

The Electro-Excitation Form Factors for Low-Lying States of ${}^7\text{Li}$ Nucleus

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Abstract

The transverse electron scattering form factors have been studied for low – lying excited states of ${}^7\text{Li}$ nucleus. These states are specified by $J^\pi T = \frac{1^-}{2} \frac{1}{2}$ (0.478MeV), $\frac{7^-}{2} \frac{1}{2}$ (4.63MeV) and $\frac{5^-}{2} \frac{1}{2}$ (6.68MeV). The transitions to these states are taking place by both isoscalar and isovector components. These form factors have been analyzed in the framework of the multi-nucleon configuration mixing of harmonic oscillator shell model with size parameter $b_{rms}=1.74\text{fm}$. The universal two-body of Cohen-Kurath is used to generate the 1p-shell wave functions. The core polarization effects are included in the calculations through effective g-factors and resolved many discrepancies with experiments. A higher configuration effect outside the 1p-shell model space, such as the 2p-shell, enhances the form factors for q-values and reproduces the data. The present results are compared with other theoretical models.
PACS: 25.30.Bf Elastic electron scattering - 25.30.Dh Inelastic electron scattering to specific states – 21.60.Cs Shell model – 27.20.+n $5 \leq A \leq 19$

Key word: Transverse electron, scattering

Introduction:

Electron scattering has been established as a successful tool for the study of elementary excitation modes of the nucleus. The basic electron scattering theory is based on the electromagnetic interaction, in which the electron interacts with the charge, current and magnetization distributions of the nucleus [1]. There have been several attempts to estimate the higher configuration effects in electron scattering through different processes. Willely in 1963 [2] calculated the contributions from the transverse electric and magnetic interactions for ${}^7\text{Li}$. The cross section of electron scattering from

the ground state of ${}^7\text{Li}$ has been measured by Van Niftrik et al. (1971) [3]. The results agree very well with results of lifetime measurements. Lichtenstadt et al. [4] measured the M1 and M3 form factors of ${}^7\text{Li}$ ($J^\pi = \frac{3^-}{2}$) ground state and the M1 and E2 form factors of the ($J^\pi = \frac{1^-}{2}$), 0.478 MeV state by 180° electron scattering. A very good agreement between the data and calculations using Cohen-Kurath (C- K) shell model amplitudes [5], was obtained by choosing an oscillator parameter value of 1.65fm and by normalizing

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the calculated form factors for both states by an overall factor of 0.92 (0.85 in the cross section). The longitudinal and the transverse form factors of the 4.63 MeV excitation ($J^\pi = \frac{7^-}{2}$) in ${}^7\text{Li}$ were measured over the momentum transfer range $0.8 \leq q \leq 4.2 \text{ fm}^{-1}$ in 1990 by Lichtenstadt et al. [6]. Booten and Van Hees (1994) [7] studied the electromagnetic properties of 1p-shell nuclei (${}^6\text{Li}$, ${}^7\text{Li}$, ${}^{10}\text{B}$, ${}^{11}\text{B}$, ${}^{14}\text{N}$ and ${}^{15}\text{N}$). The calculations included the extended $(0+2)\hbar\omega$ model space, and the effects of meson exchange current (MEC). Extension of the model space improves agreement with the transverse form factors in the beginning of 1p- shell, but towards the end of 1p- shell the situation deteriorates. Karataglidis et al. (1997) [8] used $(0+ 2+ 4)\hbar\omega$ wave functions in the analysis of the elastic and inelastic electron scattering form factors in ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei. In their results, there is remarkable agreement between experiment and theory in all of the transverse electron scattering form factors. Dakhil (1998) [9] included the contribution of the higher configurations such as 2p- shell in the calculations of elastic and inelastic form factors for ${}^6\text{Li}$, ${}^{13}\text{C}$ and ${}^{14}\text{N}$. This inclusion enhanced the form factors and reasonably reproduced the data. The effects of MEC were included in the calculations. Effective operators for the different multipoles were used to normalize the transverse elastic and inelastic form factors to the experimental data. AL-Bannaa (2001) [10] studied elastic and inelastic electron scattering from ${}^6\text{Li}$, ${}^7\text{Li}$, ${}^9\text{Be}$ and ${}^{10}\text{B}$. The core polarization effects had been calculated through the first-order perturbation theory including

