DOI: http://dx.doi.org/10.21123/bsj.2019.16.1.0111

Results on a Pre-T_2 Space and Pre-Stability

Anmar Hashim Jasim

Received 5/3/2018, Accepted 18/11/2018, Published 11/3/2019

This work is licensed under a Creative Commons Attribution 4.0 International License.

Abstract:

This paper contains an equivalent statements of a pre-T_2 space, where $\Delta = \{(x, x) | x \in X\}$ and $K = \{(x_1, x_2) \in X \times X \mid f(x_1) = f(x_2)\}$ are considered subsets of $X \times X$ with the product topology. An equivalence relation between the preclosed set Δ and a pre-T_2 space, and a relation between a pre-T_2 space and the preclosed set K with some conditions on a function f are found. In addition, we have proved that the graph C of R is preclosed in $X \times X$, if X/R is a pre-T_2 space, where the equivalence relation R on X is open.

On the other hand, we introduce the definition of a pre-stable (γpre-stable) set by depending on the concept of a pre-neighborhood, where we get that every stable set is pre-stable. Moreover, we obtain that a pre-stable (γpre-stable) set is positively invariant (invariant), and we add a condition on this set to prove the converse. Finally, a relationship between, (i) a pre-stable (γpre-stable) set and its component (ii) a pre-T_2 space and a (positively critical point) critical point, are gotten.

Key words: A preirresolute function, A pre-stable (γpre-stable) set, Preopen, Positively invariant, Positively critical point.

Introduction:

This paper consists of three sections, section one is called **"**Introduction", where the contents of this paper are explained and fundamental concepts are given. The concept of a preopen set lies in (1), where they have introduced "a preopen set A when $A \subseteq \overline{A}^o$ and its properties such as the intersection of an open set and a preopen set is preopen".

Also, "the union of any family of preopen sets is a preopen set", you can find it in (2). While "its complement (i.e. preclosed)" you see it in (1). References (1) and (3) show that, "the intersection of all preclosed sets containing A is called the preclosure of A, denoted by $\overline{A}^p$, which is the smallest preclosed set containing A". But (4) and (5) introduce a preneighborhood and a preopen function respectively. On the other side, many discussed preopen sets and their relationship with other sets, which explain in (7). By occasion, the concepts of "invariant, positively invariant and a stable set", you find it in (8). Besides, stability plays a significant role in various areas of life, for example, in the field of pharmacy as in (9), in engineering (10) and (11). In addition to mathematics (12), and (13), besides, in chemistry (14).

Mustansiriyah University, College of Science,
Mathematics Department, Baghdad, Iraq
E-mail: dr.anmar@uomustansiriyah.edu.iq

Moreover, section two has the name "A Relation Between a pre-T_2 Space and Some Sets", where we introduce an equivalent statements of a pre-T_2 space.

In this section we study the relation between a pre-T_2 space and the graph C of R, and the sets $\Delta = \{(x, x) | x \in X\}$, $K = \{(x_1, x_2) \in X \times X \mid f(x_1) = f(x_2)\}$, which are considered subsets of $X \times X$ with the product topology. On the other side, an equivalence relation between the preclosed set Δ and a pre-T_2 space, and a relation between a pre-T_2 space and the preclosed set K with some conditions on a function f are found. In addition, we have proved that the graph C of R is preclosed in $X \times X$, if X/R is a pre-T_2 space, where the equivalence relation R on X is open.

Section three is called "A Pre-Stable Set ", where we introduce the definition of a pre-stable (γpre-stable) set by depending on the concept of pre-neighborhood. We prove theorems on this set, which illustrate their characteristics. For example, we get that every stable set is pre-stable. Moreover, we have obtained that a pre-stable (γpre-stable) set is positively invariant (invariant), and we add a condition on this set to prove the converse. Finally, the relationships between, (i) a pre-stable (γpre-stable) set and its component (ii) a pre-T_2 space and a (positively critical point) critical point, are gotten.

Now we recall the following theorems and definitions that we need:

Theorem 1.1.(2). A topological space X is called pre-T_2 if and only if for each $x, y \in X$ and $x \neq y$, there exist two preopen sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

Theorem 1.2.(3). Let $(X_i)_{i\in I}$ be a family of topological spaces and $\phi \neq A_i \subseteq X_i$ for each $i \in I$. Then $\prod_{i\in I} A_i$ is preopen in $\prod_{i\in I} X_i$ if and only if A_i is preopen in X_i for each $i \in I$ and A_i is a non dense for only finitely many $i \in I$.

