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A Combinatorial Approach to Obtain the Yield Probability Distribution along a Linearly-Loaded Cantilever Beam

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Abstract:

The substantial key to initiate an explicit statistical formula for a physically specified continua is to consider a derivative expression, in order to identify the definitive configuration of the continua itself. Moreover, this statistical formula is to reflect the whole distribution of the formula of which the considered continua is the most likely to be dependent. However, a somewhat mathematically and physically tedious path to arrive at the required statistical formula is needed.

The procedure in the present research is to establish, modify, and implement an optimized amalgamation between Airy stress function for elastically-deformed media and the multi-canonical joint probability density functions for multivariate distribution completion, so that the developed distribution is to exhibit a sophisticated illustration of yield probability distribution along a cantilever beam whose structure is subjected to a linearly-distributed load. This combinatorial approach is to clarify the intensity of the stresses exerted onto the beam, to standardize the terms of stresses and their affection and to convert them into a more significant depiction of a probability distribution.

Key words: Multivariate Joint Probability Density Functions, Multi-Canonical Probability Functions, Airy Stress Function, Stress Analyses, Yield Probability Functions.

Introduction:

What is an intriguing to illustrate the sophisticated correlations between statistical distributions and engineering applications, which are widely spread in the present scientific worldwide prosperity? Particularly, engineering theories and applications may be clearly elucidated as soon as they are correlated with the corresponding statistical hypotheses, expectations, and/or probability distributions, which tend to further clarify the conclusive expressions of these engineering applications. Therefore, the vast majority of the scientific, engineering and technological applications have the appreciable trend to arrive at their final expressions significantly tied to a probability distribution and/or hypothesis, which can further establish a comprehensible definition to these applications. To particularly study the continua considered in the present research, two-dimensional problems of elasticity may be effectively solved by the Airy stress function, or other stress functions, in order to arrive at the stresses' formulae. These stresses may be thereafter presented as a distribution along one or more specified axes. Afterwards, the stress distribution may be converted into a one-variable or a multivariate probability distribution, so that it can further be developed into a pellucid perspective to describe its magnitudes. Generally, each physical phenomenon is directly related to a mathematical and/or statistical interpretation with the fact of being dependently solved by each other.

D. Yevick (2003) [1] exhibits that it is convenient to evaluate the joint probability density functions identically. Afterwards, the joint probability density functions can then be calculated between the polarization mode dissemination's first and second orders in optical fibers.

C. H. Kim (2010) [2] demonstrates that the non-linear large-deflection-state stress distribution's Airy stress function in a simply-supported plate, with movable edges, can be determined by the use of the superposition technique of the Airy stress functions for an isotropic condition, with which the movable edges' boundary conditions and the state of the large deformation are satisfied.

In the present research, a combined method of Airy stress function methodology, to evaluate the stress field, and a developed multi-canonical evaluation by the joint probability density functions is presented for a cantilever solid beam, which is linearly loaded along its total span. Charts for the stress field and the yield probability distribution, for multiple values of beam length and breadth, is to be thereafter obtained in order to recognize the points within which the yielding phenomenon is the most likely to happen.

The Combinatorial Approach:

Before presenting the medium considered mathematically, it is significant to count on assuming that the loaded beam is isotropic, i.e. every point within which has the same physical, thermal and mechanical properties. Furthermore, it is convenient to assume that the developed stresses only emanate in the xy-plane, so that there are no rules to abide regarding to evaluate the z-axis stresses and hence yielding probabilities, as well as the assumption which is based on the fact that all the developed stress field is emanated elastically, in such a way the two-dimensional Hooke's law, compatibility, and Airy stress function relation are adequately applicable.

Referring to Fig. (1), the isotropic cantilever beam, having a length L and a breadth a, is linearly loaded so that the point $(0, a/2)$ is free from the distributed load and the point $(L, a/2)$ is subjected to a distributed load whose value is qL.

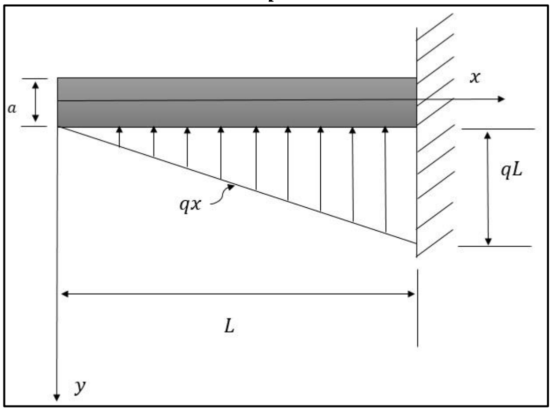

Fig. (1) The Mathematical Model [3]

From the geometry of Fig. (1), the boundary conditions can only be represented in the following equation(s)

$$\sigma_{xx}(0,\pm y)=0$$
$$\sigma_{yy}\left(0,\pm\frac{a}{2}\right)=0$$
$$\sigma_{yy}\left(x,+\frac{a}{2}\right)=qx \quad \ldots\ldots(1)$$
$$\tau_{xy}\left(x,\pm\frac{a}{2}\right)=0$$
$$\int_{-a}^{a}\tau_{xy}(0,y)dy=0$$

The stress field relations can only be written in terms of the Airy stress function, being differentiated with respect to x and/or y axes, as the following formula shows [2]

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\partial^2\Phi/\partial y^2\\ \partial^2\Phi/\partial x^2\\ \partial^2\Phi/\partial xy\end{bmatrix}\ldots\ldots(2)$$

Also, the following bi-harmonic equation must be satisfied in order to establish the appropriate mathematical and geometrical interpretation of the cantilever beam

$$\nabla^4\Phi=0\ldots(3)$$

Assuming that the Airy stress function is of the sixth order which has 25 constants, this may be illustrated in the following relation in terms of the x and y variables

$$\Phi=A_1x^2+A_2xy+A_3y^2+A_4x^3+A_5x^2y+A_6xy^2+A_7y^3+A_8x^4+A_9x^3y+A_{10}x^2y^2+A_{11}xy^3+\cdots+A_{25}y^6 \quad \ldots(4)$$

After mathematically combining the equations (1), (2), (3) and (4) in order to solve the constants which are needed to arrive at the stress field, this will lead into the following expression for the stresses induced within the cantilever beam [3]