excitations up to $6\hbar\omega$, using the modified surface delta interaction (MSDI) as a residual interaction. The inclusion of higher excited configurations enhances the form factors and brings the theoretical calculations close to the experimental data. In the present work, we follow the same sort of analysis presented in Ref. [9]. In our calculations, the space is extended to include the higher 2p-shell configuration, in which the angular part is the same and only the radial part of matrix element will be modified. The two-body interactions of C-K [5] are used in both configurations. The core polarization effects are included through effective g-factor.

Theory

The transverse form factors involving the angular momentum J , isospin T and momentum transfer q , between the initial i and final f nuclear shell model states of spin $J_{i,f}$ and isospin $T_{i,f}$ is [11],

$$|F_J^\lambda(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \langle J_f T_f | \hat{T}_{JT}^\lambda | J_i T_i \rangle \times F_{cm}(q) \times F_{fs}(q) \right|^2 \quad (1)$$

Where λ –stands the transverse magnetic or electric. The reduced matrix elements of the transverse electron scattering operator \hat{T}_{JT}^λ are expressed as the sum of the product of the one body density matrix (OBDM) $\chi_{J_i J_f}^J(J_i, J_f)$ times the single-particle matrix elements [12,13],

$$\langle f | \hat{T}_{JT}^\lambda | i \rangle = \sum_{J_i J_f} \chi_{J_i J_f}^J(J_i, J_f) \langle J_i | \hat{T}_{JT}^\lambda | J_f \rangle \quad (2)$$

Where J_i and J_f denote the single-particle initial and final states, respectively. The structure

factor $\chi_{J_i J_f}^J(J_i, J_f)$ are obtained from the work of Cohen-Kurath [5].

The finite size (f.s.) nucleon form factor is $F_{f.s.}(q) = \exp(-0.43q^2/4)$, and $F_{cm}(q) = \exp(q^2 b^2/4A)$ is the correction for the lack of translation invariance in the shell model [14, 15], where A is the mass number and b is the size parameter of Harmonic oscillator (HO). The total transverse form factor is given by: -

$$|F_J^{\lambda}(q)|^2 = \sum_{J \geq 0} \left\{ |F_J^{mag}(q)|^2 + |F_J^{ele}(q)|^2 \right\} \quad (3)$$

When the 1p-shell model space is extended to include the 2p-shell model space, the wave functions of the initial (i) and final (f) states will be written as [9]:

$$|i\rangle = \alpha |i(1p)\rangle + \sqrt{1-\alpha^2} |i(2p)\rangle \quad (4)$$

$$|f\rangle = \gamma |f(1p)\rangle + \sqrt{1-\gamma^2} |f(2p)\rangle \quad (5)$$

Where α and γ are mixing parameters. Since the C-K interaction depends on the angular parts only, the same OBDM are used for both 1p and 2p shells.

Results and Discussion

The ${}^7\text{Li}$ nucleus is well described either by shell model or an $\alpha-t$ cluster model. One can improve the nuclear wave functions by systematically analyzing the electron scattering form factors for all variety of transitions in a given nucleus. According to the many-particle shell model, the nucleus ${}^7\text{Li}$ is considered as a core of ${}^4\text{He}$ plus three nucleons distributed over the $1p_{3/2}$ and $1p_{1/2}$ orbits. The transverse form factors of the $J^\pi T = \frac{1}{2}^- \frac{1}{2}$

