)$be a modified PIP-space, if

$$\int_{-\infty}^{\infty} t dF_{x,y}(t)$$

is convergent for all $x, y \in E$, Then $(E,F,*)$ is called a modified PIP-space with mathematical expectation.

Theorem (6) [3]

Let $(E,F,*)$ be a modified probabilistic inner product space with mathematical expectation. Let

$$< x,y > = \int_{-\infty}^{\infty} t dF_{x,y}(t) \quad , \forall x,y \in E$$

then $(E, < >)$is inner product space, so that $(E, \|\ \|)$, is a normed space where $\| x \| = \sqrt{< x,x >}\quad \forall x \in E$.

Remark (7) [4]

For the definition of probabilistic inner product space, we must depend on the distribution functions that belong to Δ rather than Δ^+ because this notion should include that negative values of classical inner product space since the inner product space takes negative values; the set Δ provides therefore this part.

For all F in Δ, define the distribution function $\bar{F}$ in Δ, for all $t \in R$, as follows

$$\bar{F}(t) = \begin{cases} F(t), & if\ t \in R, but\ F(t)\ is\ symmetric \\ \iota^{-}(1 - F(t)), & if\ t \in R\backslash R^{+}, but\ F(t)\ not\ symmetric. \end{cases}$$

Where $\iota^{-}(1 - F(t))$ represents the left limit at t.

Remark (8) [6]

A notable example of a triangle function is the convolution which is define as follows

$$(F_{x,z} * F_{y,z})(t) = \int_0^\infty F_{x,z}(t - u)dF_{z,y}(u)$$

$=F_{x,z}(t_1)F_{z,y}(t_2)$

$=\prod\left(F_{x,z}(t_1), F_{z,y}(t_2)\right)$

where $t_1, t_2 \in [0, \infty]$ and satisfy

$t_1 + t_2 = t$

Remark (9) [4]

Note that for any continuous t-norm $\mathcal{T}$ defined in (2), we have

$$\prod_{\mathcal{T}}(F, G)(x) = \mathcal{T}(F(x), G(x))$$

for all $F, G \in \Delta^{+}$ and $x \in R$.

Definition (10) [7]

Let $(E, F, *)$ be a modified probabilistic inner product space, then:

1. A sequence $\{x_n\}$ in E is said to be τ-convergent to $x \in E$ if
$\forall \epsilon > 0$ and
$\forall \lambda > 0\ \exists n_0(\epsilon, \lambda)$ such that
$$F_{x_n - x x_n - x}(\epsilon) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda).$$
2. A sequence $\{x_n\}$ in E is called τ-caushy convergent if
$\forall \epsilon > 0$ and
$\forall \lambda > 0\ \exists n_0(\epsilon, \lambda)$ such that
$$F_{x_n - x_m x_n - x_m}(\epsilon) > 1 - \lambda \quad \forall n, m > n_0(\epsilon, \lambda).$$
3. $(E, F, *)$ is said to be τ −complete if each τ −caushy sequence in E is τ −convergent in E.

Definition (11) [3]

Let $(E, F, *)$ be a modified probabilistic inner product space then a linear functional T defined on E is said to be continuous if for all sequence $\{x_n\} \subseteq E$ that τ-converges to $x \in E$, then $T(x_n) \to T(x)$.

We call this type of continuity as sequentially continuous on $(E, F, *)$.

Definition (12) [3]

Let $(E, F, *)$ be a modified probabilistic inner product space with mathematical expectation, then if E is complete in $\|.\|$ then E is called probabilistic Hilbert space, where $\| x \| = \sqrt{< x, x >}\ \forall x \in E$.