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\frac{qx^3y}{4a^3}+\frac{q}{4a^3}\left(-2xy^3+\frac{6}{5}a^2xy\right)\\ -q\frac{x}{2}+qx\left(\frac{y^3}{4a^3}-\frac{3y}{4a}\right)\\ \left(\frac{3qx^2}{8a^3}+\frac{3q}{20a}\right)(a^2-y^2)-\frac{q}{8a^3}(a^4-y^4)\end{bmatrix}(5)$$

Now, it is appropriate to combine the definition of the three yield probabilities $f(x,y)$ [1] [4], with which one can determine whether the beam yields, with the stresses in terms of the distributed load intensity q (which equals to 0.001 MPa), the breadth a, the length L, and the variables x and y, so that this will lead to the following final expression [5] [4].

$$f_{xx}(x,y)=\frac{1}{(2\pi)^{\frac{3}{2}}\sigma_{xx}\sigma_{yy}\tau_{xy}(1-r^2)^{1/2}}\exp\left[-\frac{1}{2}\left(\frac{w}{\sigma_{xx}}\right)^2\right]*\exp\left[\frac{1}{2(1-r^2)}\left(\frac{w}{\sigma_{yy}}\right)^2+\left(\frac{w}{\tau_{xy}}\right)^2-2r\left(\frac{w^2}{\sigma_{yy}\tau_{xy}}\right)\right]\ldots\ldots(6)$$

$$f_{yy}(x,y)=\frac{1}{(2\pi)^{\frac{3}{2}}\sigma_{xx}\sigma_{yy}\tau_{xy}(1-r^2)^{1/2}}\exp\left[-\frac{1}{2}\left(\frac{w}{\sigma_{xx}}\right)^2\right]*\exp\left[\frac{1}{2(1-r^2)}\left(\frac{w}{\sigma_{yy}}\right)^2+\left(\frac{w}{\tau_{xy}}\right)^2-2r\left(\frac{w^2}{\sigma_{yy}\tau_{xy}}\right)\right]\ldots\ldots(7)$$

$$f_{xy}(x,y)=\frac{1}{(2\pi)^{\frac{3}{2}}\sigma_{xx}\sigma_{yy}\tau_{xy}(1-r^2)^{1/2}}\exp\left[-\frac{1}{2}\left(\frac{w}{\sigma_{xx}}\right)^2\right]*\exp\left[\frac{1}{2(1-r^2)}\left(\frac{w}{\sigma_{yy}}\right)^2+\left(\frac{w}{\tau_{xy}}\right)^2-2r\left(\frac{w^2}{\sigma_{yy}\tau_{xy}}\right)\right]\ldots\ldots(8)$$

Provided that,

$$w = {}^{1}/_{2}\left(a + L\sqrt{\frac{1-x}{y}}\right)\left(1 - \frac{xy}{\sqrt{4+y}}\right)$$
$$r = {}^{1}/_{2}\left(a + L\sqrt{\frac{1-y}{x}}\right)\left(1 - \frac{xy}{\sqrt{4+x}}\right) \quad \dots (9)$$

in such a process that these yield probabilities satisfy the following multivariate joint probability density function basic condition

$$\int_{-\infty}^{\infty} f_{xx}(x,y)dxdy = 1 \dots (10)$$

Analyses of the Results:

It has been previously demonstrated that the plane stress components, σ_{xx}, σ_{yy}, and τ_{xy}, are explicitly related to their corresponding yield probabilities f_1, f_2, and f_3, which represent f_{xx}, f_{yy}, and f_{xy} in the equations 6, 7, and 8 respectively, so that the present combinatorial analysis will importantly pose a new illustrative approach to indicate how the beam yielding process will be and which point(s) will be the most likely to undergo yielding.

The x-axis stresses σ_{xx} have been demonstrated, referring to the Figures (2), (5) and (8), that the points $\left(L, \pm\frac{a}{2}\right)$ have their maximum decreased gradually as values of y decrease from $\pm\frac{a}{2}$ to 0, and σ_{xx} values are also prone to a recognizable decay when x values fall from L until they become 0 for all values of the length L and the breadth a. Furthermore, σ_{xx} values noticeably augment when a and/or L increase. Whereas the y-axis stresses σ_{yy} are found at their maximal magnitudes at $\left(L, \frac{a}{2}\right)$ for all values of a and L. Also, σ_{yy} values increase when the values of x, y, a and/or L increase as shown in Figures (3), (6), and (9). On the other hand, the xy-plane shear stresses τ_{xy} have their maximums in the point $(x, 0)$, and their minimums in the point(s) $(x, \pm\frac{a}{2})$ for all values of x, a and L, the values of τ_{xy} also increase when x, a, and/or L increase as shown in Figures (4), (7), and (10).

Now, the idea of combining the stresses' effects with the joint probability density functions has become, in many engineering and statistical aspects, significantly substantial so that it further furnishes prestigiously illustrative methods to investigate the percentages of the stress effects statistically upon the solid beam structure. To illustrate, the three yield probabilities f_1, f_2 and f_3 effects are to be discussed in terms of the variables $x, y, a,$ and L, although the fact that the percentages of the three yield probabilities appear independent of the values of both a and L. Firstly, it is clear that about 53.5 percent the x-axis yield probability f_1 according to which the beam, for all the values of y, will likely to yield at the line $x = L$, while other lines $x = 0, x = 0.2L, x = 0.4L, x = 0.6L$ and, $x = 0.8L$ exhibit 0, 0.5, 3.5, 12 and 30.5 percent of f_1 respectively. Therefore, it appears that the beam yield point starts from the line $x = L$, and it will, depending on the intensity and the nature of the distributed load, continuously move until arriving at the line $x = 0$. Secondly, about 33 percent is the y-axis yield probability f_2 at $x = L$, at which the beam will start yielding until it reaches $x = 0$, with the possible plastic deformation and collapse if the distributed load increases. In addition, the other lines of constant values of the y-axis $x = 0, x = 0.2L, x = 0.4L, x = 0.6L$ and, as done, $x = 0.8L$ exhibit 0, 7, 13, 20 and 27 percent of f_2 respectively. Thirdly, the xy-plane shear yield probability f_3 is obviously equal to 45 percent at the line $x = L$, which tends towards yielding the beam so that it helps the yielded zone keep moving to the other points until the line $x = 0$ is reached. Moreover, the other assumed lines $x = 0.2L$, $x = 0.4L$, $x = 0.6L$ and, so do other specified lines, $x = 0.8L$ exhibit 0, 1, 8, 16 and 30 percent of the xy-plane yield probability respectively. As previously mentioned, the sum of each probability for all values of x satisfy the equation (10) [6]. Figures (11) to (19) illustrate the values, as percentages of stacked bars, of the yield probabilities f_1, f_2 and f_3.