(0.478MeV), $\frac{7}{2}^- \frac{1}{2}$ (4.63MeV) and

$\frac{5}{2}^- \frac{1}{2}$ (6.68MeV) states are calculated within the framework of shell model using Cohen - Kurath interaction [5]. The transverse form factors of the ground state and two first excited states have been measured up to the momentum transfer $q \sim 4.2 \text{ fm}^{-1}$ by Lichtenstadt

et al.[4,6], but for $\frac{5}{2}^- \frac{1}{2}$ (6.68MeV) state, the experimental data is absent. The single particle wave functions of harmonic oscillator potential with size parameter $b_{rms} = 1.74 \text{ fm}$ chosen to reproduce the root mean square charge radius, are used for all transitions considered.

The 0.478 MeV ($\frac{1}{2}^- \frac{1}{2}$) state

The transverse form factors for 0.478 MeV ($\frac{1}{2}^- \frac{1}{2}$) state are shown in Fig. (1). The individual multipoles M1 and E2 which comprise the total form factor are denoted by short-dashed and dotted curves respectively. The experimental data of Niftrik et al. [3] (circles) and Lichtenstadt et al. [4] (squares) are compared to the present results and to other models.

The M1 and E2 multipoles in the 0.478 MeV form factor are expected to have a similar q -dependence to those of the elastic M1 and M3 multipoles respectively. Since the E2 and M3 form factors peak around the location of the M1 minimum. The M1 multipole dominates form factor below 1.0 fm^{-1} . The results obtained in 1p-shell model space with free g -factors and $b_{rms} = 1.74 \text{ fm}$ (long-dashed curve) reproduce the experimental data for momentum transfers up to $q \sim 2.4 \text{ fm}^{-1}$, but at higher q - data the form factors are underestimated. This discrepancy

could partly be resolved by introduction the higher contributions as shown in Fig.(2)(solid curve). The present calculations include the admixture of 2p-shell with $\alpha = \gamma = 0.99$ and effective g- factors ($g_{s-eff}^{p/n} = 0.84 g_{s-free}^{p/n}$). The results with these parameters (solid curve) reproduce the experimental-data very well for momentum transfers up to $q \sim 3.0 \text{ fm}^{-1}$, and they are still unpredicted at high q-data. The comparison is made with results in the $2\hbar\omega$ model space of Booten et al. [7] and with the $(0+2+4)\hbar\omega$ of Karataglidis et al [8]. This compression is shown in Fig. (3). In the extended model, Booten et al. [7] (dotted curve) performed a good description of the data up to $q \approx 3.0 \text{ fm}^{-1}$.

The results of Karataglidis et al. [8] (cross symbol curve) reproduce the magnitude and shape of this form factor up to 3.0 fm^{-1} , with but a slight overestimation above 1.0 fm^{-1} . The results of the above three models are close to each other for $q \sim 3.0 \text{ fm}^{-1}$ and underestimated the higher q data as indicated in Fig. (3).

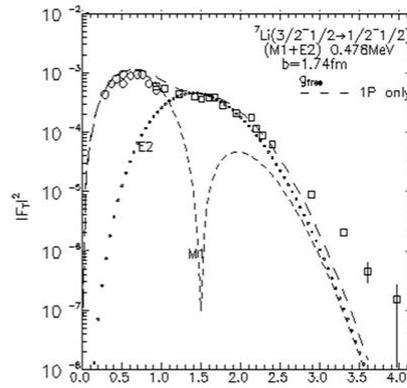


Fig. (1) Transverse inelastic electron scattering form factors to the $\frac{1}{2}^-$ (0.478 MeV) state in Li. The data of Ref. [3] (circles) and Ref. [6] (squares) are compared with the results of 1p-shell model space (dashed components are curve).The M1 and E2 shown.