Definition (13) [8]

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation , let T be a linear operator defined on $(E, F, *)$, then T is said to be F-bounded Operator if there exists a constant $c > 0$,such that

$$F_{\ Tx,Tx}(t) \geq F_{\ x,x}\left(\frac{t}{c}\right) \quad for\ all\ x \in E, t \in R.$$

Definition (14) [8]

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation and let $T: (E, F, *) \to (E, F\ *)$ be a linear operator, then T is said to be F-continuous operator at $y \in E$ if for all $\epsilon > 0$ there exists corresponding $\delta > 0$ such that for all $x \in E$ and

$$F_{Tx - Ty, Tx - Ty}(\epsilon) \geq F_{x - y, x - y}(\delta)$$

then if T is F-continuous operator at each point of E, T is F-continuous on $(E, F, *)$.

Theorem (15) [8]

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, and let T be a linear operator defined on $(E, F, *)$, if T is F-continuous operator on $(E, F, *)$, then T is sequentially continuous operator on $(E, F, *)$.

Theorem (16) [8]

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation and let $T: (E, F, *) \to (E, F\ *)$ be a linear operator, then T is F-bounded operator if and only if T is F-continuous operator.

The main results

In this section, we introduce the F-compact operators defined on probabilistic Hilbert space, and give some properties of them.

Theorem (1)

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, then for all $u, v \in E$ and $t_1, t_2 > 0$, we have

$$F_{\boldsymbol{u+v,u+v}}((t_1 + t_2)^2) \geq T\left(F_{u,u}(t_1^2), F_{v,v}(t_2^2)\right)$$

where T is any continuous t-norm satisfying $T(t,t) \geq t$ for all $t \in [0,1]$.

Proof:

Let $\lambda = -{}^{t}/_{s}$, i.e. $\lambda s + t = 0$ where $t, s > 0$, Let $p = F_{u,u}(t^2)$,

$q = F_{\lambda v,u}(\lambda ts)$ and $r = F_{\lambda v,\lambda v}(\lambda^2 s^2)$.

from conditions (MPIP-1) and (MPIP-5), we have

$$0 = F_{u+\lambda v,u+\lambda v}((\lambda s + t)^2)$$

$$= \int_0^\infty F_{u,u+\lambda v}((\lambda s + t)^2 - z) dF_{\lambda v,u+\lambda v}(z)$$

by the remark (8), we have

$$= \Pi\left(F_{u,u+\lambda v}(t^2 + t\lambda s), F_{\lambda v,u+\lambda v}(t\lambda s + \lambda^2 s^2)\right)$$

by the remark (9), we have

$$= \prod\nolimits_{\mathcal{T}}\left(F_{u,u+\lambda v}(t^2 + t\lambda s), F_{\lambda v,u+\lambda v}(t\lambda s + \lambda^2 s^2)\right)$$

$$= \mathcal{T}\left(F_{u,u+\lambda v}(t^2 + t\lambda s), F_{\lambda v,u+\lambda v}(t\lambda s + \lambda^2 s^2)\right) \quad \ldots (1)$$

on the other hand, we have by (MPIP-5)

$$F_{u,u+\lambda v}(t^2 + t\lambda s)$$

$$= \int_0^\infty F_{u,u}\big((t^2 + t\lambda s) - z\big) dF_{\lambda v,u}(z)$$

by the remark (8), we have

$$= \Pi\left(F_{u,u}(t^2), F_{\lambda v,u}(t\lambda s)\right)$$

by the remark (9), we have

$$= \prod\nolimits_{\mathcal{T}}\left(F_{u,u}(t^2), F_{\lambda v,u}(t\lambda s)\right)$$

$$= \mathcal{T}\left(F_{u,u}(t^2), F_{\lambda v,u}(t\lambda s)\right)$$

$$= \mathcal{T}(p, q) \quad \ldots \ (2)$$

also, by (MPIP-5), we have

$$F_{\lambda v,u+\lambda v}(t\lambda s + \lambda^2 s^2)$$

$$= \int_0^\infty F_{\lambda v,\lambda v}\big((t\lambda s + \lambda^2 s^2) - z\big) dF_{\lambda v,u}(z)$$

