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On Soft Turning Points

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Abstract:

The soft sets were known since 1999, and because of their wide applications and their great flexibility to solve the problems, we used these concepts to define new types of soft limit points, that we called soft turning points.
Finally, we used these points to define new types of soft separation axioms and we study their properties.

Keywords: Soft compact, Soft ideal, Soft separation axioms, Soft set and Soft turning points.

Introduction:

The concept of soft sets was first presented by Molodtsov (1) in 1999 as a common numerical instrument for managing dubious objects. Later, in 2011, Shabir and Naz (2) studied the soft spaces and they characterized soft topology on the gathering of soft sets. Consequently, they defined fundamental concepts of soft spaces, for example, soft closed set, soft open set, soft subspace and soft separation axioms. Zorlutuna el at. (3) introduced soft interior points, soft interior set, soft closure set and soft continuous function. In (4), Kharal and Ahmad gave the definition of mapping on soft classes and found some properties of image and inverse image of such mapping. Shabir and Ahmad (5), introduced separation axioms in the soft space that was defined by Zorlutuna. While the concept of soft ideal and likewise soft local function were first presented at (6,7). In 2014, Kandil *el at*. (6) presented the concept of supra generalized closed soft sets as for via soft ideal and they found their properties. Rodyna *el at*. (8), presented the semi closed and open sets modulo a soft ideal in the soft spaces. Mustafa el at. (9), presented the soft generalized closed sets and expand this idea into soft ideal topological spaces. In 2015 Aysequl and Goknur (10) introduced I_ regular, I_ normal and found a squire relations between them. The properties for instance, separation axioms and compactness to more details (11,12). Moreover, Kanal el at. (7) presented the concepts of soft normal (regular) space in view of the thoughts of soft ideals and semi open soft set .Recently, Zehra and Saziye (13) characterize open sets and soft regular generalized closed as for soft spaces and Yunus el at. (14) characterize the soft regular I_ closed and soft AI_ set and give decay of coherence in a domain Hayashi_ Samuels space and the range soft topological space. Compact space are one of the most important classes in general topological spaces (15,16,17). They well-know properties which use in many disciplines. Aygunoglu and Aygun (18), studied the soft compact around on a soft topology. In this paper, we are utilized to characterize another kind of a limit points and set up a connection amongst them and we characterize another sort of soft separation axioms in soft ideal topological space.

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Preliminaries

Through auto the proper we take E the parameters of the element of universe set. Also we take the symbol E_X be the union of E and the negation set of E ($\neg E$). Now for any subset A of E_X we define the soft set F_A over X as

$F_A = \{(e, F(e));\ e \in A, F: A \longrightarrow P(X)\}$.
where $P(X)$ is the power set of X .(10)

Definition 2.1 : (19)

The soft set F_A is called the ,

1. "Null soft set" if $F(e) = \varphi$, for all $e \epsilon A$, and obtained by $\tilde{\varphi}$.

That means $\tilde{\varphi} = \{(e, \varphi): e \in A\}$.

2. "Absolute soft set" if $F(e) = X$, for all $e \epsilon A$, and obtained by $\tilde{\tilde{X}}$.

That means $\tilde{\tilde{X}} = \{(e, X): e \in A\}$.

Definition 2.2 : (19)

Let $F_A, G_A \tilde{\tilde{\in}} SS(X)$. Then:

1. F_A is called a "soft subset" of G_A which is obtained by $F_A \tilde{\subseteq} G_A$ if $F_e \tilde{\subseteq} G_e$ for each $e \in A$.

Obviously, for all $F_A \tilde{\in} SS(X)$, then $\tilde{\varphi} \tilde{\subseteq} F_A$ and $F_A \tilde{\subseteq} \tilde{X}$.

2. The soft sets F_A and G_A are called a "soft equal" which is obtained by $F_A = G_A$ if $F_e = G_e$ for all $e \in A$.

It is clear that, $F_A = G_A$ iff $F_A \tilde{\subseteq} G_A$ and $G_A \tilde{\subseteq} F_A$.

Definition 2.3: (20)

The soft complement of a set F_A obtained by $F_A^{\tilde{c}}$ where $F_A^{\tilde{c}} : A \longrightarrow P(X)$ is a function given by $F_A^{\tilde{c}} = X \setminus F_e$ for all $e \in A$.

Clearly $: (\tilde{\varphi})^{\tilde{c}} = \tilde{X}, (\tilde{X})^{\tilde{c}} = \tilde{\varphi}$ and $(F_A^{\tilde{c}})^{\tilde{c}} = F_A$.

Definition 2.4: (19)

Let $F_A, G_A \tilde{\in} SS(X)$. We define a soft union $\tilde{\cup}$, a soft intersection $\tilde{\cap}$, and a soft difference $\tilde{\setminus}$ of two soft set F_A, G_A as follows :

1. $F_A \tilde{\cup} G_A = \{(e, F(e) \cup G(e)) : e \in A\}$.
2. $F_A \tilde{\cap} G_A = \{(e, F(e) \cap G(e)) : e \in A\}$.
3. $F_A \tilde{\setminus} G_A = \{(e, F(e) \setminus G(e)) : e \in A\}$.

Definition 2.5: (3)

Let $F_A \tilde{\in} SS(X)$, then the soft point in obtained by F_e where :

$$F_e(a) = \begin{cases} F(e) \neq \varphi & \text{if } a = e \\ \varphi & \text{if } a \neq e \end{cases}$$

For all $e \in A$.

Definition 2.6: (3)

For any sub family $\{F_{iA}\}_{i \in \lambda}$ of $SS(X)$, such that λ be any index set .

1. The union of these soft sets is the soft set $\mathcal{H}_A$, where $\mathcal{H}(e) = (\cup_{i \in \lambda} F_{i(e)}) \forall e \in A$, we write$(\tilde{\cup}_{i \in \lambda}\{F_{iA}\}) = \mathcal{H}_A$.
2. The intersection of these soft sets is the soft set G_A where $G(e) = (\cap_{i \in \lambda} F_{i(e)}) \forall e \in A$, we write$(\tilde{\cap}_{i \in \lambda}\{F_{iA}\}) = G_A$.

Proposition 2.7: (20) (De_ Morgan's Laws).

If $F_A, G_A \tilde{\in} SS(X)$. Then

1. $(F_A \tilde{\cup} G_A)^{\tilde{c}} = F_A^{\tilde{c}} \tilde{\cap} G_A^{\tilde{c}}$.
2. $(F_A \tilde{\cap} G_A)^{\tilde{c}} = F_A^{\tilde{c}} \tilde{\cup} G_A^{\tilde{c}}$.

Definition 2.8: (2)

For any family of soft sets over the universal set X with parameters A is called soft topology $\tilde{T}$, if satisfy the following properties :

1. $\tilde{\varphi}, \tilde{X} \tilde{\in} \tilde{T}$.
2. $\forall \$_1, \$_2 \tilde{\in} \tilde{T} \rightarrow \$_1 \tilde{\cap} \$_2 \tilde{\in} \tilde{T}$.
3. $\$_\lambda \tilde{\in} \tilde{T} \ \forall\ \lambda \in \Lambda \rightarrow \tilde{\cup}_{\lambda \in \Lambda} \$_\lambda \tilde{\in} \tilde{T}$.

The triplet $(\tilde{X}, \tilde{T}, A)$ is called a soft topological space over X.

The members of $\tilde{T}$ are called soft open sets and a soft set F_A is called soft closed iff the soft complement $(\tilde{X} \tilde{\setminus} F_A)$ is soft open.

Definition 2.9: (2)

In soft space $(\tilde{X}, \tilde{T}, A)$ we can define the soft sub space for a non_ empty set Y of X by $\tilde{T}_Y = \{V_A ; \exists\, F_A \tilde{\in} \tilde{T} \ni V_A = F_A \tilde{\cap} \tilde{Y}\}$ which obtained by $(\tilde{Y}, \tilde{T}_Y, A)$.