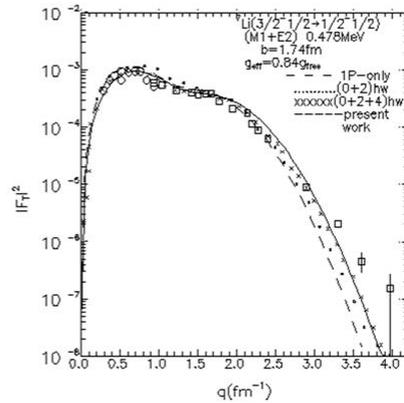


Fig. (2) Transverse inelastic electron scattering form factors to the $\frac{1}{2}^-$ (0.478 MeV) state in Li. The present results in (1p+2p)-shell (solid curve) and in 1p-shell model space (dashed curve) are compared with the data of Ref.[3](circles) and Ref.[6](squares).

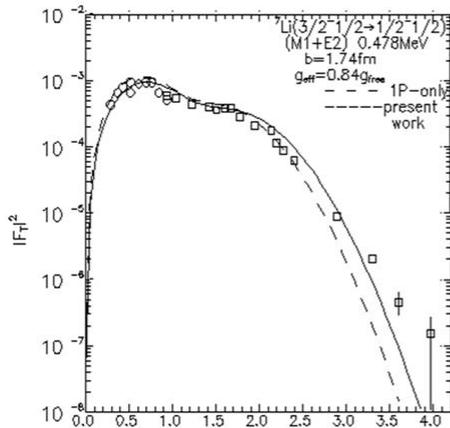


Fig. (3) Same as caption to Fig. (2), the data of Ref.[3,6] are compared to The present results (solid curve), and to the $(0+2)\hbar\omega$ results of Ref. [7] (dotted curve), and to the $(0+2+4)\hbar\omega$ results of Ref. [8] (cross symbol curve).

The 4.63 MeV $(\frac{7^-}{2} \frac{1}{2})$ state

The calculated transverse form factors of the 4.63 MeV

$(J^\pi T = \frac{7^-}{2} \frac{1}{2})$ state are presented in

Fig. (4). The contribution multipoles are the E2 and M3 components. The E2 (solid curve) and M3 (cross-symbol curve) multipoles are dominated and peak around $q \sim 1.5 \text{ fm}^{-1}$.

The E2 multipole shows diffraction minimum at $q = 0.4 \text{ fm}^{-1}$. Both E2 and M3 have a similar q -dependence for q up to 0.6 fm^{-1} . Same behavior can be noted in the results of Dubach et al. [15], Booten et al. [7] and Karataglidis et al [8]. The 1p-shell results with free g -factors and $b_{\text{ms}}=1.74 \text{ fm}$ (dashed curve) reproduce the peak position of data but show large discrepancies over all range of momentum transverse. The experimental data of Lichtenstadt et al [6] for the 4.63 MeV state are measured to cover the range of $0.8 < q < 4.2 \text{ fm}^{-1}$. This data are compared with present results and with that of Booten et al. [7] and Karataglidis et al.

[8] as shown in Fig.(5). The inclusion of 2p-shell with $\alpha = \gamma = 0.94$, and effective g -factors ($g_{s\text{-eff}}^{p/n} = 0.85 g_{s\text{-free}}^{p/n}$) may provide sizable corrections to the 1p-shell model space results. This inclusion is indicated by solid curve in Fig. (6). The present results reproduce the experimental data up to $q \sim 3.0 \text{ fm}^{-1}$ and underestimate the higher q - data. Similar results obtained by Booten et al. [7] (dotted curve). The results of Booten et al. not only predict the form factors fairly well up to momentum transfer of $q \sim 3.0 \text{ fm}^{-1}$, but also bring theory in a reasonable accord with the experiment. The data are much better reproduced in the $2\hbar\omega$ - model space.

The results of Karataglidis et al. [8] reproduce the peak magnitude and position of the data. However, the results underpredict the data above 2.0 fm^{-1} . This is due in part to the form factor being dominated by the M3 transition. The M3 form factor dominates, with the E2 contribution being a factor of 2 less.