by the remark (8), we have

$$= \Pi\left(F_{\lambda v,u}(t\lambda s), F_{\lambda v,\lambda v}(\lambda^2 s^2)\right)$$

by the remark (9), we have

$$= \prod\nolimits_{\mathcal{T}}\left(F_{\lambda v,u}(t\lambda s), F_{\lambda v,\lambda v}(\lambda^2 s^2)\right)$$

$$= \mathcal{T}\left(F_{\lambda v,u}(t\lambda s), F_{\lambda v,\lambda v}(\lambda^2 s^2)\right)$$

$$= \mathcal{T}(q, r) \quad \ldots \ (3)$$

substituting (2) and (3) in (1), we get

$$0 = \mathcal{T}\big(\mathcal{T}(p,q), \mathcal{T}(q,r)\big) = \mathcal{T}\big(p, \mathcal{T}(q, \mathcal{T}(q,r)\big)$$

$$= \mathcal{T}\big(p, \mathcal{T}(\mathcal{T}(q,q), r)\big)$$

$$\geq \mathcal{T}\big(p, \mathcal{T}(q,r)\big)$$

$$= \mathcal{T}\big(p, \mathcal{T}(r,q)\big) \qquad = \mathcal{T}(\mathcal{T}(p,r), q)$$

$r = F_{\lambda v,\lambda v}(\lambda^2 s^2) = F_{v,v}(s^2)$

$q = F_{\lambda v,u}(\lambda ts) = 1 - F_{v,u}(ts+)$

on the other hand

$\mathcal{T} \geq W(p,q) = \max(p + q - 1, 0)$ $\forall p, q \in [0,1]$, since $\mathcal{T}(t,t) \geq t$, for any t in $[0,1]$.

which implies

$$0 \geq \mathcal{T}(\mathcal{T}(F_{u,u}(t^2), F_{v,v}(s^2)), 1 - F_{v,u}(ts+))$$

$$\geq \mathcal{T}\left(F_{u,u}(t^2), F_{v,v}(s^2)\right) + 1 - F_{v,u}(ts+) - 1$$

$$F_{u,v}(ts+)$$

$$\geq \mathcal{T}\left(F_{u,u}(t^2), F_{v,v}(s^2)\right) \ \ldots . (4)$$

For any given $u, v \in E$ and $t_1, t_2 > 0$, let $c = F_{u,u}(t_1^2), d = F_{u,v}(t_1 t_2)$, $e = F_{v,v}(t_2^2)$, we have

$$F_{u+v,u+v}((t_1 + t_2)^2)$$

$$= \mathcal{T}\big(\mathcal{T}(c,d), \mathcal{T}(d,e)\big)$$

$$= \mathcal{T}\big(\mathcal{T}(c,d), \mathcal{T}(e,d)\big)$$

$$= \mathcal{T}\left(c, \mathcal{T}\big(e, \mathcal{T}(d,d)\big)\right)$$

$$\geq \mathcal{T}\big(c, \mathcal{T}(e,d)\big)$$

by(4), we get

$$\mathcal{T}(\mathcal{T}(c,e), d)$$

$$\geq \mathcal{T}\big(\mathcal{T}(c,e), \mathcal{T}(c,e)\big)$$

$$\geq \mathcal{T}(c,e)$$

$$F_{\boldsymbol{u+v,u+v}}((t_1 + t_2)^2)$$

$$\geq \mathcal{T}\left(F_{u,u}(t_1^2), F_{v,v}(t_2^2)\right)$$

■

Definition (2)

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, a subset X of E is called Probabilistic Bounded set if there exist $t \in R\ /\{0\}$ and $a \in (0,1)$, such that $F_{x,x}(t) > 1 - a$ for any x in X.

Definition (3)

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, the probabilistic closure $\bar{X}$ of a subset X of E is the set of all $y \in E$, such that there exists a sequence $\{x_n\} \subseteq X$ that is τ- convergent to y.