Definition 2.10.

In soft space $(\tilde{X}, \tilde{T}, A)$, and $F_A \tilde{\in} SS(X)$ is called the

1. Soft interior of the soft set F_A is obtained by $int(F_A)$ is defined as $(int(A)) = \tilde{\cup}\{\mathcal{H}_A : \mathcal{H}_A$ is a soft open set $\mathcal{H}_A \tilde{\subseteq} F_A\}$. Thus $int(F_A)$ is the largest soft open set contained in F_A .(1)
2. Soft closure of the soft set F_A is obtained by $c\ell(F_A)$ is define as $(c\ell(F_A)) = \tilde{\cap}\{\mathcal{H}_A : \mathcal{H}_A$ is a soft closed set and $F_A \tilde{\subseteq} \mathcal{H}_A\}$. Clearly $c\ell(F_A)$ is the smallest soft closed over X which contains F_A . (2)

Definition 2.11.

In soft space $(\tilde{X}, \tilde{T}, A)$ and for F_A be a soft set over X. Then

1. F_A is soft open iff $int(F_A) = F_A$. (1)
2. F_A is soft closed iff $c\ell(F_A) = F_A$. (2)

Definition 2.12:(21)

Let $(\tilde{X}, \tilde{T}, A)$ be a soft topological space and let $F_A \tilde{\in} SS(X)$ and $F_e \in \tilde{X}$. Then F_A is called a Soft neighbourhood of F_e, if there exists $G_A \tilde{\in} \tilde{T}$ such that $F_e \tilde{\in} G_A \tilde{\subseteq} F_A$.

The family of all soft neighborhood of $Ƒ_{ȩ}$ is symbolized by $\mathcal{N}_{Ƒ_{ȩ}}$.

Definition 2.13: (18)

Let $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ be a soft space. If ∃ a soft finite sub covering of every soft open covering of the soft space $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$, then this soft space is called "soft compact space".

Definition 2.14: (5)

Let $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ be a soft topological space over X and $Ƒ_{ȩ1}, Ƒ_{ȩ2} \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $Ƒ_{ȩ1} \neq Ƒ_{ȩ2}$. If ∃ at least one soft open set $Ƒ_{1A}$ or $Ƒ_{2A}$ such that $Ƒ_{ȩ1} \tilde{\tilde{\in}} Ƒ_{1A}$, $Ƒ_{ȩ2} \tilde{\tilde{\notin}} Ƒ_{1A}$ or $Ƒ_{ȩ2} \tilde{\tilde{\in}} Ƒ_{2A}$, $Ƒ_{ȩ1} \tilde{\tilde{\notin}} Ƒ_{2A}$, then $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ is called a"soft T_0_space".

Theorem 2.15: (5)

Every soft sub space of ST_0_space is ST_0_ space.

Definition 2.16.(2)

Let $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ be a soft space over X, $Ƒ_{ȩ} \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $Ƒ_{ȩ} \tilde{\tilde{\notin}} G_A$ and G_A be a soft closed set in $\tilde{\tilde{X}}$. If ∃ soft open sets $Ƒ_{1A}$ and $Ƒ_{2A}$ such that $Ƒ_{ȩ} \tilde{\tilde{\in}} Ƒ_{1A}$, $G_A \tilde{\tilde{\subseteq}} Ƒ_{2A}$ and $Ƒ_{1A} \tilde{\tilde{\cap}} Ƒ_{2A} = \tilde{\varphi}$, then $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ is called a "soft regular space".

Definition 2.17: (4)

Let X and Y be two initial universe sets and let A, B be sets of parameters .Let $u: X \longrightarrow Y$ and $p: A \longrightarrow B$ be a functions .Then a soft function $f_{pu}: SS(X)_A \longrightarrow SS(Y)_B$ is defined as :

For a soft set $Ƒ_A$ in $SS(X)_A$, then $(f_{pu}(Ƒ_A), B) = p(A) \subseteq B$ is a soft set in $SS(Y)_B$ given by:

$$f_{pu}(Ƒ_A)(\beta) = \begin{cases} u\left(\bigcup_{\alpha \in p^{-1}(\beta) \tilde{\cap} A} Ƒ(\alpha)\right) & \text{if } p^{-1}(\beta) \tilde{\cap} A \neq \varphi \\ \varphi & \text{other wies} \end{cases}$$

for $\beta \in B \subseteq Y$. $(f_{pu}(Ƒ_A), B)$ is called a "soft image" of a soft set $Ƒ_A$.

Definition 2.18: (4)

Let $f_{pu}: SS(X)_A \longrightarrow SS(Y)_B$ be a soft function and let G_B be a soft set in $SS(Y)_B$. Let $u: X \longrightarrow Y$ and $p: A \longrightarrow B$ be a functions.

Then $\left(f^{-1}{}_{pu}(G_B), D\right)$, $D = p^{-1}(B)$ is soft set in $SS(X)_A$, defined as:

$$\left(f^{-1}{}_{pu}(G_B), \alpha\right) = \begin{cases} u^{-1}\left(G(p(\alpha))\right), & p(\alpha) \in B \\ \varphi & \text{other wise} \end{cases}$$

for $\alpha \in D \subseteq X$.

$\left(f^{-1}{}_{pu}(G_B), \alpha\right)$ is called a soft inverse image of G_B.

Definition 2.19.

Let $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ and $(\tilde{\tilde{Y}}, \tilde{\tilde{\delta}}, B)$ be a soft spaces. Let $u: X \longrightarrow Y$ and $p: A \longrightarrow B$ be functions and let $f_{pu}: SS(X)_A \longrightarrow SS(Y)_B$ be a soft function. Then

1. The soft function f_{pu} is soft continuous iff $f^{-1}{}_{pu}(G_B) \tilde{\tilde{\in}} \tilde{\tilde{T}}$ for each $G_B \tilde{\tilde{\in}} \tilde{\tilde{\delta}}$.(5)
2. The soft function f_{pu} is soft open if $f_{pu}(Ƒ_A) \tilde{\tilde{\in}} \tilde{\tilde{\delta}}$ for each $(Ƒ_A) \tilde{\tilde{\in}} \tilde{\tilde{T}}$.(19)

Soft Ideal Topological Space

In this section, we are going to present the ideas of soft ideal base and soft maximal ideal and concentrate some imperative properties .

Definition 3.1: (22)

The sub family $\tilde{\tilde{J}}_A \tilde{\tilde{\subseteq}} SS(X)$ is called a "soft ideal" if satisfy the following:

1. $Ƒ_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_A$ and $G_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_A$ then $Ƒ_A \tilde{\tilde{\cup}} G_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_A$.
2. $Ƒ_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_A$ and $G_A \tilde{\tilde{\subseteq}} Ƒ_A$ then $G_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_A$.

Easily we see that the intersection of any family of soft ideals over X is also soft ideal, but the union of two soft ideals over a set X may be not necessary a soft ideal for example if we take X of $\mathcal{R}$ and $A = \mathcal{N}$, $Ƒ_{1A}(1) = \{1\}$, $Ƒ_{1A}(n) = \varphi \ \ \forall n \in \mathcal{N} \setminus \{1\}$

$Ƒ_{2A}(2) = \{2\}$, $Ƒ_{2A}(n) = \varphi \ \ \forall n \in \mathcal{N} \setminus \{2\}$

$\tilde{\tilde{J}}_{1A} = \{\tilde{\varphi}, Ƒ_{1A}\}$, $\tilde{\tilde{J}}_{2A} = \{\tilde{\varphi}, Ƒ_{2A}\}$ are soft ideal over $\mathcal{R}$, but $\tilde{\tilde{J}}_{1A} \tilde{\tilde{\cup}} \tilde{\tilde{J}}_{2A}$ is not soft ideal. The image of soft ideal under any soft function is soft ideal.