The results of Booten et al.[7] are close to that of Karataglidis et al.[8] and they significantly departed from the present results. A perfect description of the data in the present model is achieved up to $q \sim 3.0 \text{ fm}^{-1}$.

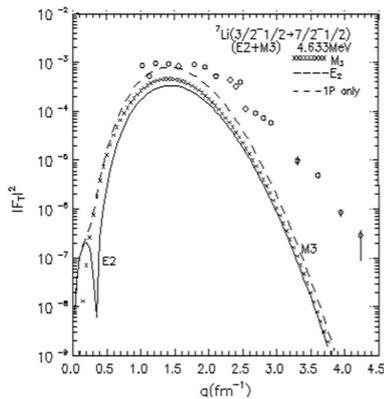


Fig. (4) Transverse form factor for $(\frac{5^-}{2} \frac{1}{2})$ state in ${}^7\text{Li}$. The E2 and M3 components are displayed. The data of Ref. [6] are compared to the total form factor calculated in 1p-shell model space (dashed curve).

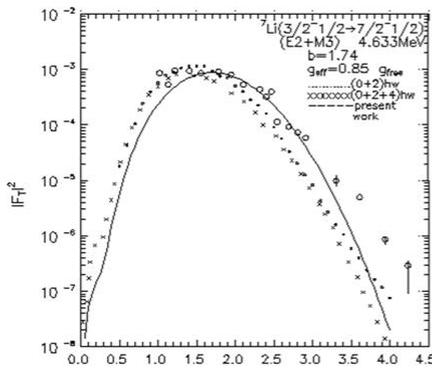


Fig. (5) Transverse $(E2+M3)$ form factor for $(\frac{5^-}{2} \frac{1}{2})$ state in ${}^7\text{Li}$. The data of Ref. [6] are compared to the present (1p+2p) results (solid curve), $(0+2+4)\hbar\omega$ results of Ref. [8] (cross symbol curve).

The 6.68 MeV $(\frac{5^-}{2} \frac{1}{2})$ state

2. The transverse form factors for the transition to the 6.68 MeV $(\frac{5^-}{2} \frac{1}{2})$ state are displayed in Fig. (6). The contributions of M1, E2, and M3 multipoles are indicated by solid, dotted and cross symbol curves respectively.

These multipoles peak around same location of about 1.5 fm^{-1} , but with different values of maxima. The M3 multipole has minor contribution to the total form factor, while the E2 and M1 multipoles are dominant and their maxima give the values 4×10^{-4} and 1.5×10^{-4} respectively. Furthermore, the results show that the E2 and M1 multipoles would have similar q-dependences. No diffractive structure was found in the transverse form factor. There is no experimental data to be compared with.

3. The 1p-shell results with free g-factors and $b_{\text{rms}}=1.74 \text{ fm}$ (dashed curve) has the same position peak of multipoles but with different magnitude about 6×10^{-4} . The inclusion of 2p-shell admixture with $\alpha=\gamma=0.99$ and effective g-factors $(g_{s-\text{eff}}^{p/n} = 0.99 g_{s-\text{free}}^{p/n})$ is indicated by solid curve in the Fig. (7). The maximum of the form factor is slightly shifted towards a higher value of q, and increased as well as α and γ decreased. Same behavior obtained by Dubach et al. [15]. Their results show a diffraction minimum at $q \sim 3.2 \text{ fm}^{-1}$ for M1 and E2 multipoles and at $q \sim 1.6 \text{ fm}^{-1}$ for M3 multipole. In the total form factor, the higher multipole (E4) fills in the minimum near $q \sim 5 \text{ fm}^{-1}$ due to exchange-current effects in the lower multipoles. A study of this form factor provides a good test for our approach.