Theorem 1.3.(6).(2). A function from a topological space X into a topological space Y is called:
(i) Preirresolute if and only if the inverse image of any preopen set in Y is a preopen set in X.
(ii) Preirresolute if and only if the inverse image of any preclosed set in Y is a preclosed set in X.

Definition 1.4.(2). A function from a topological space X into a topological space Y is called almost preopen if and only if the direct image of any preopen set in X is also a preopen set in Y.

Theorem 1.5.(2). Every homeomorphism function is:
(i) An almost preopen function.
(ii) A preirresolute function.

Definition 1.6.(8).
(i) A set $S \subset X$ is called positively invariant, if $xt \in S$ $\forall x \in S$, $\forall t \in R^+$.
(ii) A set $S \subset X$ is called invariant, if $xt \in S$ $\forall x \in S$, $\forall t \in R$.
(iii) A point $x \in X$ is called a critical point, if $x = xt$ $\forall t \in R$

Relation between a Pre-T_2 Space and Some Sets:

This section is devoted to discuss a relation between a pre-T_2 space and some sets, which is defined on a product topological space. Besides, we include theorems that we have proved in the following, where $\Delta = \{(x,x) | x \in X\}$ and $K = \{(x_1, x_2) \in X\times X \mid f(x_1) = f(x_2)\}$ are considered subsets of $X\times X$ with the product topology.

Theorem 2.1. Let X be a topological space. A subset A of X is preclosed if and only if $\forall x \notin A$ $\exists$ a preopen subset B of X, which contains x and $B \cap A = \emptyset$.
Proof: Clear.

Theorem 2.2. A topological space X is pre-T_2 if and only if the set Δ is preclosed.
Proof: $\Rightarrow$
Let X be a pre-T_2 space and $(x, y) \notin \Delta$, that is $x \neq y$.
Since X is pre-T_2, then $\exists$ A, B $\in PO(X\times X)$ s.t. $x \in A$, $y \in B$ and $A \cap B = \emptyset$.
So $A \times B \in PO(X\times X)$ and $(x, y) \in A \times B$.
Suppose that

$$(A \times B) \cap \Delta = \emptyset .$$

Since if $(z, z) \in (A \times B) \cap \Delta$, then $z \in A \cap B$ which leads to a contradiction.
Hence by 2.1, we get that Δ is a preclosed subset of $\times X$.
$\Leftarrow$ Let Δ be a preclosed subset of $X\times X$.
Let $x, y \in X$ with $\neq y$. Then $(x, y) \notin \Delta$
Since Δ is a preclosed subset of $\times X$, then by 2.1 $\exists$ a preopen subset B of $X \times X$ s.t. $(x, y) \in B$ and $B \cap \Delta = \emptyset$.
Put

$$B = \bigcup_{i\in I} (U_i \times V_i) .$$

Since B is a preopen set, then

$$B \subseteq \overline{\bigcup_{\iota\in I} (U_\iota \times V_\iota)}^{o}$$

$$\boldsymbol{B \subseteq \bigcup_{i\in I} (\overline{U_\iota}^{o} \times \overline{V_\iota}^{o})}$$

Since $(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{B}$, $\exists \boldsymbol{j} \in \boldsymbol{I}$ $\boldsymbol{s.t.}$ $(\boldsymbol{x}, \boldsymbol{y}) \in \overline{\boldsymbol{U_j}}^{o} \times \overline{\boldsymbol{V_j}}^{o}$
Let $\boldsymbol{U_j \times V_j = U \times V}$
Let $\boldsymbol{A}$ be the set which contains $(\boldsymbol{x}, \boldsymbol{y})$ and $(\boldsymbol{U \times V})$
So $\boldsymbol{A \subseteq \overline{U}^{\circ} \times \overline{V}^{\circ}, A \in PO(X \times X)}$
But $\subseteq \boldsymbol{B}$, so $\boldsymbol{A} \cap \Delta = \emptyset$. $(\boldsymbol{i.e.} (\boldsymbol{U \times V}) \cap \Delta = \emptyset)$
Suppose that $\boldsymbol{y \notin U}$.
If $\boldsymbol{y \in U}$ and since $\boldsymbol{y} \in \overline{\boldsymbol{V}}^{\circ}$, we get that

