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On the Connection between the Dynamical System and the Ellis Compactification with Transitive Pointed System

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Abstract:

In this paper, some relations between the flows and the Enveloping Semi-group were studied. It allows to associate some properties on the topological compactification to any pointed flows. These relations enable us to study a number of the properties of the principles of flows corresponding with using algebric properties. Also in this paper proofs to some theorems of these relations are given.

Keywords: Dynamical system, semi-topological semi-group, isomorphisim of H-systems, monoidal compactification, transitive pointed systems, inclusive transitive pointed systems.

Introduction:

Historically the Enveloping semi-group which is a closure of the set of continuous functions on a compact space X was used to study the dynamical system as given by R.Ellis [1]. In 1991 A. Lisan characterized what is called transitive flows which is related with the right topological monidal compactification of the system [2]. In 1992 J. D. Lawson showed that many typical of lows can be recognized by restrictions on the minimal ideals of the enveloping semi-group[3]. In 2013, N. Hindman studied the intersection between the stone-cech compactification with the additive and multiplication operations to present conditions on system that makes the closure of the minimal ideal of stone-cech compactification a right ideal in the case of multiplication operation [4]. Recently in 2014 L. Jones pointed out some close relation between the theory of right topological semi-group and the theory of systems on compact spaces.

The Enveloping semi-group has been established to be an essential tool in the abstract theory of topological dynamical systems. We study this with respect to enveloping semi-group. We are looking to what we called inclusive compactification which is topologically and algebraically isomorphic to the enveloping semi-group, study this by using semi-group property of the universal compactification and give some properties related to these concepts.

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Preliminaries:

Let H be a semi-group given with Hausdorff space. For $m, n \in H$ we define $f_n(m) = mn$ with product mn [5]. The function f_n is a right continuous. The semigroup H is called the right topology. If all f_n are continuous then the semigroup is semi topological [6]. Given H be a semi-topological semi-group, an H-system or H-flow is a triple (H, X,g), where X is a compact Hausdorff space and $g: H \times X \to X$ is separately continuous[5]. Let $g(h, x) = hx$, and g is defined an action if $(hr)x = h(rx)$ for all $h, r \in H \; and \; x \in X$ [7].

Suppose X and Y be two H-systems, a continuous function $T: X \to Y$ satisfying $T(hx) = hT(x)$ for all $h \in H, x \in X$ is called a homomorphism. $g: X \to Y$ is an isomorphism H-systems if it is onto homeomorphism[8]. We define the orbit of y for $y \in X$ to be the set $Hy = \{hy: h \in H\}$ and we will denote $\mho(y)$ to be the topological closure of Hy. If there is a $y \in X$ s.t $\{y\} \cup \mho(y)$ =X we call it the system pointed transitive[4].

Definition(2.1):[3]

A monoidal right topological compactification of H (where H has an identity, otherwise one can attach an identity to H) is a pair (T, f) s.t

(a) T is Hausdorff compact right topological semi-group which contains an identity.

(b) f is a homomorphism continuous which takes H into T.

(c) Let e be the identity element of T, $f(H) \cup \{e\}$ is dense in T.
(d) f takes the identity of H to the identity of T.
(e) $f(h) \subseteq \{x \in T: g_x(y) = xy \; is \; continuous\}$.
By standard compactification arguments there exists an inclusive right topological compactification (CH, j) of H which is distinguished by the universal property that if $f: H \to T$ is a right topological compactification of H, then there exist a unique continuous hommommorphism a $F: CH \to T$ s.t $f = F \circ j$.

Theorem(2.2): Let (X, q) be a pointed transitive H-system and (Y, p) be a pointed H-system. Then there is at most one homomorphism from (X, q) to (Y, p).

Proof: See Proposition 1-1 in[4].