The soft ideal $\tilde{\tilde{J}}_A$ is called "proper soft ideal" if $\tilde{\tilde{X}} \tilde{\tilde{\notin}} \tilde{\tilde{J}}_A$.

Definition 3.2.

The subfamily $\tilde{\tilde{J}}_{0A} \tilde{\tilde{\subseteq}} SS(X)$ is called soft ideal base if satisfy the following :

1. $\tilde{\tilde{X}} \tilde{\tilde{\notin}} \tilde{\tilde{J}}_{0A}$.
2. If $Ƒ_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_{0A}$ and $G_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_{0A}$, then there exists $\mathcal{H}_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_{0A}$ such that $(Ƒ_A \tilde{\tilde{\cup}} G_A) \tilde{\tilde{\subseteq}} \mathcal{H}_A$.

Observe that if $\tilde{\tilde{X}} \neq (Ƒ_A \tilde{\tilde{\cup}} G_A) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{0A}$ for each $Ƒ_A$ and G_A in $\tilde{\tilde{J}}_{0A}$, then $\tilde{\tilde{J}}_{0A}$ is soft ideal base over X and so any soft ideal over X is soft ideal base.

Also for any soft ideal $\tilde{\tilde{J}}_A$ over X and with condition $(Ƒ_A \tilde{\tilde{\cup}} G_A) \neq \tilde{\tilde{X}} \ \forall \ G_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_A$ and $Ƒ_A \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$, the family, $\tilde{\tilde{J}}_{0A} = \{Ƒ_A \tilde{\tilde{\cup}} G_A : G_A \tilde{\tilde{\in}} \tilde{\tilde{J}}_A\}$ is soft ideal base .So, the soft ideal $\tilde{\tilde{J}}_A$ generated by soft ideal base is of the form $\tilde{\tilde{J}}_{A} = \left\{ F_{A} : \exists\, G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{0A} \ni F_{A} \tilde{\tilde{\subseteq}} G_{A} \right\}$. There is some properties of soft ideal base that, if $Y \subseteq X$ and $\tilde{\tilde{J}}_{0A}$ is a soft ideal base over Y, then its soft ideal base over X. Then image of soft ideal base under soft mapping is soft ideal base.

Definition 3.3.
Let $\tilde{\tilde{J}}_{A}$ and $\tilde{\tilde{\mathcal{J}}}_{A}$ be two soft ideals over X. Then $\tilde{\tilde{J}}_{A}$ is said to be"finer then"$\tilde{\tilde{\mathcal{J}}}_{A}$ if and only if $\tilde{\tilde{\mathcal{J}}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{J}}_{A}$. Any soft ideal over X is finer then $\tilde{\tilde{J}}_{A} = \{\tilde{\tilde{\varphi}}\}$. A non_empty soft set F_{A} is called finite if A is finite on $\exists a_1, \dots, a_n \in A \ni F(a_i) \neq \varphi \ \forall\, i = 1, \dots, n$. From this property we state the following theorem.

Definition 3.4.
Let $\tilde{\tilde{J}}_{A}$ be a soft ideal over X. Then $\tilde{\tilde{J}}_{A}$ is said to be "maximal soft ideal" over X if and only if $\tilde{\tilde{J}}_{A}$ is not contained any other soft ideal over X.
(i.e) $\tilde{\tilde{J}}_{A}$ is maximal soft ideal over X if and only if for every proper soft ideal $\tilde{\tilde{\mathcal{J}}}_{A}$ over X such that $\tilde{\tilde{J}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{A}$ then $\tilde{\tilde{J}}_{A} = \tilde{\tilde{\mathcal{J}}}_{A}$.

Theorem 3.5.
Let X be a set. Every soft ideal over X is contained in a maximal soft ideal over X.

Theorem 3.6.
Let X be any set and let $\tilde{\tilde{J}}_{A}$ be soft ideal over X such that $\left(\cup_{G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}} G_{A} \right) = \tilde{\tilde{X}}$. Then $\tilde{\tilde{J}}_{A}$ is finer then the finite soft ideal over X. Where the finite soft ideal $\tilde{\tilde{\mathcal{J}}}_{A} = \{F_{A} : F_{A} \text{ is finite}\}$.

Proof: Is directed from Definitions (3.1) and (3.3) . ■

There are many properties of maximal soft ideal, the following theorems show that.

Theorem 3.7.
The following statements are equivalent:

1. A soft ideal $\tilde{\tilde{J}}_{A}$ over X is maximal soft ideal.
2. For each $F_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$, then either $F_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ or $F_{A}^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$.
3. $\tilde{\tilde{J}}_{A}$ Contains all those soft sub sets F_{A} of $\tilde{\tilde{X}}$ which $\left(F_{A} \tilde{\cup} G_{A} \right) \neq \tilde{\tilde{X}}$ for each $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$.
4. For any two soft sub sets F_{A} and G_{A} of $\tilde{\tilde{X}}$ such that $(F_{A} \tilde{\cap} G_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$, we have either $F_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ or $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$.

Proof:

1) $\Longrightarrow$ 2). If $F_{A}^{\tilde{\tilde{c}}} \tilde{\tilde{\notin}} \tilde{\tilde{J}}_{A}$, then $\left(F_{A} \tilde{\cup} G_{A} \right) \neq \tilde{\tilde{X}}$ for each $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ because if there exists $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ such that $(F_{A} \tilde{\cap} G_{A}) = \tilde{\tilde{X}}$, then $F_{A}^{\tilde{\tilde{c}}} \tilde{\tilde{\subseteq}} G_{A}$ and so by Definition (3.1) we have $F_{A}^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ contraction. Let $\tilde{\tilde{\mathcal{J}}}_{A}$ be a soft ideal generated by soft ideal base $\left\{ F_{A} \tilde{\cup} G_{A} : G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A} \right\}$. Since $F_{A} \tilde{\tilde{\subseteq}} (F_{A} \tilde{\cup} G_{A})$ for each $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ then $F_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A} \rightarrow 1$. To show that $\tilde{\tilde{J}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{A}$. Let $D_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. Since $D_{A} \tilde{\tilde{\subseteq}} (F_{A} \tilde{\cup} G_{A})$, then $D_{A} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$ and we have $\tilde{\tilde{J}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{A}$. But $\tilde{\tilde{J}}_{A}$ is maximal soft ideal, then $\tilde{\tilde{J}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{A}$. By (1) $F_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$.

2) $\Longrightarrow$ 1). Let $\tilde{\tilde{\mathcal{J}}}_{A}$ be a proper soft ideal over X and $\tilde{\tilde{J}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{A}$. To prove that $\tilde{\tilde{\mathcal{J}}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{J}}_{A}$. Suppose $\tilde{\tilde{\mathcal{J}}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{J}}_{A}$, then there exists $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$, such that $G_{A} \tilde{\tilde{\notin}} \tilde{\tilde{J}}_{A}$. Then by hypothesis $G_{A}^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. But $\tilde{\tilde{J}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{A}$. So $G_{A}^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$. Thus $\left(G_{A} \tilde{\cup} G_{A}^{\tilde{\tilde{c}}} \right) = \tilde{\tilde{X}} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$. But this contradiction because $\tilde{\tilde{X}} \tilde{\tilde{\notin}} \tilde{\tilde{\mathcal{J}}}_{A}$, so that $\tilde{\tilde{\mathcal{J}}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{J}}_{A}$ we have $\tilde{\tilde{J}}_{A} = \tilde{\tilde{\mathcal{J}}}_{A}$. Therefore $\tilde{\tilde{J}}_{A}$ is maximal soft ideal.

1) $\Longleftrightarrow$ 3). The prove is directly by the Definitions (3.3) and (3.4).