$$\boldsymbol{U \cap V \neq \emptyset}$$

Hence, $(\boldsymbol{U \times V}) \cap \Delta \neq \phi$. Which leads to a contradiction
By the same way $\boldsymbol{x \notin V}$.
Now either $\boldsymbol{x \notin U}$, $\boldsymbol{y \notin V}$.
So there are preopen sets $\boldsymbol{U_1}$ and $\boldsymbol{V_1}$ in $\boldsymbol{X}$ with $\in \boldsymbol{U_1}$, $\boldsymbol{y \in V_1}$
$\boldsymbol{U \cup U_1}$ and $\boldsymbol{V \cup V_1}$ are preopen subsets of $\boldsymbol{X}$ which contains x and y respectively. By 1.2, we have

$$(\boldsymbol{U \cup U_1}) \times (\boldsymbol{V \cup V_1}) \in \boldsymbol{X_1 \times X_2}$$

$$(\boldsymbol{U \cup U_1}) \cap (\boldsymbol{V \cup V_1}) = \emptyset .$$

Or $\boldsymbol{x \in U}$, $\boldsymbol{y \notin V}$. There exists a preopen set $\boldsymbol{V_1}$ which contains y.
$V \cup V_1$ is a preopen subset of X and $U \cap (V \cup V_1) = \emptyset$
By the same way if $x \notin U, y \in V$.
If $x \in U, y \in V$, then U and V are preopen sets, and $U \cap V = \emptyset$
So X is pre-T_2 space.

Remark 2.3. From the proof 2.2, we get that every preopen subset B of $X \times X$ which contains (x, y) with
$B = \cup_{i\in I} (U_i \times V_i)$, there exists a preopen set A which contains (x, y) and $U \times V$ with

$$A \subseteq \overline{U}^{\circ} \times \overline{V}^{\circ}.$$

Theorem 2.4. Let f be a preirresolute function from a topological space X to a pre-T_2 space Y. Then the set K is preclosed.

Proof: Let $(x,y) \notin K. i.e\ f(x) \neq f(y)$.
Since Y is pre-T_2, then there exist A and B preopen subsets of Y such that $f(x) \in A$, $f(y) \in B$, and $A \cap B = \emptyset$.
Since f is preirresolute, then $f^{-1}(A)$ and $f^{-1}(B)$ are preopen subsets of X.
So by 1.2, we get that

$$f^{-1}(A) \times f^{-1}(B) \in PO(X \times X),$$

where $(x, y) \in f^{-1}(A) \times f^{-1}(B)$.
Suppose that

$$(f^{-1}(A) \times f^{-1}(B)) \cap K = \emptyset.$$

Since if $(z_1, z_2) \in (f^{-1}(A) \times f^{-1}(B)) \cap K$, then $f(z_1) = f(z_2)$ and $f(z_1) \in A$, $f(z_2) \in B$. that is $A \cap B \neq \emptyset$, which leads to a contradiction.
So, K is a preclosed subset of $X \times X$.

Theorem 2.5. Let f be a homeomorphism function from a topological space X to a topological space Y such that the set K is preclosed. Then Y is a pre-T_2 space.

Proof: Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$.
Since f is onto, then there exist $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$.
i.e $(x_1, x_2) \notin K$.
By 2.1 and 2.3, we get that there exists $A \in PO(X \times X)$ which contains (x_1, x_2) and $U \times V$ such that
$A \subseteq \overline{U}^{\circ} \times \overline{V}^{\circ}$ and $\cap\ K = \emptyset$.
So