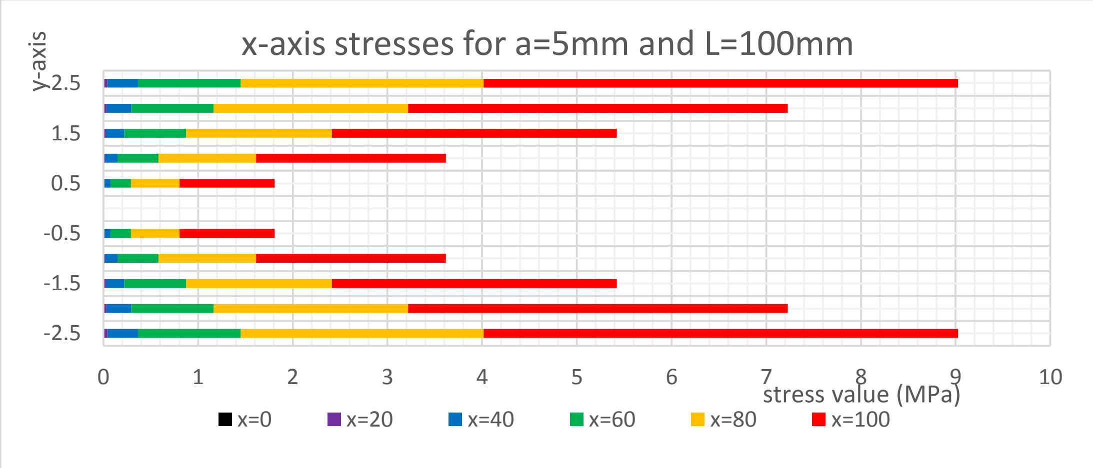

Fig. (2) X-Axis Stresses for a=5mm and L=100mm

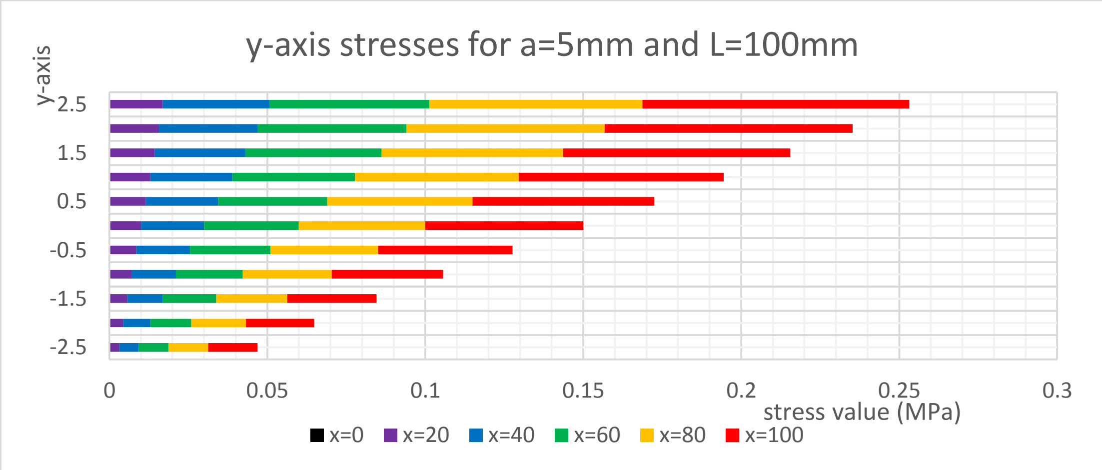

Fig. (3) Y-Axis Stresses for a=5mm and L=100mm

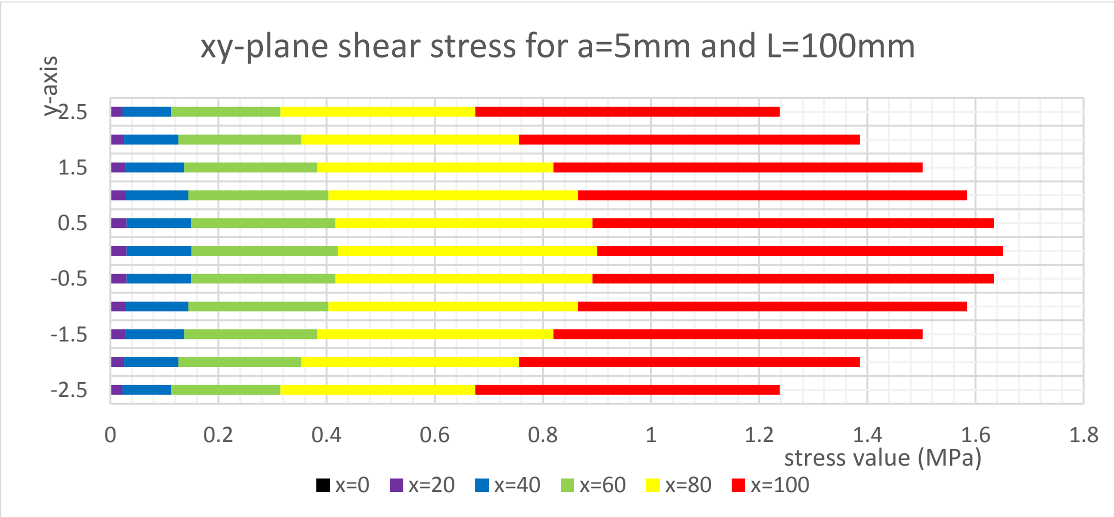

Fig. (4) XY- Plane Shear Stress for a=5mm and L=100mm

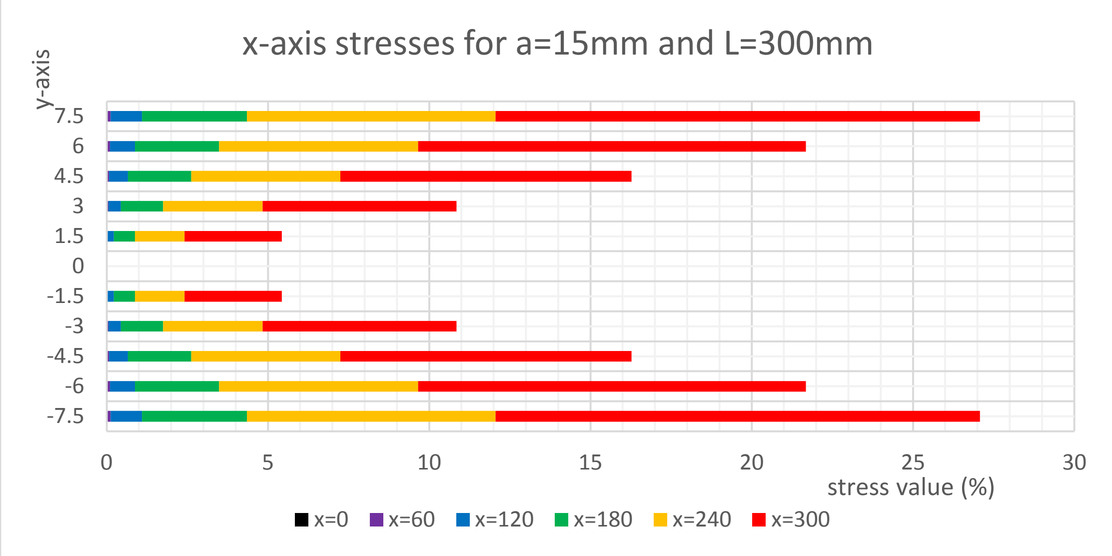

Fig. (5) X-Axis Stresses for a=15mm and L=300mm

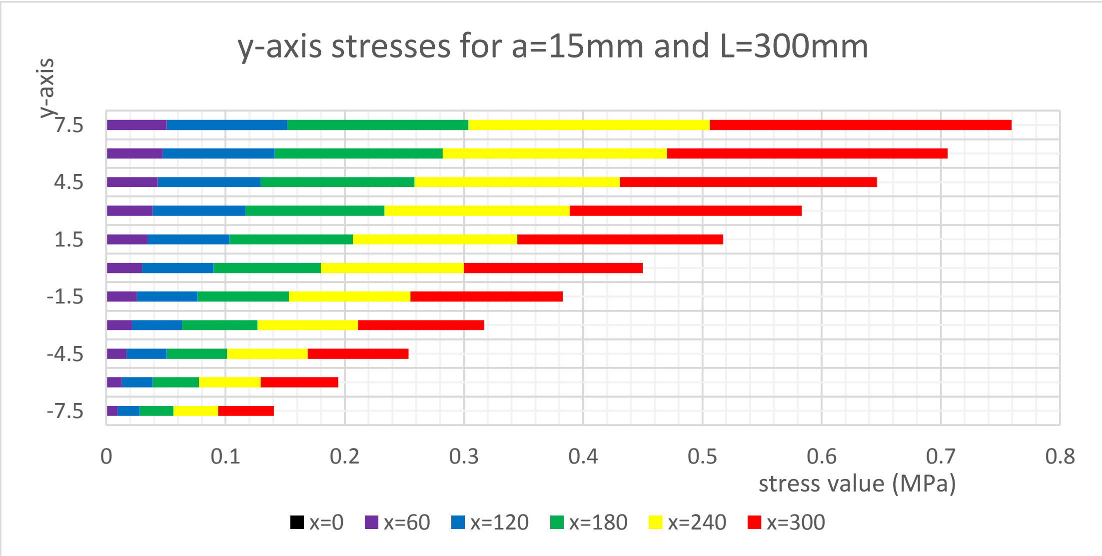

Fig. (6) Y-Axis Stresses for a=15mm and L=300mm

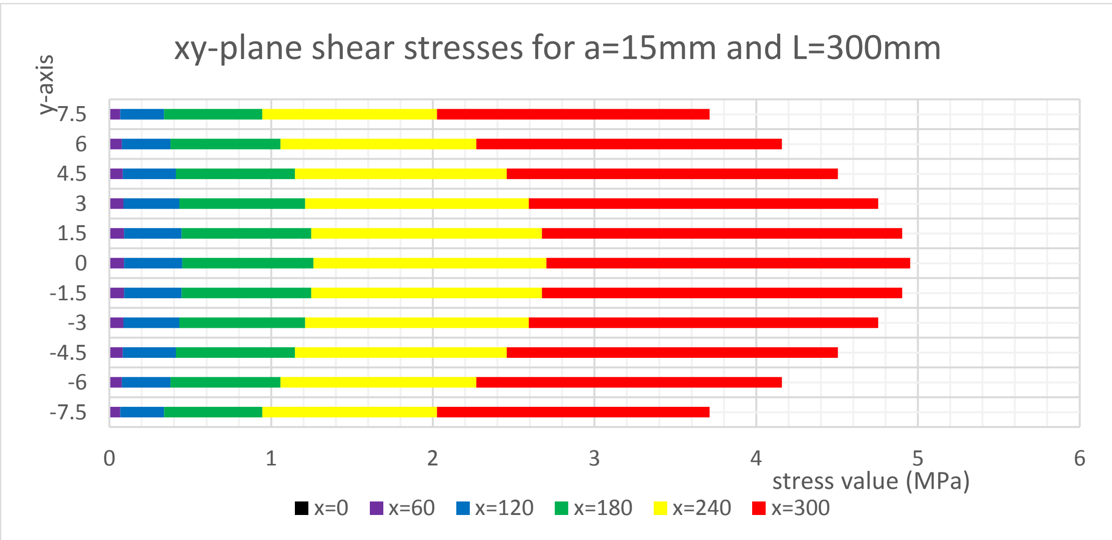