If $X = \bar{X}$ then we call X a probabilistic closed set.

Definition (4)

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, a subset X of E is called Probabilistic Compact set if each sequence $\{x_n\} \subseteq X$ has τ-convergent subsequence.

In the following, the compact operator will be defined on probabilistic Hilbert space, and call it the F-compact operator.

Definition (5) (F-compact operator)

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, a linear operator $T: (E, F, *) \to (E, F *)$ is called an F-compact operator if for any Probabilistic Bounded subset X of E, then $T(X)$ is relatively probabilistic compact.

Theorem (6)

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, a linear operator $T: (E, F, *) \to (E, F *)$ is an F-compact operator if and only if for all Probabilistic Bounded sequence $\{x_n\} \subseteq E$ then $\{T(x_n)\}$ has τ-convergent subsequence.

Proof:

Assume that T is an F-compact operator, to prove for any probabilistic bounded sequence $\{x_n\} \subseteq E$ then $\{T(x_n)\}$ has τ- convergent subsequence.

let $\{x_n\}$ be a probabilistic bounded sequence in E, since T is an F-compact operator then by definition (5), $\overline{T(x_n)}$ is a probabilistic compact set for all $n \geq 1$. Then $\{T(x_n)\}$ has τ- convergent subsequence.

Conversely, assume that for any probabilistic bounded sequence $\{x_n\} \subseteq E$ then $\{T(x_n)\}$ has τ- convergent subsequence.

to prove that T is an F-compact operator.

let X be a probabilistic bounded subset of E, and let $\{x_n\} \subseteq \overline{T(X)}$, then by definition (3), there is a sequence $\{y_n\}$ in $T(X)$ such that for given $\epsilon > 0, \lambda > 0, \exists n_0(\epsilon, \lambda)$ such that

$$F_{y_n - x_n, y_n - x_n}\left(\left({\epsilon}/{2}\right)^2\right) = F_{x_n - y_n, x_n - y_n}\left({\epsilon}/{2})^2\right) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda).$$

since $y_n \subseteq T(X)$ then $y_n = T(z_n)$ for some z_n in X, also, z_n is probabilistic bounded sequence, then by assumption $T(z_n)$ has τ-convergent subsequence $\{y_{n_k}\} = \{T(z_{n_k})\}$, thus for all $\epsilon > 0$, $\lambda > 0$, $\exists n_1(\epsilon, \lambda)$ such that

$$F_{y_{n_k} - y, y_{n_k} - y}\left(\left({\epsilon}/{2}\right)^2\right) > 1 - \lambda \quad \forall n_k > n_1(\epsilon, \lambda)$$

for some $y \in E$.

$$F_{x_{n_k} - y, x_{n_k} - y}(\epsilon) = F_{x_{n_k} - y + y_{n_k} - y_{n_k}, x_{n_k} - y + y_{n_k} - y_{n_k}}(\epsilon)$$

by theorem (1), and by choosing $\mathcal{T}(a, b) = \min(a, b)$, we get

$$\geq \min\left(F_{x_{n_k} - y_{n_k}, x_{n_k} - y_{n_k}}\left(\left({\epsilon}/{2}\right)^2\right), F_{y_{n_k} - y, y_{n_k} - y}\left(\left({\epsilon}/{2}\right)^2\right)\right) > 1 - \lambda$$

$$\forall n_k > n_2(\epsilon, \lambda) = \max\{n_0(\epsilon, \lambda), n_1(\epsilon, \lambda)\},$$

thus T is an F-compact operator.

■

Remark (7)

The set of all convergent sequences is a linear subspace of sequences space.