$$(U \times V) \cap K = \emptyset.$$

Suppose that $x_2 \notin U$.
Since if $x_2 \in U$, and we have $x_2 \in \overline{V}^{\circ}$.
That is $U \cap V \neq \emptyset$, so there exists $z \in U \cap V$.
Hence $(z,z) \in U \times V$.
Therefore $(z,z) \in (U \times V) \cap K$ which leads to a contradiction.
By the same way $x_1 \notin$ V.
Now either $x_1 \notin U$ and $x_2 \notin V$.
As in the steps of proof 2.2, we get that $U \cup U_1$ and $V \cup V_1$ are preopen subsets of X which contains $x_1$and x_2 respectively. To prove $hat\ ((U \cup U_1) \times (V \cup V_1)) \cap K = \emptyset$.
Since
$(U \times V) \cap K = \emptyset$,
then
$f(U) \cap f(V) = \emptyset$.
But f is open, so $f(U)$ and $f(V)$ are open subsets of Y.
$\overline{f(U)}^{\circ} \cap f(V) = \emptyset$.
Because of f is continuous, then $f(\overline{U}^{\circ}) \subseteq \overline{f(U)}^{\circ}$.
Hence $f(\overline{U}^{\circ}) \cap f(V) = \emptyset$.
By tha same way $f(U) \cap f(\overline{V}^{\circ}) = \emptyset$.
Now if $((U \cup U_1) \times (V \cup V_1)) \cap K \neq \emptyset$.
That is there exist $(x_1, a) \in ((U \cup U_1) \times (V \cup V_1)) \cap K$.
Either there exists $a \in V$ and $f(a) = f(x_1)$.
But $x_1 \in \overline{U}^{\circ}$, therefore $f(x_1) \in f(\overline{U}^{\circ}) \cap f(V)$ which leads to a contradiction.
Or there exists $(b, x_2) \in ((U \cup U_1) \times (V \cup V_1)) \cap K$. And by the same way we get a contradiction.
Hence,

$$((U \cup U_1) \times (V \cup V_1)) \cap K = \emptyset.$$

Since f is a homeomorphism, then by 1.6(i), we get that $f(U \cup U_1)$ and $f(V \cup V_1)$ are preopen subsets of Y which contains $y_1$and y_2 respectively.
Suppose that

$$f(U \cup U_1) \cap f(V \cup V_1) = \emptyset$$

If there exists $z \in f(U \cup U_1) \cap f(V \cup V_1)$, then there exists $a_1 \in U \cup U_1$ such that $f(a_1) = z$ and there exists $b_1 \in V \cup V_1$ such that $f(b_1) = z$.
Hence $(a_1, b_1) \in K$.
So
$(a_1, b_1) \in [(U \cup U_1) \times (V \cup V_1)] \cap K$,
which leads to a contradiction.
Therefore
$f(U \cup U_1) \cap f(V \cup V_1) = \emptyset$.
Or $x_1 \in U$ and $x_2 \notin V$.
There existsV_1which contains x_2. So $V \cup V_1$ is a preopen subset of X.
Since f is a homeomorphism, then by 1.6(i), we get that $f(U)$ and $f(V \cup V_1)$ are preopen subsets of Y which contains y_1 and y_2 respectively and

$$f(U) \cap f(V \cup V_1) = \emptyset.$$

Similarly if $x_1 \notin U$ and $x_2 \in V$.
If $x_1 \in U$ and $x_2 \in V$, then $f(U)$ and $f(V)$ are preopen subsets of Y which contain $y_1$and $y_2$respectively and

$$f(U) \cap f(V) = \emptyset.$$

Hence Y is a pre-T_2 space.

Corollary 2.6. Let f be a homeomorphism function from a topological space X in to a topological space Y. Then Y is a pre-T_2 space if and only if the set K is preclosed.
Proof: Clear

Theorem 2.7. Let X be a topological space and R be an equivalence relation on X where R is open. Then the graph C of R is preclosed in $X \times X$ if X/R is a pre-T_2space.
Proof:
Let $\psi: X \to X/R$ be a quotient function.
Let Δ be a diagonal set of $X/R \times X/R$. Then

$$(\psi \times \psi)^{-1}(\Delta) = C.$$

Since X/R is pre-T_2 , then by 2.2, we get that Δ is a preclosed subset of $X/R \times X/R$.
So by 1.3(ii) and 1.6(ii), we get that $(\psi\times\psi)^{-1}(\Delta)$ is a preclosed subset of $X\times X$.
Hence C is a preclosed subset of $X\times X$.

Pre-Stable Set:

In this section we introduce a pre-stable set, a γpre-stable set, and a positively critical point, where we get some results, that illustrate their characteristics. The section includes what we have obtained as follows:

Definition 3.1. We say that a set S is pre-stable if every pre-neighborhood U of S has a positively invariant pre-neighborhood V of S.