Fig. (7) XY-Plane Shear Stresses for a=15mm and L=300mm

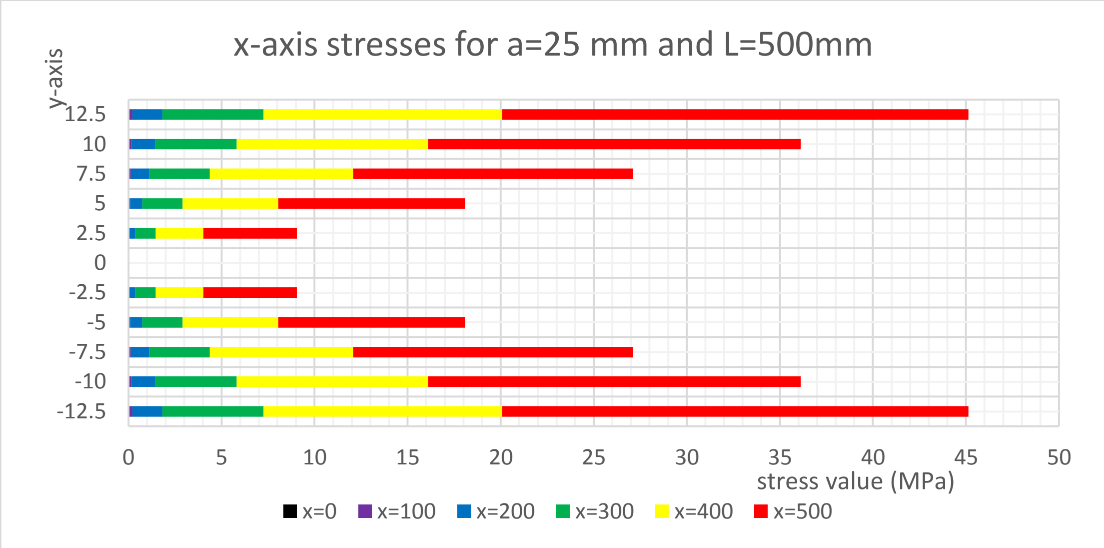

Fig. (8) X-Axis Stresses for a=25 mm and L=500mm

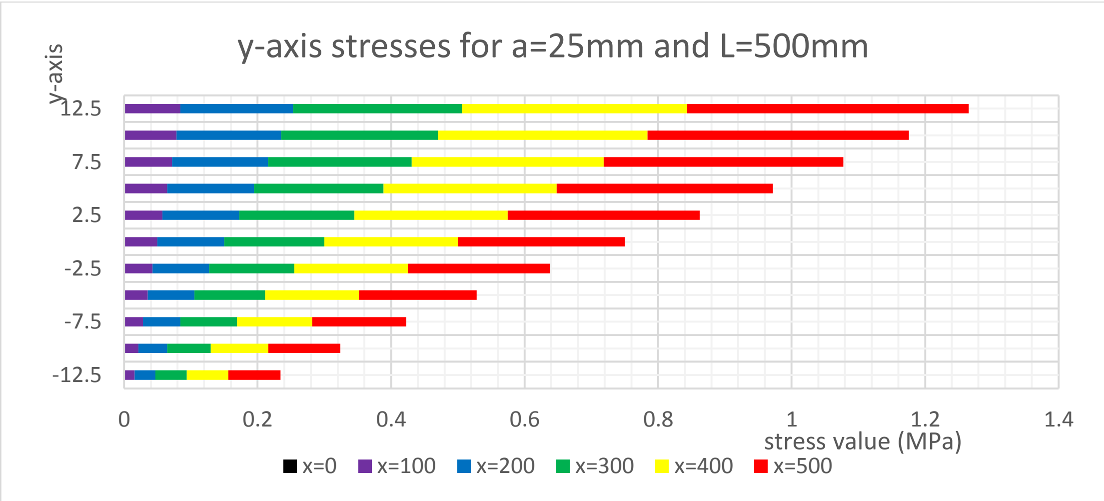

Fig. (9) Y-Axis Stresses for a=25mm and L=500mm

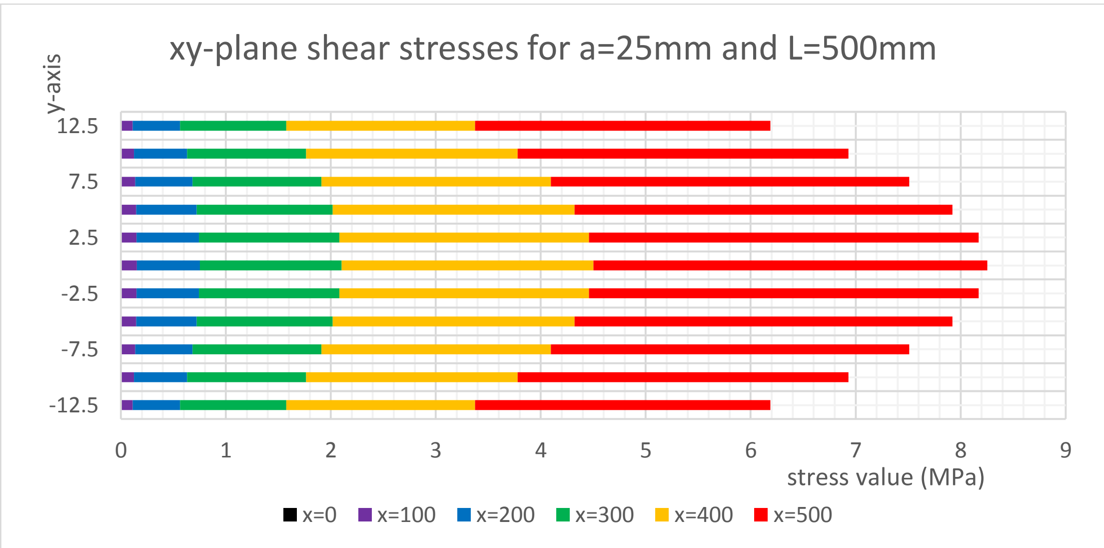

Fig. (10) XY-Plane Shear Stresses for a=25mm and L=500mm

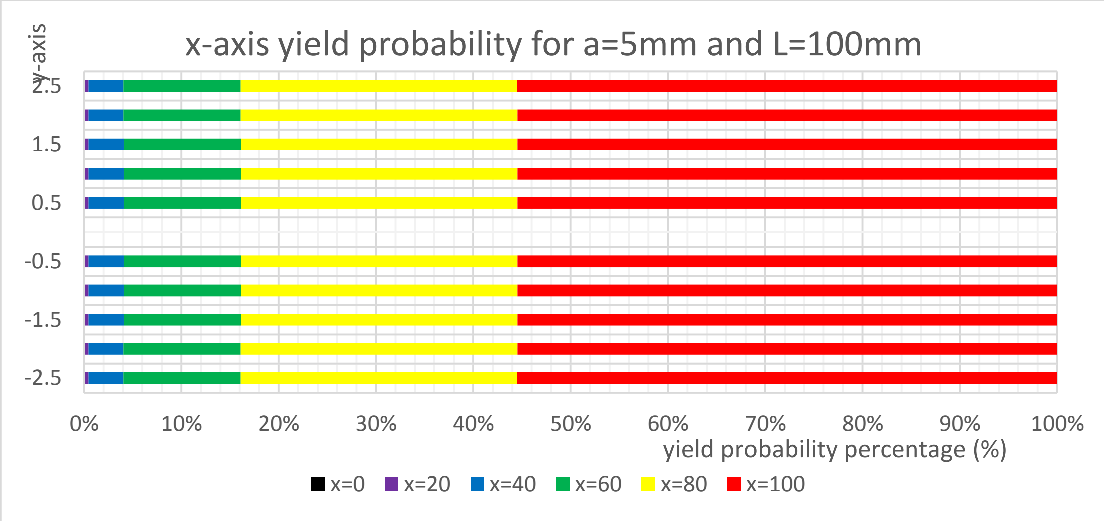

Fig. (11) X-Axis Yield Probability for a=5mm and L=100mm

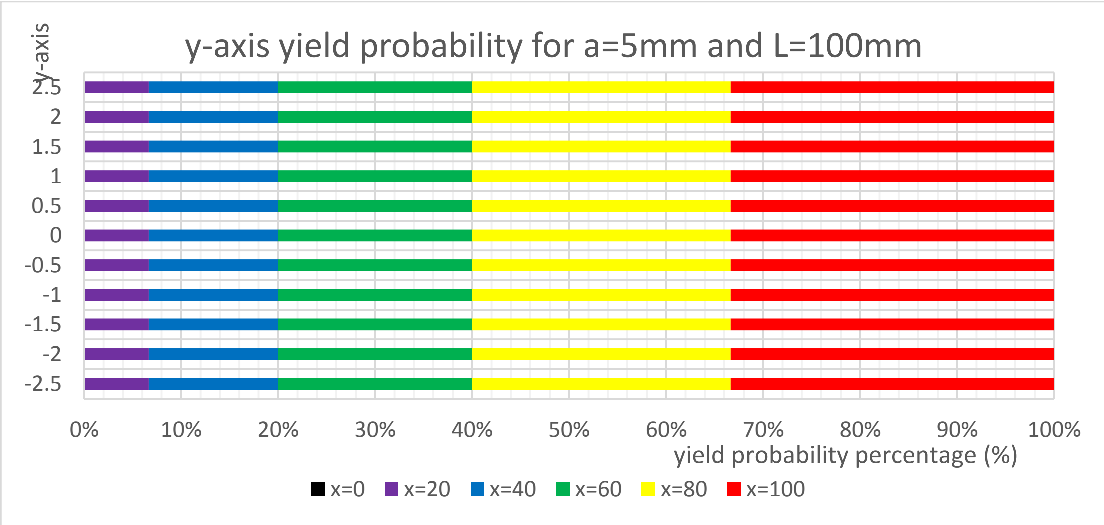