1) $\Longrightarrow$ 4). Let $\tilde{\tilde{J}}_{A}$ is maximal soft ideal over X and let $F_{A}, G_{A} \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $(F_{A} \tilde{\cap} G_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. If possible that $F_{A} \tilde{\tilde{\notin}} \tilde{\tilde{J}}_{A}$ and $G_{A} \tilde{\tilde{\notin}} \tilde{\tilde{J}}_{A}$. Then $F_{A} \neq \tilde{\tilde{X}}$ and $G_{A} \neq \tilde{\tilde{X}}$ because if $F_{A} = \tilde{\tilde{X}}$, then $(F_{A} \tilde{\cap} G_{A}) = \left(\tilde{\tilde{X}} \tilde{\cap} G_{A} \right) = G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$, which is contracted and similar if $G_{A} = \tilde{\tilde{X}}$. Let $\tilde{\tilde{\mathcal{J}}}_{A} = \left\{ \mathcal{H}_{A} : (\mathcal{H}_{A} \tilde{\cap} F_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A} \right\}$. Then $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$ because $(F_{A} \tilde{\cap} G_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. To show that $\tilde{\tilde{\mathcal{J}}}_{A}$ is a soft ideal over X .

1. Since $\left(\tilde{\tilde{X}} \tilde{\cap} F_{A} \right) = F_{A}$ and $F_{A} \tilde{\tilde{\notin}} \tilde{\tilde{J}}_{A}$. So $\tilde{\tilde{X}} \tilde{\tilde{\notin}} \tilde{\tilde{\mathcal{J}}}_{A}$.
2. Let $\mathcal{H}_{A} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$ and $D_{A} \tilde{\tilde{\subseteq}} \mathcal{H}_{A}$. Imply that $(\mathcal{H}_{A} \tilde{\cap} F_{A}) \tilde{\tilde{\subseteq}} \tilde{\tilde{J}}_{A}$ and $(D_{A} \tilde{\cap} F_{A}) \tilde{\tilde{\subseteq}} (\mathcal{H}_{A} \tilde{\cap} F_{A})$. But $\tilde{\tilde{J}}_{A}$ is a soft ideal then $(D_{A} \tilde{\cap} F_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. So $D_{A} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$.
3. Let $\mathcal{H}_{1A}, \mathcal{H}_{2A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$, then $(\mathcal{H}_{1A} \tilde{\cap} F_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ and $(\mathcal{H}_{2A} \tilde{\cap} F_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. Since $\tilde{\tilde{J}}_{A}$ is a soft ideal, then $(\mathcal{H}_{1A} \tilde{\cap} F_{A}) \tilde{\cup} (\mathcal{H}_{2A} \tilde{\cap} F_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. So $(\mathcal{H}_{1A} \tilde{\cup} \mathcal{H}_{2A}) \tilde{\cap} F_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ and we have $(\mathcal{H}_{1A} \tilde{\cup} \mathcal{H}_{2A}) \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$.

To prove that $\tilde{\tilde{J}}_{A} \tilde{\tilde{\subseteq}} \tilde{\tilde{J}}_{A}$. Let $D_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$. Since$(D_{A} \tilde{\cap} F_{A}) \tilde{\tilde{\subseteq}} D_{A}$, then $(D_{A} \tilde{\cap} F_{A}) \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$and so $D_{A} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{A}$. Therefore $\tilde{\tilde{J}}_{A} = \tilde{\tilde{\mathcal{J}}}_{A}$. Hence either $F_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$ or $G_{A} \tilde{\tilde{\in}} \tilde{\tilde{J}}_{A}$.

4) $\Longrightarrow$ 1). Let $\tilde{\tilde{\mathcal{J}}}_{A}$ be a soft ideal satisfy the condition, then $\tilde{\varphi} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$. But $\tilde{\varphi} = (F_A \,\tilde{\tilde{\cap}}\, F_A{}^{\tilde{\tilde{c}}})$ for any soft sub set F_A of $\tilde{\tilde{X}}$, then by part 4 either $F_A \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$ or $F_A{}^{\tilde{\tilde{c}}} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$. By part1 we have $\tilde{\tilde{\mathcal{J}}}_{A}$ is maximal soft ideal over X. ■

Corollary 3.8.
If $\tilde{\tilde{\mathcal{J}}}_{A}$ is maximal soft ideal over X and $F_{1A} \,\tilde{\tilde{\cap}}\, F_{2A} \,\tilde{\tilde{\cap}}\, \ldots \,\tilde{\tilde{\cap}}\, F_{nA} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$, that at last one $F_{iA} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$ $(i = 1, \ldots, n)$.

Corollary 3.9.
If $\tilde{\tilde{\mathcal{J}}}_{A}$ is maximal soft ideal over X and $F_{1A}, F_{2A}, \ldots, F_{nA}$ are soft sub sets of $\tilde{\tilde{X}}$ such that $\left\{F_{iA}{}^{\tilde{\tilde{c}}} : i = 1, \ldots, n\right\}$ covers $\tilde{\tilde{X}}$, then some $F_{iA} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$ $(i = 1, \ldots, n)$.

Proportion 3.10.
Let $\wp$ be a collection of sub sets of $\tilde{\tilde{X}}$ such that for all $n \in \mathcal{N}$ and $F_{1A}, F_{2A}, \ldots, F_{nA} \,\tilde{\tilde{\in}}\, \wp$ we have $\tilde{\tilde{\cup}}_{i=1}^{n} F_{iA} \neq \tilde{\tilde{X}}$. Then $\tilde{\tilde{\mathcal{J}}}_{A} = \left\{F_A \,\tilde{\tilde{\subseteq}}\, \tilde{\tilde{\mathcal{J}}}_{A} : \text{there exists } F_{1A}, F_{2A}, \ldots, F_{nA} \,\tilde{\tilde{\subseteq}}\, F_{iA}\right\}$ is a soft ideal over X containing $\wp$. Indeed $\tilde{\tilde{\mathcal{J}}}_{A}$ is the soft ideal generated by $\wp$.

Proportion3.11.
Let $\wp$ be a collection of subsets of $\tilde{\tilde{X}}$ with the finite intersection property. Then $\wp^{\tilde{\tilde{c}}} = \left\{F_A{}^{\tilde{\tilde{c}}} : F_A \,\tilde{\tilde{\in}}\, \wp\right\}$ for a soft sub base for soft ideal.

Soft Turning Points.

In this section we build up meaning of soft turning point via soft points knowledge by Shabir (2) and we discover association with soft separation axiom. Also we characterized another idea of soft separation axioms .

Definition 4.1.

1. If $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ be a soft space with soft ideal $\tilde{\tilde{\mathcal{J}}}_{A}$. Then a soft point F_e is called a"soft turning point" if $F_A{}^{\tilde{\tilde{c}}} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$ for each $F_A \,\tilde{\tilde{\in}}\, N_{F_e}$.
2. If $\tilde{\tilde{\mathcal{J}}}_{0A}$ be a soft ideal base over X and $F_e \,\tilde{\tilde{\in}}\, \tilde{\tilde{X}}$. Then a soft point F_e is called a "soft turning point" of $\tilde{\tilde{\mathcal{J}}}_{0A}$ if and only if F_e is a soft turning point of $\tilde{\tilde{\mathcal{J}}}_{A}$ where $\tilde{\tilde{\mathcal{J}}}_{A}$ is a soft ideal generated by $\tilde{\tilde{\mathcal{J}}}_{0A}$.