Proof:

Since $\{x_n\}$ is τ-convergent to $x \in E$ then $\forall\epsilon > 0$ and $\forall\lambda > 0$, $\exists n_0(\epsilon, \lambda)$ such that

$$F_{x_n - x, x_n - x}\left(\left({}^{\epsilon}/_{2}\right)^2\right) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda).$$

Since $\{y_n\}$ is τ-convergent to $y \in E$ then $\forall\epsilon > 0$ and $\forall\lambda > 0$, $\exists n_1(\epsilon, \lambda)$ such that

$$F_{y_n - y, y_n - y}\left(\left({}^{\epsilon}/_{2}\right)^2\right) > 1 - \lambda \quad \forall n > n_1(\epsilon, \lambda)$$

$$F_{(x_n+y_n)-(x+y),(x_n+y_n)-(x+y)}(\epsilon)$$
$$= F_{(x_n-x)+(y_n-y),(x_n-x)+(y_n-y)}(\epsilon)$$

by theorem (1), and by choosing $\mathcal{T}(a,b) = \min(a,b)$, we get

$$\geq \min\left(F_{x_n-x,x_n-x}\left(\left({}^{\epsilon}/_{2}\right)^2\right), F_{y_n-y,y_n-y}\left(\left({}^{\epsilon}/_{2}\right)^2\right)\right)$$
$$> 1 - \lambda$$
$$\forall n_k > n_2(\epsilon, \lambda) = \max\{n_0(\epsilon, \lambda), n_1(\epsilon, \lambda)\},$$

thus $\{x_n + y_n\}$ is τ-convergent sequence to $x + y$.

To prove $\{\alpha(x_n)\}$ is τ-convergent sequence to αx for all $\alpha \in R|\{0\}$

since $\{x_n\}$ is τ-convergent to $x \in E$ then $\forall\epsilon > 0$ and $\forall\lambda > 0$, $\exists n_0(\epsilon, \lambda)$ such that

$$F_{x_n-x,x_n-x}(\epsilon) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda).$$

If $\alpha > 0$, then

$$F_{\alpha x_n-\alpha x,\alpha x_n-\alpha x}(\epsilon) = F_{x_n-x,x_n-x}\left({}^{\epsilon}/_{\alpha^2}\right)$$
$$= F_{x_n-x,x_n-x}(\epsilon) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda).$$

If $\alpha < 0$, then

$$F_{\alpha x_n-\alpha x,\alpha x_n-\alpha x}(\epsilon)$$
$$= 1 - F_{x_n-x,\alpha x_n-\alpha x}\left({}^{\epsilon}/_{\alpha} +\right)$$
$$= 1 - \left[1 - F_{x_n-x,x_n-x}\left(\left({}^{\epsilon}/_{\alpha^2}\right)\right)\right]$$
$$= F_{x_n-x,x_n-x}(\epsilon) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda)$$

thus $\{\alpha(x_n)\}$ is τ-convergent sequence to αx.

■

Theorem (8)

Let T_1 and T_2 be F-compact operators, then $T_1 + T_2$, αT_1 are F-compact operators for all $\alpha \in R|\{0\}$.

Proof:

Let $\{x_n\}$ be a probabilistic bounded sequence in E, since T_1 is an F-compact operator, then $\{T_1(x_n)\}$ has τ-convergent subsequence $\{T_1(z_n)\}$, and since T_2 is an F-compact operator then $\{T_2(x_n)\}$ has τ- convergent subsequence $\{T_2(z_n)\}$. suppose $T_1(z_n)$ is τ-convergent to $x \in E$, that is $\forall\epsilon > 0$ and $\forall\lambda > 0$, $\exists n_0(\epsilon, \lambda)$ such that

$$F_{T_1(z_n)-x,T_1(z_n)-x}\left(\left({}^{\epsilon}/_{2}\right)^2\right) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda)$$

suppose $T_2(z_n)$ is τ-convergent to $y \in E$, that is $\forall\epsilon > 0$ and $\forall\lambda > 0$, $\exists n_1(\epsilon, \lambda)$ such that

$$F_{T_2(z_n)-y,T_2(z_n)-y}\left(\left({}^{\epsilon}/_{2}\right)^2\right) > 1 - \lambda \quad \forall n > n_1(\epsilon, \lambda)$$

by the Remark (7), we get

$$F_{(T_1(z_n)+T_2(z_n))-(x+y),(T_1(z_n)+T_2(z_n))-(x+y)}(\epsilon) > 1 - \lambda$$
$$\forall n > n_2(\epsilon, \lambda) = \max\{n_0(\epsilon, \lambda), n_1(\epsilon, \lambda)\}$$

then $\{T_1(z_n) + T_2(z_n)\}$ is τ-convergent subsequence to $x + y$.