Fig. (12) Y-axis Yield Probability for a=5mm and L=100mm

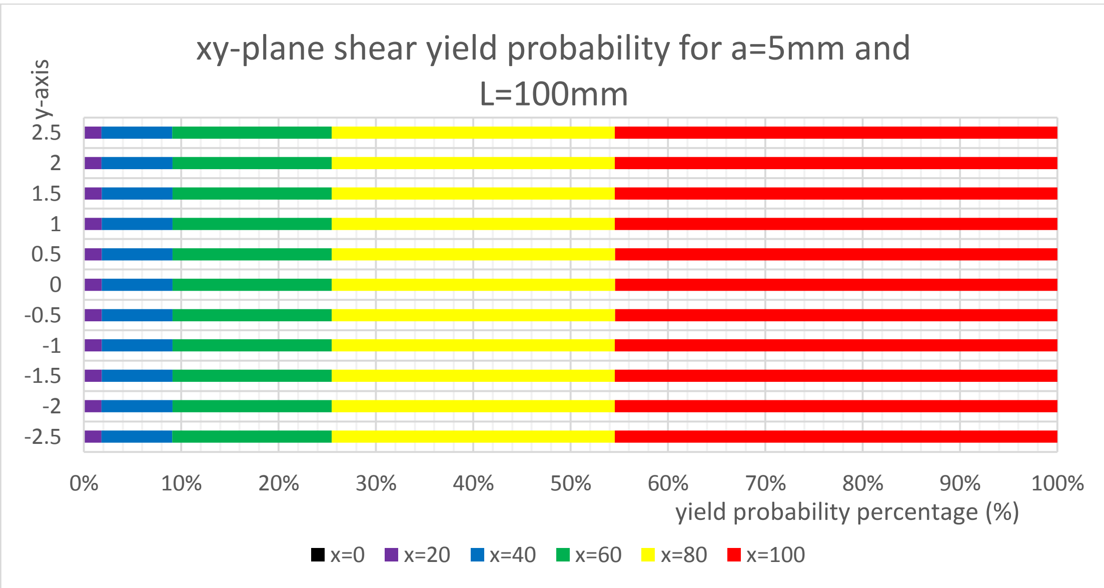

Fig. (13) XY-Plane Yield Probability for a=5mm and L=100mm

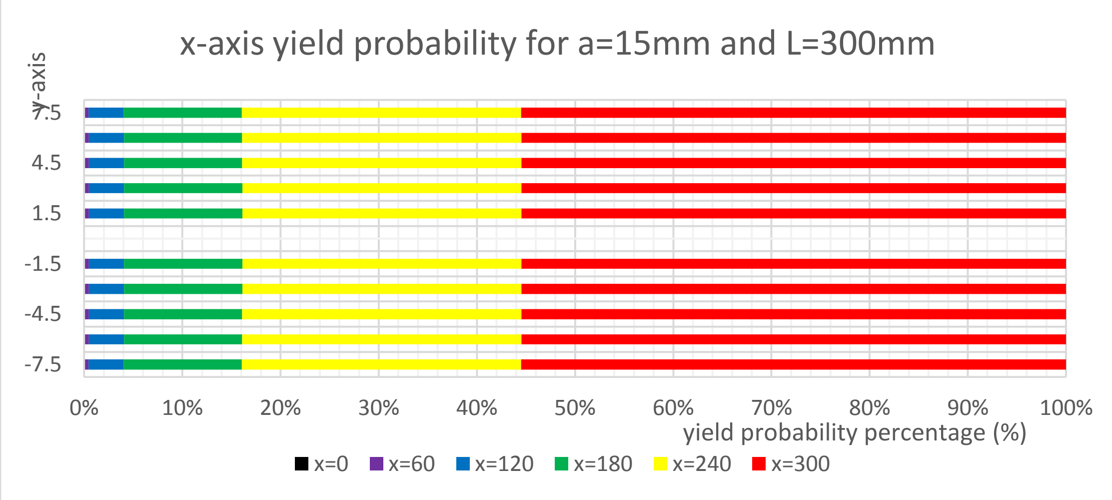

Fig. (14) X-Axis Yield Probability for a=15mm and L=300mm

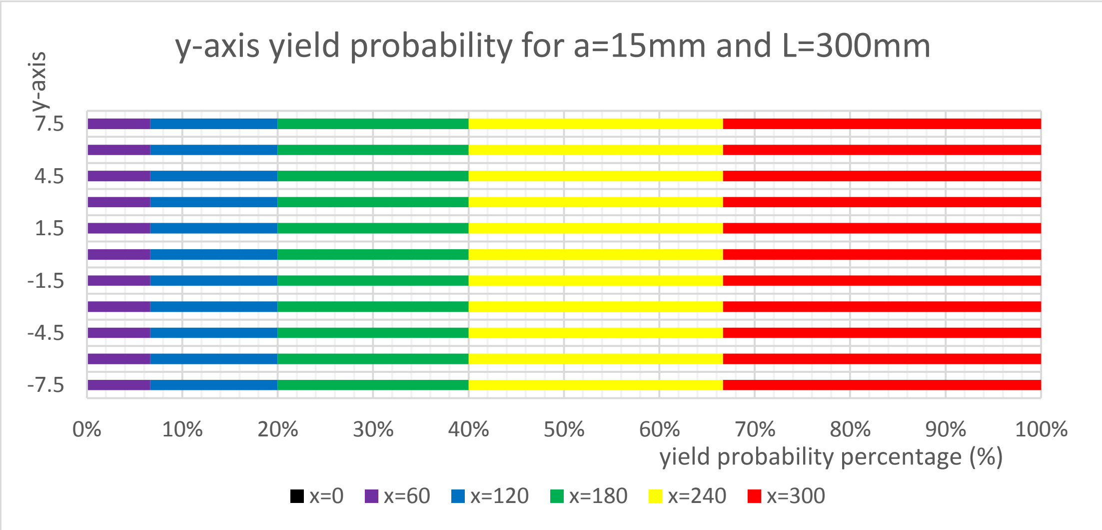

Fig. (15) Y-Axis Yield Probability for a=15mm and L=300mm

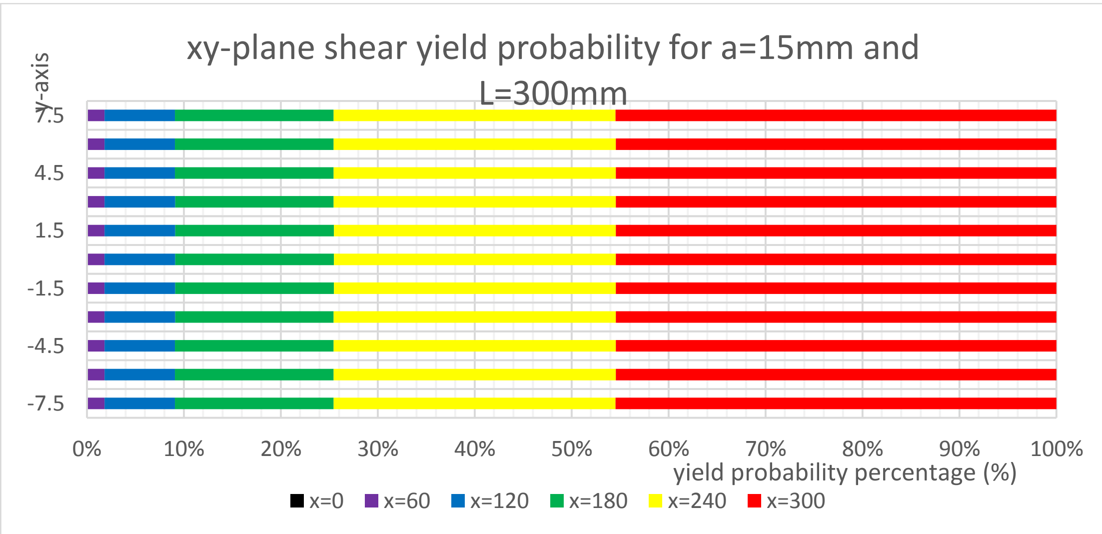

Fig. (16) XY-Plane Yield Probability for a=15mm and L=300mm

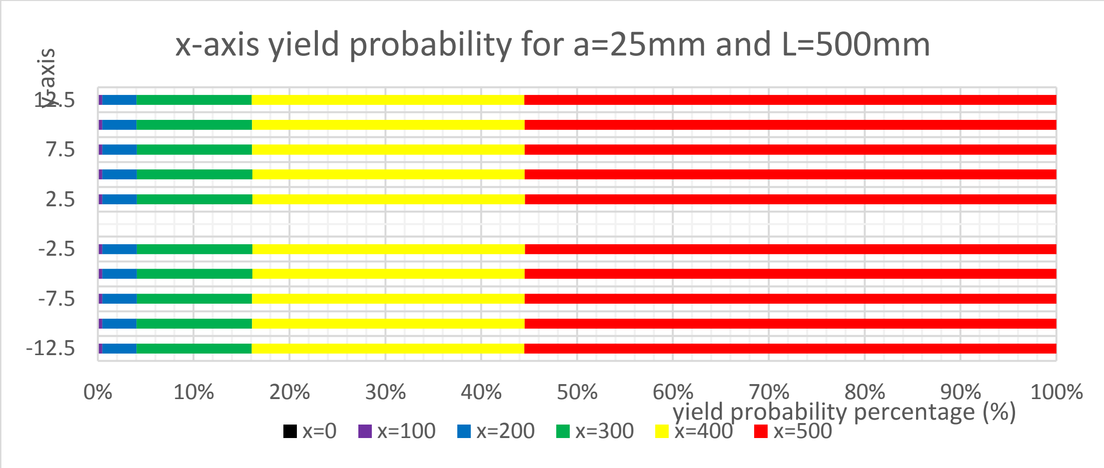