Example 4.2.
Let X be a universal set such that $X = \{h_1, h_2\}$ and let $A \subseteq E_X$ be a parameters such that $A = \{e_1, e_2\}$. Let $\tilde{\tilde{T}} = \left\{\tilde{\varphi}, \tilde{\tilde{X}}, F_{1A}, F_{2A}, F_{3A}\right\}$ be a soft topological space and let $\tilde{\tilde{\mathcal{J}}}_{A} = \{\tilde{\varphi}, F_{1A}, F_{2A}, F_{3A}\}$,where
$F_{1A} = \{(e_1, \{h_2\}), (e_2, \varphi)\}$,
$F_{2A} = \{(e_1, \varphi), (e_2, \{h_2\})\}$ and
$F_{3A} = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$.
The all soft points F_e on $\tilde{\tilde{X}}$
$F_{1e1} = \{(e_1, \{h_1\}), (e_2, \varphi)\}$, $F_{2e1} = \{(e_1, \{h_2\}), (e_2, \varphi)\}$, $F_{3e1} = \{(e_1, X), (e_2, \varphi)\}$, $F_{4e2} = \{(e_1, \varphi), (e_2, \{h_1\})\}$, $F_{5e2} = \{(e_1, \varphi), (e_2, \{h_2\})\}$ and $F_{6e2} = \{(e_1, \varphi), (e_2, X)\}$.
Since $F_{2e1} = F_{1A}$, $F_{5e2} = F_{2A}$ and $N_{F_{2e1}} = \left\{\tilde{\tilde{X}}, F_{1A}, F_{3A}\right\}$, $N_{F_{5e2}} = \left\{\tilde{\tilde{X}}, F_{2A}, F_{3A}\right\}$
But $D_A = F_{1A}{}^{\tilde{\tilde{c}}} = \{(e_1, \{h_1\}), (e_2, X)\} \,\tilde{\tilde{\notin}}\, \tilde{\tilde{\mathcal{J}}}_{A}$
, $F_A = F_{2A}{}^{\tilde{\tilde{c}}} = \{(e_1, X), (e_2, \{h_1\})\} \,\tilde{\tilde{\notin}}\, \tilde{\tilde{\mathcal{J}}}_{A}$ and
$G_A = F_{3A}{}^{\tilde{\tilde{c}}} = \{(e_1, \{h_1\}), (e_2, \{h_1\})\} \,\tilde{\tilde{\notin}}\, \tilde{\tilde{\mathcal{J}}}_{A}$
Since $N_{F_{3e1}} = N_{F_{4e2}} = N_{F_{1e1}} = N_{F_{6e2}} = \left\{\tilde{\tilde{X}}\right\}$, but $\tilde{\tilde{X}}^{\tilde{\tilde{c}}} = \varphi \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$
Thus
F_{1e1} ,F_{3e1},F_{4e2} and F_{6e2} are soft turning points of $\tilde{\tilde{\mathcal{J}}}_{A}$ but F_{2e1},F_{5e2} are not soft turning point of $\tilde{\tilde{\mathcal{J}}}_{A}$.

Proposition 4.3.
Let $\tilde{\tilde{\mathcal{J}}}_{0A}$ be a soft ideal base over X and $F_e \,\tilde{\tilde{\in}}\, \tilde{\tilde{X}}$. Then F_e is a soft turning point of $\tilde{\tilde{\mathcal{J}}}_{0A}$ if and only if for each $F_A \,\tilde{\tilde{\in}}\, N_{F_e}$ implies $F_A{}^{\tilde{\tilde{c}}} \,\tilde{\tilde{\subseteq}}\, \mathcal{H}_A$ for some $\mathcal{H}_A \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{0A}$.

Proposition 4.4.
The soft space$(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ is a soft compact if for each a soft ideal $\tilde{\tilde{\mathcal{J}}}_{A}$ over X there exists a finer soft ideal $\tilde{\tilde{\mathcal{J}}}_{A}$ which has a soft turning point.

Proof :
Suppose $\tilde{\tilde{X}}$ be a soft compact. Let $\tilde{\tilde{\mathcal{J}}}_{A}$ be a soft ideal over X and let $K_A = \left\{c\ell(F_A) : F_A{}^{\tilde{\tilde{c}}} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}\right\}$, then K_A is a family of soft closed sets with the finite intersection proper because if there exists $F_{1A}{}^{\tilde{\tilde{c}}}, \ldots, F_{nA}{}^{\tilde{\tilde{c}}} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$ such that $\left(\tilde{\tilde{\cap}}_{i=1}^{n} c\ell(F_{iA})\right) = \tilde{\varphi}$, then $\left(\tilde{\tilde{\cup}}_{i=1}^{n} int\left(F_{iA}{}^{\tilde{\tilde{c}}}\right)\right) = \tilde{\tilde{X}}$, so$\left(\tilde{\tilde{\cup}}_{i=1}^{n} F_{iA}{}^{\tilde{\tilde{c}}}\right) = \tilde{\tilde{X}} \,\tilde{\tilde{\in}}\, \tilde{\tilde{\mathcal{J}}}_{A}$contradiction. Let $F_e \,\tilde{\tilde{\in}}\, \left(\tilde{\tilde{\cap}} K_A\right)$. Then for

each $Ƒ_{Ą}^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{J}}}_{Ą}$ and each $\mathcal{V}_{Ą} \tilde{\tilde{\in}} N_{Ƒ_{ę}}$, we have $(Ƒ_{Ą} \tilde{\tilde{\cap}} \mathcal{V}_{Ą}) \neq \tilde{\tilde{\varphi}}$ as $Ƒ_{ę} \tilde{\tilde{\in}} c\ell(Ƒ_{Ą})$, so $\left(Ƒ_{Ą}^{\tilde{\tilde{c}}} \tilde{\tilde{\cup}} \mathcal{V}_{Ą}^{\tilde{\tilde{c}}}\right) \neq \tilde{\tilde{X}}$, as $Ƒ_{ę} \tilde{\tilde{\in}} c\ell(Ƒ_{Ą})$. Thus from Properties (3.10).$\left(\tilde{\tilde{\mathcal{J}}}_{Ą} \tilde{\tilde{\cup}} \left(N_{Ƒ_{ę}}\right)^{\tilde{\tilde{c}}}\right)$ from subbase for soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$, $\tilde{\tilde{\mathcal{J}}}_{Ą} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{Ą}$ and $Ƒ_{ę}$ is a soft turning point for a soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$.

Conversely. Let $\wp$ be a collection of soft closed sets with finite intersection properties. Then by Properties (3.11), $\mathcal{H}_{Ą} = \left\{\mathcal{H}_{Ą}; \mathcal{H}_{Ą}^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} \wp\right\}$ from subbase for soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$. By hypothesis there exists a soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$ finer then $\tilde{\tilde{\mathcal{J}}}_{Ą}$ and has a soft turning point of $\tilde{\tilde{\mathcal{J}}}_{Ą}$, say $Ƒ_{ę}$. So for each $U_{Ą} \tilde{\tilde{\in}} N_{Ƒ_{ę}}$ and each $\mathcal{V}_{Ą}^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} \mathcal{H}_{Ą}$ we must have $\left(U_{Ą}^{\tilde{\tilde{c}}} \tilde{\tilde{\cup}} \mathcal{V}_{Ą}^{\tilde{\tilde{c}}}\right) \neq \tilde{\tilde{X}}$, thus $(U_{Ą} \tilde{\tilde{\cap}} \mathcal{V}_{Ą}) \neq \tilde{\tilde{\varphi}}$ and we have $Ƒ_{ę} \tilde{\tilde{\in}} c\ell(Ƒ_{Ą})$. Since $\mathcal{V}_{Ą}$ is soft closed, then $c\ell(\mathcal{V}_{Ą}) = \mathcal{V}_{Ą}$, so we have $Ƒ_{ę} \tilde{\tilde{\in}} \mathcal{V}_{Ą}$. So $Ƒ_{ę} \tilde{\tilde{\in}} \tilde{\tilde{\bigcap}}\wp$. Thus $\tilde{\tilde{\bigcap}}\wp \neq \tilde{\tilde{\varphi}}$. Therefore $\tilde{\tilde{X}}$ is soft compact. ■

Corollary 4.5.

The soft space$\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, Ą\right)$ is a soft compact if every maximal soft ideal over X has a soft turning point .