thus $T_1 + T_2$ is an F-compact operator.

To prove αT_1 is an F-compact operator for any α in $R|\{0\}$.

let $\{y_n\}$ be a probabilistic bounded sequence in E, since T_1 is an F-compact operator, then $\{T_1(y_n)\}$ has τ-convergent subsequence $\{T_1(z_n)\}$.

suppose $T_1(z_n)$ is τ-convergent to $x \in E$, that is $\forall\epsilon > 0$ and $\forall\lambda > 0$, $\exists n_0(\epsilon, \lambda)$ such that

$$F_{T_1(z_n)-x,T_1(z_n)-x}(\epsilon) > 1 - \lambda \quad \forall n > n_0(\epsilon, \lambda)$$

by the Remark (7),

we get $F_{\alpha(T_1(z_n)-x),\alpha(T_1(z_n)-x)}(\epsilon) > 1 - \lambda$

$$\forall n > n_0(\epsilon, \lambda), \ \forall \alpha \in R|\{0\}.$$

thus αT_1 is an F-compact operator

■

Theorem (9)

Let $(E, F, *)$ be a modified probabilistic Hilbert space with mathematical expectation, and let $T_1 : (E, F, *) \to$

$(E, F*)$ be an F-compact operator and let $T_2: (E, F, *) \to (E, F*)$ be an F-continuous operator, then $T_2 T_1$ and $T_1\, T_2$ are an F-compact operators.

Proof:

Let $\{y_n\}$ be a probabilistic bounded sequence in E, since T_1 is an F-compact operator, then $\{T_1(y_n)\}$ has subsequence $\{T_1(y_{n_k})\}$, which is τ–convergent to $y \in E$.

since T_2 is F- continuous operator, then by theorem (15), T_2 is sequentially continuous on $(E, F, *)$, that is $\left\{T_2\left(T_1(y_{n_k})\right)\right\}$ is τ–convergent to $T_2(y)$, thus $T_2 T_1$ is an F-compact operator.

To prove $T_1 T_2$ is an F-compact operator, and let $\{y_n\}$ be a probabilistic bounded sequence in E, then by definition (2) there exist $t_0 \in R/\{0\}$, $a_0 \in (0,1)$, such that $F_{y_n, y_n}(t_0) > 1 - a_0$ for all $n \geq 1$

since T_2 is F- continuous operator, then by theorem (16), T_2 is an F-bounded operator, thus $\exists\, c > 0$ such that

$$F_{T_2 y_n, T_2 y_n}(t_0) \geq F_{y_n, y_n}\left({}^{t_0}/_{c}\right) > 1 - a_0 \quad \forall\, n \geq 1$$

then $\{T_2(y_n)\}$ is a probabilistic bounded sequence in E, but T_1 is F-compact, then $\{T_1(T_2(y_n))\}$ has τ–convergent subsequence.

thus $T_1 T_2$ is an F-compact operator. ■

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المؤثر المرصوص في فضاء هيلبرت الاحتمالي

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الخلاصة:

هذا البحث يقدم تعريف المؤثر المرصوص في فضاء هيلبرت الاحتمالي وبعض الخصائص الرئيسية لهذا المؤثر.

الكلمات المفتاحية: المجموعات المقيدة الاحتمالية، التقارب من نوع τ، فضاء هيلبرت الأحتمالي، المؤثر المرصوص.