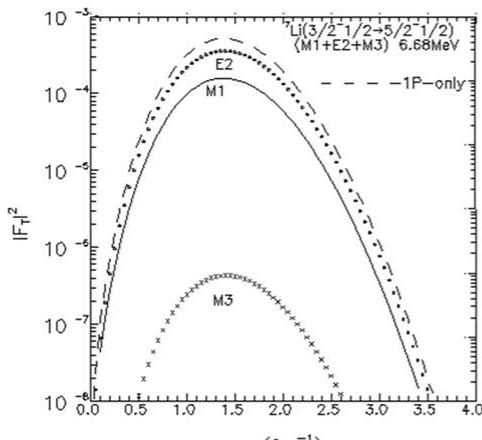


Fig. (6) Multipole decomposition of the transverse form factors of $\frac{5^-}{2} \frac{1}{2}$ (6.68 MeV) state in ${}^7\text{Li}$. The total form factor in 1p-shell model space denoted by dashed curve.

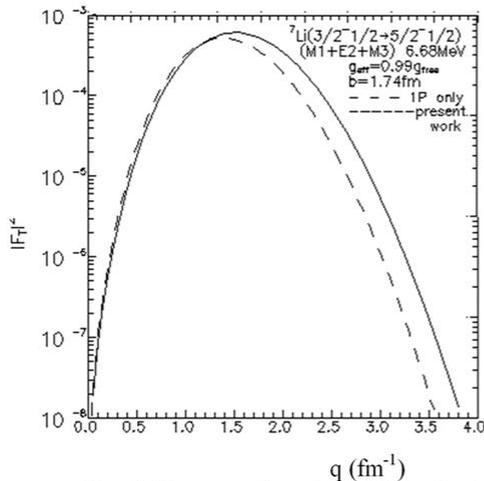


Fig. (7) Transverse form factor of the (1p + 2p)-shell model (solid curve) and of 1p-shell model space (dashed curve).

4. Conclusions

The present results show many conclusions which can be summarized as follows:-

1 The 1p-shell model space results with b_{rms} and free g-factors can account

for the data only as far as $q \leq 2.0 \text{ fm}^{-1}$.

2 The effective g-factors given in the text make a reduction to the form factors and do not reproduce the higher q- data.

3 The inclusion of higher orbit contribution gives a remarkable improvement in the form factors.

4 The higher q-data for the transition to $J^\pi T = \frac{7^-}{2} \frac{1}{2}$ states in ${}^7\text{Li}$ beyond 3.0 fm^{-1} needs inclusion higher orbits contributions beyond that of 2p-shell, in order to be described satisfactorily.

5 The study of all transitions that the data are absent gives good improvement for the validity of our approach.

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عوامل التشكل للاثارة الالكتروونية للحالات المحفزة الواطنة لنوى ${}^7\text{Li}$

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الخلاصة

تم دراسة عوامل التشكل المستعرضة للاستطارة الالكتروونية من الحالات المحفزة الواطنة للنواة ${}^7\text{Li}$. عينت هذه الحالات بأعداد الكم $\frac{7}{2}^{-}\frac{1}{2}$ (0.478MeV) و $\frac{5}{2}^{-}\frac{1}{2}$ (6.68MeV). الانتقالات لهذه الحالات تشمل المركبات المتجهة وغيرا لمتجهة في فضاء البرم النظيري. تم تحليل عوامل التشكل في إطار أنموذج القشرة ذي التشكيلات المختلطة للنويات المتعددة للمتذبذب التوافقي مع معلمات الحجم $b_{rms}=1.74\text{fm}$. أستخدم تفاعل الجسيمين ل-Cohen-Kurath للحصول على الدالة الموجية للغلاف Ip. ادخل تأثير استقطاب القلب من خلال عوامل - g الفعالة وقد حلت العديد من التباينات مع التجارب. إدخال تأثير مساهمات المدارات العالية خارج غلاف الفضاء Ip كالغلاف -2p أدت إلى زيادة عوامل التشكل لقيم q وتطابقها مع البيانات. قورنت النتائج الحالية مع نتائج نماذج أخرى.