Remark(2.3): Let (X, q) and (Y, p) be two pointed transitive systems, if $h: (X, q) \to (Y, p)$ and $r: (Y, p) \to (X, q)$ are homomorphism, then h and r are inverse homeomorphism and therefore H-system is an isomorphism[9].

Remark(2.4): Let (H, X, D) be an H-system. For each $h \in H$, let D^* denote the continuouse mapping $x \longmapsto hx$, we fram the Ellis semigroup by capturing the closure of $H(D) = \{D^*: h \in H\}$ in the Cartesian product $X \times X$ given with the product topology.

Main results:

In this part, we present the new definitions and theorems related with the study and the relations between the dynamical system the enveloping semi-group.

Definition(3.1):[2]
Consider (T_1, d_1) and (T_2, d_2) be two monidal compactifications of H. We define the C-homomorphism $K: T_1 \to T_2$ which is a continuous homomorphism identity preserving such that $K \circ d_1 = d_2$.

Lemma(3.2): Let (T_1, d_1) and (T_2, d_2) be monidal compactification then K is a homomorphism of pointed H-system iff $K: (T_1, d_1) \to (T_2, d_2)$ is a C-homomorphism.

Proof: If K is a homomorphism pointed of H-systems, then for $h \in H, n \in T_1$ we get $K(d_1(h)n) = K(hn) = hK(n) =$
$d_2(h)K(n) \ldots . (1)$.
Take n=1, we have $K \circ d_1 = d_2$ and thus $K \circ d_1(h) = d_2(h)$ for all $h \in H$. Let $n, m \in T_1$. Since $m \in T_1$, then there is a net $\{d_1(h_\alpha)n$ converge to nm. On the other hand, $d_1(h_\alpha)n = d_2(h_\alpha)K(n) by \, (1)$
$= K\big(d_2(h_\alpha)\big)K(n)$.
But by continuity of K we get $K\big(d_1(h_\alpha)\big)$ convergence to K(m) and so $K\big(d_1(h_\alpha)\big)K(n)$. Thus $K(mn) = K(m)K(n)$.

Conversely, by definition (2), $K \circ d_1 = d_2$. Therefore
$K(nx) = K(d_1(h)n) = K\big(d_1(h)\big)k(x) =$
$d_2(h)K(x)$. Hence K is a homomorphism of pointed H-system.

Remark(3.3): T can be considered as an H-system by defining $T(h, t) = j(h)t$ where H is topological semi-group and (T, j) be a monoidal compactification.

Lemma(3.4): Consider (H, X, D) as an H-system then
(a) $f \longmapsto f(q): V(X) \to X$ where $q \in X$, is a homomorphisim of H-systems, and a homomorphisim of pointed H-systems, from $(V(X), I_x)$, where I_x is the identity function.
(b) If (T, j) is a monoidal compactification of H defined on part (a) from $(V(T), 1_T)$ to the flow $(T, 1)$ is an isomorphic, where 1 is the identity of T.

Proof: For part (a). Let $R: V(X) \to X$ which is defined by $R(f) = f(q) \; for \; q \in X$, then $R(hf) = (hf)(q) = sf(q) = hR(f)$ for all $h \in H$ and $f \in V(x)$. Also $R(Id_x) = I_x(q) = q$. Thus R is homomorphisim pointed H-system. For part (b) is left for the reader.

Remark(3.5): Let (H, X) and (H, Y) be two systems, if K is surjective homomorphisim from X to Y map then $R: V(X) \to V(Y)$ is unique homomorphisim which is defined by $R(a) = K(a(x)$, where $x \in X$ satisfies $K(x) = y$.

Theorem (3.6): Let $R: V(X) \to Y$ be the flow homomorphisim $h \longmapsto hq$ Then R is an isomorphism of flows iff (X, q) is isomorphic to the flow coming from some monidal compaction (T, j).