Theorem 3.2. Every stable set is pre-stable.
Proof: Clear.

Theorem 3.3. A set S is positively invariant if it is pre-stable.
Proof:
Let $x \in S, and\ U$ is a pre-neighborhood of x, such that $U \subseteq S$.
Since S is pre-stable, then there exists a positively invariant pre-neighborhood V of x and $V \subseteq U$.
So

$$xt \in V \subseteq S \quad \forall t \in R^+$$

Therefore, S is positively invariant.

Definition 3.4. We say that a set S is γpre-stable if every pre-neighborhood U of S has an invariant pre-neighborhood V of S.

Corollary 3.5. A set S is invariant if it is γpre-stable.
Proof: Clear.

Definition 3.6. We say that a point $x \in X$ is a positively critical point, if $x = xt \quad \forall t \in R^+$.

Theorem 3.7. If a set S is positively invariant, where every point in S is a positively critical point, then S is pre-stable.
Proof:
Suppose that $x \in S$, and suppose that U is a pre-neighborhood of x such that $U \subseteq S$.
Since S is positively invariant, then

$$xt \in S \quad \forall t \in R^+$$

But we have x a positively critical point, so $x = xt \quad \forall t \in R^+$
Hence, it is easy to find a positively invariant pre-neighborhood V of x with $V \subseteq U$
Which means that a set S is pre-stable.

Corollary 3.8. If a set S is invariant, where every point in S is a critical point, then S is γpre-stable.
Proof: Clear.

Theorem 3.9. A set S is pre-stable if and only if every component of S is pre-stable.
Proof: Set
$S = \cup_i S_i\ , \quad i = 1,2, \dots, n.$
Now let S be pre-stable.
Let $x_i \in S, and\ U_i$ be a pre-neighborhood of x_i such that $U_i \subseteq S_i$, $i = 1, \dots, n.$
Since S is pre-stable, then there exists a positively invariant pre-neighborhood V_i of x_i and $V_i \subseteq U_i$, $i = 1,2, \dots, n.$ So

$$x_i t_i \in V_i \subseteq U_i\ , \quad \forall t_i \in R^+ , i = 1,2, \dots, n.$$

Which means that S_i is pre-stable.
Conversely, suppose that S_i is pre-stable.
Let $x \in S, and\ U$ is a pre-neighborhood of , such that

$$U \subseteq S_i \subseteq S\ , \qquad i = 1,2, \dots, n.$$

Since S_i is pre-stable, then there exist positively invariant pre-neighborhood V of x and $V \subseteq U$.
That means $xt \in V \subseteq S \quad \forall t \in R^+$, therefore S is pre-stable.

Corollary 3.10. A set S is γpre-stable if and only if every component of S is γpre-stable.
Proof: Clear.

Theorem 3.11. (i) A point $x \in X$ is not a critical point, if X is a pre- T_2 space.
(ii) A point $x \in X$ is not a positively critical point, if X is a pre- T_2 space.
Proof: (i)Clear, since if x is a critical point, then $x = xt \quad \forall\ t \in R$.
There are pre-neighborhoods U and V of x and xt respectively.
But $U \cap V \neq \emptyset$, which is a contradiction with X.
Hence $x \in X$ is not a critical point.
By the same way (ii) $x \in X$ is not a positively critical point, with replace R by R^+.

Acknowledgements:
I gratefully acknowledge the staff of the Mathematics Department/ College of Science/ Mustansiriyah University for supporting this work.

Conflicts of Interest: None.

References:

1. Mashhour AS, Abd El-Monsef ME, El-Deeb SN. On Pretopological Spaces. Bull. Math. De la soc. R.s.de Roumanie. 1984;28(78):39-45.
2. Al-Khazraji RB. On Semi P open sets. Thesis submitted to the college of education / Ibn Al-Haitham. Baghdad. Baghdad University;2004.