Proof:

Directly from Theorem (4.4), if $\tilde{\tilde{X}}$ is soft compact, then for every maximal soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$ there exists a soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$ finer then $\tilde{\tilde{\mathcal{J}}}_{Ą}$ $\left(\tilde{\tilde{\mathcal{J}}}_{Ą} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{Ą}\right)$ and there exists a soft turning point of a soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$. But $\tilde{\tilde{\mathcal{J}}}_{Ą}$ is maximal soft ideal, then $\tilde{\tilde{\mathcal{J}}}_{Ą} = \tilde{\tilde{\mathcal{J}}}_{Ą}$ and we have $\tilde{\tilde{\mathcal{J}}}_{Ą}$ is a soft turning point.

Conversely. Suppose any maximal soft ideal has a soft turning point. To show that $\tilde{\tilde{X}}$ is soft compact. Let $\tilde{\tilde{\mathcal{J}}}_{Ą}$ is a soft idea over X, then by Theorem (3.5), there exists a maximal soft ideal $\tilde{\tilde{\mathcal{J}}}_{Ą}$ such that $\left(\tilde{\tilde{\mathcal{J}}}_{Ą} \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{J}}}_{Ą}\right)$.

By hypothesis $\tilde{\tilde{\mathcal{J}}}_{Ą}$ has a soft turning point. So by Theorem (4.4) $\tilde{\tilde{X}}$ is soft compact. ■

Theorem 4.6.

For any soft space $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, Ą\right)$, the following statements are equivalent:

1. $\tilde{\tilde{X}}$ is ST_0_space.
2. For each two distinct soft points $Ƒ_{ę1} \neq Ƒ_{ę2}$, we have $c\ell\,(Ƒ_{ę1}) \neq c\ell(Ƒ_{ę2})$.
3. For each two distinct soft points $Ƒ_{ę1} \neq Ƒ_{ę2}$, $Ƒ_{ę1}$ is not a soft turning point of $\mathcal{J}_{Ƒ_{2Ą}}$ or $Ƒ_{ę2}$ is not a soft turning point of $\mathcal{J}_{Ƒ_{1Ą}}$.

Proof:

1) ⟺ 2). By directed by the Definition (2.14).

2) ⟹ 3). Let $Ƒ_{ę1}, Ƒ_{ę2} \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $Ƒ_{ę1} \neq Ƒ_{ę2}$. Since $\tilde{\tilde{X}}$ is ST_0_space then $Ƒ_{ę1}$ is not a soft turning point of $\mathcal{J}_{Ƒ_{2Ą}}$ or $Ƒ_{ę2}$ is not soft turning point of $\mathcal{J}_{Ƒ_{1Ą}}$. Suppose $Ƒ_{ę1}$ is not a soft turning point of $\mathcal{J}_{Ƒ_{2Ą}}$ then there is $Ƒ_{Ą} \tilde{\tilde{\in}} N_{Ƒ_{ę1}}$ such that $Ƒ_{Ą}^{\tilde{\tilde{c}}} \tilde{\tilde{\notin}} \mathcal{J}_{Ƒ_{1Ą}}$, so $Ƒ_{ę2} \tilde{\tilde{\notin}} Ƒ_{Ą}$ and we have $Ƒ_{ę2} \tilde{\tilde{\in}} Ƒ_{Ą}^{\tilde{\tilde{c}}}$. $Ƒ_{Ą}^{\tilde{\tilde{c}}}$ is soft closed set containing $Ƒ_{ę2}$ but not $Ƒ_{ę1}$. We get $c\ell(Ƒ_{ę2}) \tilde{\tilde{\subseteq}} Ƒ_{Ą}^{\tilde{\tilde{c}}}$ and $Ƒ_{ę1} \tilde{\tilde{\notin}} Ƒ_{Ą}^{\tilde{\tilde{c}}}$ which implies that $Ƒ_{ę1} \tilde{\tilde{\notin}} c\ell(Ƒ_{ę2})$. So we have $Ƒ_{ę1} \tilde{\tilde{\in}} c\ell(Ƒ_{ę1})$ but $Ƒ_{ę1} \tilde{\tilde{\notin}} c\ell(Ƒ_{ę2})$. Therefore $c\ell(Ƒ_{ę1})$ and $c\ell(Ƒ_{ę2})$ are distinct.

3) ⟹ 2). Let $Ƒ_{ę1}$ and $Ƒ_{ę2}$ are two distinct soft points in $\tilde{\tilde{X}}$ with $c\ell(Ƒ_{ę1}) \neq c\ell(Ƒ_{ę2})$ we have to prove $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, Ą\right)$ is ST_0_space .Then we can find one point $Ƒ_{ę3}$ in $\tilde{\tilde{X}}$ such that it belongs to one of the closures above say $c\ell(Ƒ_{ę1})$. Claim $Ƒ_{ę1}$ is not in $c\ell(Ƒ_{ę2})$. In fact, suppose $Ƒ_{ę1} \tilde{\tilde{\in}} c\ell(Ƒ_{ę2})$. Then $Ƒ_{ę1} \tilde{\tilde{\subseteq}} c\ell(Ƒ_{e2})$ and $c\ell(Ƒ_{ę1}) \tilde{\tilde{\subseteq}} c\ell\left(c\ell(Ƒ_{ę2})\right) = c\ell(Ƒ_{ę2})$. So $Ƒ_{ę3} \tilde{\tilde{\in}} c\ell(Ƒ_{ę1}) \tilde{\tilde{\subseteq}} c\ell(Ƒ_{ę2})$, which is a contradiction . Accordingly $Ƒ_{ę1} \tilde{\tilde{\notin}} c\ell(Ƒ_{ę2})$ and consequently $Ƒ_{ę1} \tilde{\tilde{\in}} (c\ell(Ƒ_{ę2})^{\tilde{\tilde{c}}}$ also since $c\ell(Ƒ_{ę2})$ is soft closed. Hence $(c\ell(Ƒ_{ę2})^{\tilde{\tilde{c}}}$ is soft open set in $\tilde{\tilde{X}}$ containing $Ƒ_{ę1}$but not $Ƒ_{ę2}$. Since $Ƒ_{ę2} \tilde{\tilde{\notin}} (c\ell(Ƒ_{ę2})^{\tilde{\tilde{c}}}$ then $c\ell(Ƒ_{ę2}) \tilde{\tilde{\notin}} \mathcal{J}_{Ƒ_{2Ą}}$. Hence $Ƒ_{ę1}$ is not a soft turning point of a soft ideal $\mathcal{J}_{Ƒ_{2Ą}}$ and we have $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, Ą\right)$ is ST_0_space.■

Theorem4.7.

The property of a space being a ST_0_spaceis preserved under injection, soft open and hence is a soft topological property .