Proof: Clearly (X, q) must be a pointed H-system. Let R be an isomorphic of flows. Thus (X, q) is an isomorphic to the Ellis compactification. On the other hand, assume that (X, q) is isomorphic for some monidal compactification (T, j). We can define an isomorphism $Z: V(X) \to V(Y)$ as in remark (4). Therefore (X, q) is isomorphic to its Ellis compactification $(V(X), I_x)$.

Definition(3.7):
A pointed transitive H-system (X, p) is said to be inclusive if there is a homomorphisim from (X, p) to any other pointed H-system.

Definition(3.8):[9]
Let$(H, +)$ be a semigroup, for any $A \subseteq H$ and $x \in H$ we define $-x + A = \{h \in H: x + h \in A\}$. Given any two ultrafilters $p, q \in CH$ we define their sum by $p + q = \{A \subseteq H \,/\, \{x \in H \,/\, -x + A \in q\} \in p\}$.
In the next Theorem, we will see how the inclusive right topological compactification of H led to the inclusive pointed transitive H-flow $(CH, \overline{e})$.

Theorem (3.9): Let (CH, j) be the inclusive right topological compactification of H. Let we define $K: H \times CH \to CH$ by $k(n, q) = j(n)q$ where the right hand side is added in CH. Then (H, CH, K) is an H-system, and the pointed H-system $(CH, \overline{e})$ where $\overline{e}$ is the base point.

Proof: Note that by definition of filter addition above we have $K(n, q) = j(n) + q = f_{j(n)}(q)$ where $f_{j(n)}$right continuous. Thus K is separately continuous. Since j is a homomorphism, $K(n_1 + n_2, q) = j(n_1 + n_2)q = j(n_1) + j(n_2)x = j(n_1)j(n_2)x = j(n_1)n_2q = K(n_1, n_2q)$. Therefore, K is defined as an action of H on CH. Since $j(e) = \overline{e}$ then $K(e, x) = K(e)x = \overline{e}x = ex = x$ acts as an identity on CH. Also $K(H \times \{\overline{e}\}) \cup \{\overline{e}\} = j(H)\overline{e} \cup \{\overline{e}\} = j(H) \cup \{\overline{e}\}$ is dense in CH, implies the action is a point transitive. To show $(CH, \overline{e})$ is inclusive. Let (Y, p) be a pointed H-system. Thus, the extended action $\overline{K}: CH \times Y \to Y$ is right continuous action of CH on Y. Define $\tau: CH \to Y$ by $\tau(t) = tp$, where the right hand side is the extended action. Clearly g preserves distinguished points. Finally, for $\kappa \in H$ and since j is a homomorphisim then $\tau(\kappa x) = \tau(j(\kappa)x)$

$$= \tau(j(\kappa)j(r)) \quad where\ j(r) = x$$
$$= \tau\big(j(\kappa + r)\big)$$
$$= j(\kappa + r)q$$
$$= j(\kappa)j(r)q$$
$$= \kappa\tau(r)$$
$$= \kappa\tau\big(j(r)\big)$$
$$= \kappa\tau(x).$$

Thus g is a homomorphism of H-system and the proof is complete.

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العلاقة بين النظام الديناميكي مع تراص ايليس والنظام الديناميكي المتعدي الحاوي على نقطة اساسية

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الخلاصة:

في هذا البحث تمت دراسة بعض العلاقات بين الأنظمة الديناميكية وبين مايسمى Enveloping Semi-group . حيث العلاقة بينهما سوف تتيح لنا ربط الخصائص الموجودة بين الفضاءات التي تمت توسعتها لتكون فضاءا متراصا وبين النظام الديناميكي الذي يحوي على نقطة اساسية. هذه الروابط سوف تسمح لدراسة جوانب كثيرة من نظريات التدفقات او النظام الديناميكي من خلال منظور الخصائص الجبرية.

الكلمات المفتاحية: النظام الديناميكي، شبه التبولوجي شبه الزمرة، تماثل الأنظمة H، نظام النقطة المتعدي، انظمة ذات قاعدة شاملة.