3. Mohammed S, Elatrash, S. F. Preopen and Semi Open in T_o-Alexandroff Space. El al., J. Al-Aqsa Unv.2006:10(S.E.):396-414.
4. Navalagi GB. Pre-neighborhoods. The Mathematics Education. ISSN:0047-6269, 1998; 32(4): 201-206. Available from. https://www.researchgate.net/publication/230688959_Pre-Neighbourhoods
5. Mashhour AS, Abd El-Monsef ME, El-Deeb SN. On Precontinuous and Weak Precontinuous functions. Proc. Math.Phys.Soc. Egypt; 1982;53:47-53.
6. Reilly IL, and Vamanmurthy MK. On α-continuity in Topological Spaces. Acta. Math. Hungar. 1985; 45(1-2);27-32.
7. Carlos C, Ennis R, Margot S, Jose S. Minimal open sets on generalized topological space. Proyecciones Journal of Mathematics Vol. 36, No 4, pp. 739-751, December 2017. Universidad Catolica del Norte Antofagasta - Chile.
8. Bhatia NP, and Szego GP. Stability Theory of Dynamical Systems;1970.12,13,58 p.
9. Suthar N., and Choudhary M. A review on stability studies of pharmaceutical products. ,IJAPBR. 2017; 2(3): 67-75
10. Farzin T, Peter MD, Dragan N, Ying T. A local characterization of Lyapunov functions on Riemannian manifolds (math.OC) [internet]. 2014;Jun24. Available from. arXiv:1311.0078v2
11. Saeed M. Stability and Control in Complex Networks of Dynamical Systems. Electronic Theses and Dissertations. 692. University of Central Florida[internet]. 2015. Available from. http://stars.library.ucf.edu/etd/692
12. Ruszewski, A. Practical Stability and Asymptotic Stability of Interval Fractional Discrete-time Linear State-space System. In: Szewczyk, R., Zieliński, C., Kaliczyńska, M. (eds.) Recent Advances in Automation, Robotics and Measuring Techniques. AISC, vol. 267, pp. 217–228. Springer, Heidelberg (2014).
13. Mikolaj B, Andrzej R. Robust Stability Check of Fractional Discrete-Time Linear Systems with Interval Uncertainties. Springer International Publishing Switzerland 2015. Available from. https://link.springer.com/chapter/10.1007/978-3-319-09900-2_18
14. Mohamed H, Hamid M, Tilal E. A Stability-Indicating RP-HPLC-UV Method for Determination and Chemical Hydrolysis Study of a Novel Naproxen Prodrug. Journal of Chemistry. Volume 2017, Article ID 5285671, 10 pages. Published 25 October 2017 https://doi.org/10.1155/2017/5285671.

نتائج على الفضاء $\text{Pre-}T_2$ و الاستقرارية-pre

انمار هاشم جاسم

قسم الرياضيات، كلية العلوم، الجامعة المستنصرية.

الخلاصة:

يحتوي هذا البحث بعض المكافئات لفضاء $\text{pre-}T_2$ ، باعتبار ان $K = \{(x_1, x_2) \in X \times X \mid f(x_1) = f(x_2)\}$ و $\Delta = \{(x, x) \mid x \in X\}$ هي مجموعات جزئية من التبولوجية الجدائية $X \times X$. حيث حصلنا على علاقة تكافئ بين المجموعة Δ مغلقة - pre و الفضاء X $\text{pre-}T_2$ ، و علاقة بين الفضاء $\text{pre-}T_2$ و المجموعة K مغلقة -pre مع بعض الشروط على الدالة f . علاوة لذلك، وجدنا بان بيان C للعلاقة R يكون مغلقة - pre في $X \times X$ اذا كان الفضاء $\text{pre-}T_2$ و علاقة التكافئ R على X مفتوحة.

من ناحية اخرى ، قدمنا تعريف المجموعة المستقرة- pre (المستقرة -γpre) باستخدام مفهوم الجوار-pre حيث حصلنا بان كل مجموعة مستقرة تكون مستقرة - pre. و المجموعة المستقرة-pre (المستقرة -γpre) تكون موجبة invariant (invariant) وقد وضعنا شرط على هذه المجموعة لبرهنة العكس. اخيرا حصلنا على علاقة بين (i) المجموعة S مستقرة- pre (المستقرة -γpre) وعناصرها(ii) الفضاء $\text{pre-}T_2$ والنقطة الحرجة (النقطة الحرجة الموجبة).

الكلمات المفتاحية : الدالة preirresolute ، المجموعة المستقرة- pre (المستقرة -γpre)، المفتوحة - pre ، الموجبة invariant ، النقطة الحرجة الموجبة.