Proof :

Let $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, Ą\right)$ be ST_0_space and let $\not{f}$ be injection open mapping of $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, Ą\right)$ subjective another soft space $\left(\tilde{\tilde{Y}}, \tilde{\tilde{\delta}}, B\right)$. Then we want show that $\left(\tilde{\tilde{Y}}, \tilde{\tilde{\delta}}, B\right)$ is also a ST_0_space. Let $Ƒ_{ę1}, Ƒ_{ę2}$ be any two distinct points of $\tilde{\tilde{Y}}$. Since $\not{f}$ is onto soft mapping , there exists distinct points $Ƒ_{ę3}, Ƒ_{ę4}$ of $\tilde{\tilde{X}}$ such that $\not{f}(Ƒ_{ę3}) = Ƒ_{ę1}$ and $\not{f}(Ƒ_{ę4}) = Ƒ_{ę2}$. Since

$\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A\right)$ is ST_0_space, then F_{e3} is not a soft turning point of $\mathcal{I}_{F_{e4}}$ or F_{e4} is not soft turning point of $\mathcal{I}_{F_{e3}}$. Suppose F_{e3} is not a soft turning point of $\mathcal{I}_{F_{e4}}$, then there is a T_ soft open nhd F_A of F_{e1} such that $F_A^{\tilde{\tilde{c}}} \tilde{\tilde{\notin}} \mathcal{I}_{F_{e4}}$, so $F_{e4} \tilde{\tilde{\notin}} F_A$ .Since f is an open and injective mapping, $f(F_A)$ is a δ_ soft open set containing $f(F_{e3}) = F_{e1}$ and not containing $f(F_{e4}) = F_{e2}$. In other words, $f(F_A)$ is a δ_ soft open nhd of F_{e1} not containing F_{e2}. So that $(f(F_A))^{\tilde{\tilde{c}}} \tilde{\tilde{\notin}} \mathcal{I}_{F_{e2}}$ and we have F_{e1} is not a soft turning point of a soft ideal $\mathcal{I}_{F_{e2}}$ over Y. Hence $\left(\tilde{\tilde{Y}}, \tilde{\tilde{\delta}}, B\right)$ is also ST_0_space. Since the property of being ST_0_space is preserved under injective, onto open soft mapping, if is a certainly preserved under homeomorphisms. Hence it is a soft topological property.

Also, we can take another prove to Theorem (2.15) by using properties of the soft turning points, as follows :

Let $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A\right)$ be a ST_0_space we prove that every subspace $\tilde{\tilde{Y}}$ of $\tilde{\tilde{X}}$ is $ST\mathcal{I}_0$_space. Let F_{e1},F_{e2} be two distinct points of $\tilde{\tilde{Y}}$. Since $\tilde{\tilde{Y}} \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$, then F_{e1},F_{e2} are also distinct points of $\tilde{\tilde{X}}$. Again since $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A\right)$ is a ST_0_space, then F_{e1} is not a soft turning point of a soft ideal $\mathcal{I}_{F_{2A}}$ over X or F_{e2} is not a soft turning point of a soft ideal $\mathcal{I}_{F_{1A}}$ over X. Suppose F_{e1} is not a soft turning point of a soft ideal $\mathcal{I}_{F_{2A}}$. So that $\exists$ T_soft open sets G_A such that $F_{e1} \tilde{\tilde{\in}} G_A$ but $G_A^{\tilde{\tilde{c}}} \tilde{\tilde{\notin}} \mathcal{I}_{F_{e2}}$. Then G'_A=($G_A \tilde{\tilde{\cap}} \tilde{\tilde{\delta}}$) is T_Y_soft open set such that $F_{e1} \tilde{\tilde{\in}} G'_A$ but $F_{e2} \tilde{\tilde{\notin}} G'_A$. So that $F_{e1} \tilde{\tilde{\in}} G'_A$ but $(G'_A)^{\tilde{\tilde{c}}} \tilde{\tilde{\notin}} \mathcal{I}_{YF_{e2}}$ where $\mathcal{I}_{YF_{e2}}$ is soft ideal over Y. Hence F_{e1} is not soft turning point of a soft ideal $\mathcal{I}_{YF_{2A}}$ over Y, so $\left(\tilde{\tilde{Y}}, \tilde{\tilde{T}}_Y, A\right)$ is ST_0_space.

Lemma 4.8.
Let $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A\right)$ be a soft space and $\tilde{\tilde{\mathcal{I}}}_A$ be a soft ideal base over X , then $int\left(\tilde{\tilde{\mathcal{I}}}_{0A}\right) = \{int(F_A) = F_A \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{I}}}_{0A}\}$ is a soft ideal over X.

Theorem 4.9.
Let $\tilde{\tilde{X}}$ be a soft space , and $\tilde{\tilde{\mathcal{I}}}_A$ be a soft ideal base over X . If F_e is a soft turning point of $int(\tilde{\tilde{\mathcal{I}}}_{0A})$ then F_e is a soft turning point of $\tilde{\tilde{\mathcal{I}}}_{0A}$,the converse may be not true.

Proof:
Let F_e be a soft turning point of $int(\tilde{\tilde{\mathcal{I}}}_{0A})$. Then by Properties (4.3), for each $G'_A \tilde{\tilde{\in}} N_{F_e}$ there exists $int(F_A) \tilde{\tilde{\in}} int\left(\tilde{\tilde{\mathcal{I}}}_{0A}\right)$ such that $(G'_A)^{\tilde{\tilde{c}}} \tilde{\tilde{\subseteq}} int\left(\tilde{\tilde{\mathcal{I}}}_A\right)$. Since $int(F_A) \tilde{\tilde{\subseteq}} F_A$, then $(G'_A)^{\tilde{\tilde{c}}} \tilde{\tilde{\subseteq}} F_A$, i.e. for each $G'_A \tilde{\tilde{\subseteq}} N_{F_e}$ there exists $F_A \tilde{\tilde{\in}} \tilde{\tilde{\mathcal{I}}}_{0A}$ such that $(G'_A)^{\tilde{\tilde{c}}} \tilde{\tilde{\subseteq}} F_A$. Again by Properties (4.3), we get F_e is a soft turning point of $\tilde{\tilde{\mathcal{I}}}_{0A}$.■

By Theorem (4.9) and by Definition (2.10) part 1 we get the following result :
If we take Let $X = \{h_1, h_2\}$ and $A = \{e_1, e_2\}$ .Let $\tilde{\tilde{T}} = \left\{\tilde{\tilde{\varphi}}, \tilde{\tilde{X}}, F_A, G_A, \mathcal{H}_A, D_A\right\}$ be a soft topological space and where
$F_A = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$, $G_A = \{(e_1, X), (e_2, \{h_2\})\}$,
$\mathcal{H}_A = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$ and
$D_A = \{(e_1, \varphi), (e_2, \{h_2\})\}$

The all soft points F_e on $\tilde{\tilde{X}}$ are
$F_{1e1} = \{(e_1, \{h_1\}), (e_2, \varphi)\}$, $F_{2e1} = \{(e_1, \{h_2\}), (e_2, \varphi)\}$, $F_{3e1} = \{(e_1, X), (e_2, \varphi)\}$, $F_{4e2} = \{(e_1, \varphi), (e_2, \{h_1\})\}$, $F_{5e2} = \{(e_1, \varphi), (e_2, \{h_2\})\}$ and $F_{6e2} = \{(e_1, \varphi), (e_2, X)\}$
and $N_{F_{1e1}} = \left\{\tilde{\tilde{X}}, F_A, G_A\right\}$, $N_{F_{2e1}} = \left\{\tilde{\tilde{X}}, \mathcal{H}_A, G_A\right\}$, $N_{F_{3e1}} = \left\{\tilde{\tilde{X}}, G_A\right\}$,
$N_{F_{4e2}} = N_{F_{6e2}} = \left\{\tilde{\tilde{X}}\right\}$ and
$N_{F_{5e2}} = \left\{\tilde{\tilde{X}}, F_A, G_A, \mathcal{H}_A, D_A\right\}$.
Let $\tilde{\tilde{\mathcal{I}}}_{0A} = \left\{\tilde{\tilde{\varphi}}, F_A^{\tilde{\tilde{c}}}, G_A^{\tilde{\tilde{c}}}, \mathcal{H}_A^{\tilde{\tilde{c}}}\right\}$ is soft ideal base, so each soft points in $\tilde{\tilde{X}}$ are soft turning point of $\tilde{\tilde{\mathcal{I}}}_{0A}$.But
$int\left(\tilde{\tilde{\mathcal{I}}}_{0A}\right) = \{\varphi\}$.
So F_{4e2},F_{6e2} are soft turning point of $int\left(\tilde{\tilde{\mathcal{I}}}_{0A}\right)$ but F_{1e1} ,F_{2e1},F_{3e1},F_{5e2} are not soft turning points of $int\left(\tilde{\tilde{\mathcal{I}}}_{0A}\right)$

Theorem 4.10.
For any soft space $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A\right)$, we have the following statements are equivalent:

1. $\tilde{\tilde{X}}$ is soft regular space.
2. For any soft ideal base $\tilde{\tilde{J}}_{0A}$, F_e is a soft turning point of $\tilde{\tilde{J}}_{0A}$ if and only if F_e is a soft turning point of $int(\tilde{\tilde{J}}_{0A})$.