Fig. (17) X-Axis Yield Probability for a=25mm and L=500mm

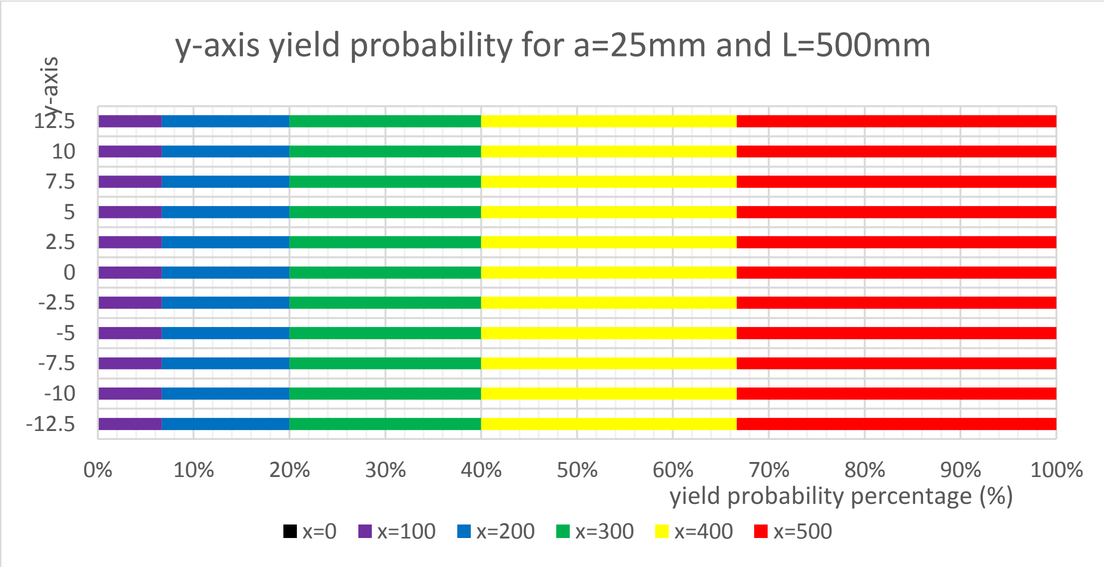

Fig. (18) Y-Axis Yield Probability for a=25mm and L=500mm

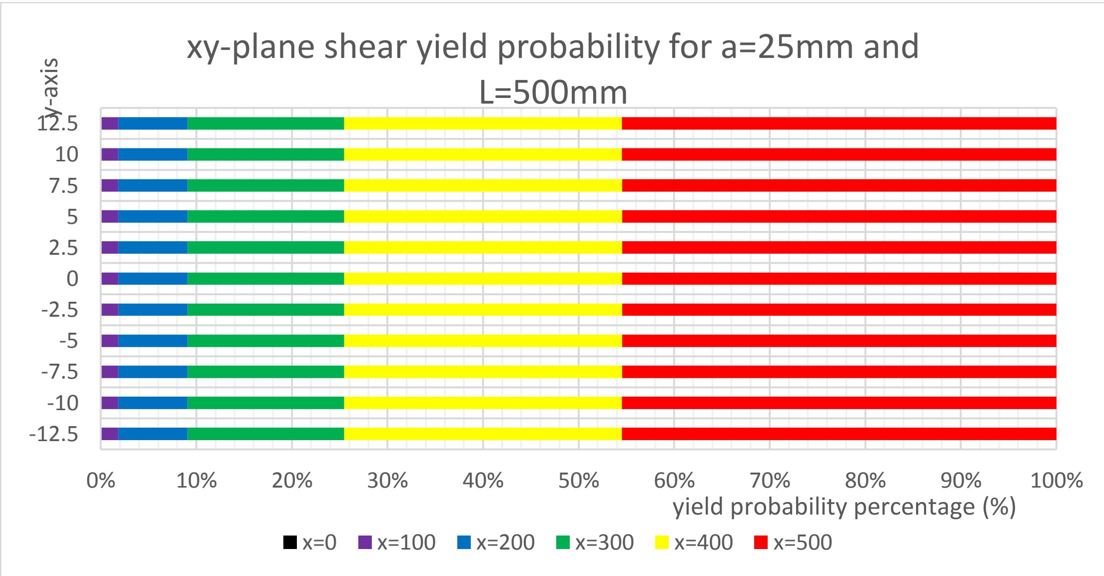

Fig. (19) XY-Plane Yield Probability for a=25mm and L=500mm

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طريقة توفيقية لحساب توزيع احتمالات الخضوع على طول عتبة مثبتة من جانب واحد محملة بصورة خطية

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الخلاصة:

ان المفتاح الجوهري لتكوين علاقة احصائية واضحة لاوساط معلومة فيزيائيا هو اعتماد صيغ مشتقة لغرض معرفة التأسيس النهائي لهذه الاوساط. علاوة على ذلك ستعكس هذه العلاقة الاحصائية التوزيع الكامل للتعبير الذي تعتمد عليه الاوساط المذكورة بأكبر نسبة ممكنة. بالرغم من احتياج طريق شاق رياضيا وفيزيائيا لغرض الوصول الى التعبير الاحصائي المطلوب.

طريقة العمل للبحث الحالي هو لتأسيس، تعديل و استكمال دمج بين دوال الاجهاد لأيري للاوساط المشوهة بصورة مرنة و دوال الاحتمالات المشتركة الاحصائية متعددة الاسس لاكمال التوزيع المتعدد في المتغيرات، وبطريقة ممكن فيها ان يبين التوزيع المتكون توضيح متطور لتوزيع احتمالات الخضوع على طول عتبة مثبتة من طرف واحد والتي يتعرض هيكلها الى اجهاد موزع بصورة خطية. ان هذه الطريقة الدمجية ستوضح قوة الاجهادات المسلطة على العتبة، لتوحيد قياس حدود الاجهادات وتأثيرها و تحويلها الى توضيح اكثر اهمية على شكل توزيع احتمالية

الكلمات المفتاحية: دوال الكثافة الاحتمالية المشتركة متعددة المتغيرات، دوال الاحتمالات متعددة الاسس، دالة اجهاد أيري، تحليلات الاجهاد، دوال احتمالات الخضوع