Proof:

1) $\Longrightarrow$ 2). Let $\tilde{\tilde{X}}$ be a soft regular space and let $F_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$. Then for each $F_A \tilde{\tilde{\in}} N_{F_e}$ there is $G_A \tilde{\tilde{\in}} \tilde{\tilde{T}}$ such that $F_e \tilde{\tilde{\in}} G_A \tilde{\tilde{\subseteq}} \; c\ell(G_A) \tilde{\tilde{\subseteq}} F_A$.

So that for each $F_A \tilde{\tilde{\in}} N_{F_e}$ there is $G_A \tilde{\tilde{\in}} \tilde{\tilde{T}}$ such that $F_A^{\tilde{\tilde{c}}} \tilde{\tilde{\subseteq}} int(G_A^{\tilde{\tilde{c}}})$ …1

So that if F_e is a soft turning point of $\tilde{\tilde{J}}_{0A}$, then there is $\mathcal{H}_A \tilde{\tilde{\subseteq}} \tilde{\tilde{J}}_{0A}$ such that $G_A^{\tilde{\tilde{c}}} \tilde{\tilde{\subseteq}} \mathcal{H}_A$ and we have $int(G_A^{\tilde{\tilde{c}}}) \tilde{\tilde{\subseteq}} int(\mathcal{H}_A)$. From(1) we have $F_A^{\tilde{\tilde{c}}} \tilde{\tilde{\subseteq}} int(\mathcal{H}_A)$. So that F_e is a soft turning point of $int(\tilde{\tilde{J}}_{0A})$.

Conversely. Direct from Theorem (4.8).

2) $\Longrightarrow$ 1). Suppose that $\tilde{\tilde{X}}$ is not a soft regular space. Then $\exists$ a point $F_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ and a soft open set $F_A \tilde{\tilde{\in}} N_{F_e}$ with the property.

If $G_A \tilde{\tilde{\in}} N_{F_e}$ then $c\ell(G_A)$ is not contained in F_A. F_e is a soft turning point of a soft ideal base ($i.e.$) $G_A \tilde{\tilde{\in}} N_{F_e}$, then $F_A^{\tilde{\tilde{c}}}$ is not contained in $(G_A^{\tilde{\tilde{c}}})$. $\tilde{\tilde{J}}_{0A}$=$\{G_A: G_A^{\tilde{\tilde{c}}} \tilde{\tilde{\in}} N_{F_e}\}$, but by Lemma (4.7) F_e is not a soft turning point of $\tilde{\tilde{J}}_{0A}$ contradiction with 2. Therefore $\tilde{\tilde{X}}$ is a soft regular space.■

Definition 4.11.

A soft space $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ is called"S$T\mathcal{J}_1$_space" iff $\forall F_{e1}, F_{e2} \tilde{\tilde{\in}} \tilde{\tilde{X}}$, $F_{e1} \neq F_{e2}$ then F_{e1} is not a soft turning point of $\mathcal{J}_{F_{2A}}$ or F_{e2} is not a soft turning point of $\mathcal{J}_{F_{1A}}$.

Theorem 4.12.

The property of a space being a S$T\mathcal{J}_1$_space is preserved under injective soft open and hence is a soft topological property.

Proof:

The prove is directed by the Definition (4.11) and Theorem (4.7).■

Theorem 4 .13.

Every soft sub space of a S$T\mathcal{J}_1$_space is a S$T\mathcal{J}_1$_space.

Proof:

The prove is directed by the Definition (4.11) and Theorem (2.15).■

Theorem 4.14.

Let $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ be a soft space. $\tilde{\tilde{X}}$ is a S$T\mathcal{J}_1$_space if and only if for each $F_{e1}, F_{e2} \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $F_{e1} \neq F_{e2}$, F_{e1} and F_{e2} are soft closed sets in $\tilde{\tilde{X}}$.

Definition 4.15.

A soft space $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ is called"S$T\mathcal{J}_{\frac{1}{2}}$_space" if for any two distinct soft points $F_{e1}, F_{e2} \tilde{\tilde{\in}} \tilde{\tilde{X}}$, there exists a soft ideal $\tilde{\tilde{J}}_A$ over X such that F_{e1} is not a soft turning point of $\tilde{\tilde{J}}_A$ or F_{e2} is not a soft turning point of $\tilde{\tilde{J}}_A$.

Note 4.16.

From above Definition (2.14) and Definition (4.11). Every soft $T\mathcal{J}_1$_space is soft T_0_space and every soft T_0_space is soft $T\mathcal{J}_{\frac{1}{2}}$_$space$.

Example 4.17.

Let $X = \{h_1, h_2\}$,$A = \{e\}$ and $\tilde{\tilde{T}}$=$\{\tilde{\tilde{\varphi}}, \tilde{\tilde{X}}, F_{1A}, F_{2A}\}$, where F_{1A}, F_{2A} are open soft sets over X where $F_{1A} = \{(e, \{h_1\})\}$,$F_{2A} = \{(e, \{h_2\})\}$.

Then $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ are soft T_0_space, soft $T\mathcal{J}_1$_space and soft$T\mathcal{J}_{\frac{1}{2}}$_space.

Theorem 4.18.

The property of a space being a S$T\mathcal{J}_{\frac{1}{2}}$_space is preserved under injective soft open and hence is a soft topological property.

Proof:

The prove is directed by the Definition (4.15) and a Theorem (4.7). ■

Theorem 4.19.

Every soft sub space of a S$T\mathcal{J}_{\frac{1}{2}}$_space is a S$T\mathcal{J}_{\frac{1}{2}}$_space.

Proof:

The prove is directed by the Definition (4.15) and a Theorem (2.15). ■

Theorem 4.20.

Let F_A be a soft compact subset of a S$T\mathcal{J}_{\frac{1}{2}}$_space $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$ and suppose $F_{1A} \tilde{\tilde{\in}} F_A^{\tilde{\tilde{c}}}$. Then $\exists$ soft open sets $G_A, \mathcal{H}_A \ni F_{e1} \tilde{\tilde{\in}} G_A$,$F_A \tilde{\tilde{\subseteq}} \mathcal{H}_A$, $G_A \tilde{\tilde{\cap}} \mathcal{H}_A = \tilde{\tilde{\varphi}}$.

Corollary 4.21.

Let $\tilde{\tilde{X}}$ be a soft compact S$T\mathcal{J}_{\frac{1}{2}}$_space $(\tilde{\tilde{X}}, \tilde{\tilde{T}}, A)$, $F_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ and $\tilde{\tilde{J}}_{0A}$ be a soft ideal base over X. F_e is a soft turning point of $\tilde{\tilde{J}}_{0A}$ if and only if F_e is a soft turning point of $int(\tilde{\tilde{J}}_{0A})$.

Conflicts of Interest: None.

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حول نقاط التحول المرنة

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الخلاصة:

المجموعات المرنة عرفت منذ 1999،وبسبب تطبيقاتها الواسعة ومرونتها الكبيرة لحل المشاكل ، استخدمنا هذه المفاهيم لتحديد انواع جديدة من النقاط المرنة التي أطلقنا عليها نقاط التحول المرنة . و أخيراً استخدمنا تلك النقاط لتعريف انواع جديدة من بديهيات الفصل المرنة و دراسة أهم خواصها.

الكلمات المفتاحية: المرصوصة المرنة، المرنة المثالية، بديهيات الفصل المرنة، المجموعات المرنة ونقاط التحول المرنة .