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Numerical Solution of Fractional Volterra-Fredholm Integro-Differential Equation Using Lagrange Polynomials

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Abstract:

In this study, a new technique is considered for solving linear fractional Volterra-Fredholm integro-differential equations (LFVFIDE's) with fractional derivative qualified in the Caputo sense. The method is established in three types of Lagrange polynomials (LP's), Original Lagrange polynomial (OLP), Barycentric Lagrange polynomial (BLP), and Modified Lagrange polynomial (MLP). General Algorithm is suggested and examples are included to get the best effectiveness, and implementation of these types. Also, as special case fractional differential equation is taken to evaluate the validity of the proposed method. Finally, a comparison between the proposed method and other methods are taken to present the effectiveness of the proposal method in solving these problems.

Key words: Fractional Volterra-Fredholm Integro-Differential Equations, Lagrange polynomials.

Introduction:

Fractional integro-differential equations (FIDE's) occur in many applications in the sciences (physics, engineering, finance, biology) (1). In most of the problems the analytical solution cannot be found, and hence finding a good approximate solution using numerical methods will be very helpful (2).

Many researchers studied and discussed the numerical solution of FVFIDE's. Mittal *and* Nigam (1) in 2014 used Adomian decomposition approach to find numerical solution to FIDE's of Volterra type with Caputo fractional derivative. Huang *et al* (3) in 2011 used Taylor expansion series for solving (approximately) a class of linear fractional integro-differential equations including two types Fredholm and Volterra. Mohammed (2) in 2014 investigated numerical solution of LFIDE's by the least squares method with the aid of shifted Chebyshev polynomial. Maleknejad *et al* (4) in 2013 presented a numerical scheme, based on the cubic B-spline wavelets for solving fractional integro-differential equations. Mohamed *et al* (5) in 2016 introduced an analytical method, called homotopy analysis transform method (HATM) which is a combination of HAM and Laplace decomposition method, this scheme is applied to linear and nonlinear fractional integro-differential equations.

Shwayyea *and* Mahdy (6) in 2016 investigated the numerical solution of linear fractional integro-differential equations by the least squares method with the aid of shifted Laguerre polynomial. Oyedepo *et al* (7) in 2016 proposed two numerical methods for solving FIDE's the proposed methods are the least squares method with the aid of Bernstein polynomials function as the basis. Senol *and* Kasmaei (8) in 2017 developed with perturbation-iteration algorithm to obtain approximate solutions of some FIDEs. Alkan *and* Hatipoglu (9) in 2017 study sinc-collocation method for solving Volterra-Fredholm integrodifferential equations of fractional order. Syam (10) in 2017 modified the version of the fractional power series method to extract an approximate solution of the model. The method is a combination of the generalized fractional Taylor series and the residual functions. Hamoud *and* Ghadle (11) in 2018 applied the Adomian decomposition and the modified Laplace Adomian decomposition methods to find the approximate solution of a nonlinear FVFIDE. Hamoud *et al* (12) in 2018 studied the existence and uniqueness theorems for FVFIDE's.

Recently, many researchers have been using Lagrange polynomials to get numerical solution to

different types of problems. Wang and Wang (13) in 2013 used Lagrange collocation method to solve Volterra–Fredholm integral equations, this method transforms the system of the linear integral equations into matrix form via Lagrange collocation points. Mustafa and Muhammad (14) in 2014 introduced a numerical method for solving linear Volterra-Fredholm integro-differential equations of the 1st order using three types of Lagrange polynomial including OLP, MLP and BLP. Mustafa and Ghanim (15) in 2014 used Lagrange polynomials for solving linear Volterra-Fredholm integral equations by three types including OLP, MLP and BLP. Liu *et al* (16) in 2017 solved the two-dimensional linear Fredholm integral equations of the second kind by combining the meshless barycentric Lagrange interpolation functions and the Gauss-Legendre quadrature formula. Pan and Huang (17) in 2017 presented a modified barycentric rational interpolation method for solving two-dimensional integral equations. Tian and He (18) in 2018 used barycentric rational interpolation collocation method to solve higher-order boundary value problems. Wu *et al* (19) in 2018 find numerical solution of a class of nonlinear partial differential equations using Barycentric interpolation collocation method.

This study aims to find numerical solutions of LFFVIDE of the following form:

$$D^\alpha u(x) = q(x)u(x) + f(x) + \int_a^x k_1(x,t)u(t)dt + \int_a^b k_2(x,t)u(t)dt, \quad 0 < \alpha < 1 \quad (1)$$

with initial condition

$$u(a) = u_0 \quad (2)$$

Where $D^\alpha u(x)$ denote the 'Caputo fractional derivative' of $u(x)$; $f(x), q(x), k_1(x, t)$ and $k_2(x, t)$ are continuous functions, x and t are real variables in $[a, b]$ and $u(x)$ is the indefinite function to be determined using OLP, MLP and BLP.

Section (2), introduces some necessary definitions and mathematical preliminaries which are required for establishing our results. While section (3) presents the derivation of the proposed methods. Section (4) proposes the general algorithm for the method. Test examples are given in section (5) including general and special cases of LFFVIDE to improve the capability of the proposed method to solve various type of equation in addition with LFFVIDE, in all the test examples $u(x)$ is chosen in such a way that we know the exact solution. The exact solution is used only to show that the numerical solution obtained with our method is true.

Preliminaries

There are different definitions of 'fractional integral' sometimes defined in $(0, \infty)$ and sometimes called the left- sided integrals are given below:

Definition 1

The 'Caputo definition' of the fractional-order derivative of function $f: [a, b] \rightarrow R$ is defined as:

$$D^\alpha f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{f^{(m)}(t)}{(x-t)^{\alpha+1-m}} dt \quad (3)$$

where α is the order of the derivative, $m - 1 < \alpha < m$ and $m \in \mathbb{Z}^+$ is the smallest integer greater than α (7).

Definition 2

Define OLP for a $n+1$ data points $\{(x_0, u_0), (x_1, u_1), \dots, (x_n, u_n)\}$ as follows (20):

$$p_n(x) = \sum_{j=0}^n u_j L_{n,j}(x) \quad (4)$$

such that:

$$L_{n,j}(x) = \prod_{\substack{k=0 \\ j \neq k}}^n \frac{(x-x_k)}{(x_j-x_k)} \quad (5)$$

Definition 3

Define MLP for $n+1$ data points $\{(x_0, u_0), (x_1, u_1), \dots, (x_n, u_n)\}$ as follows (21):

$$p_n(x) = \sum_{j=0}^n w_j u_j M_{n,j}(x) \quad (6)$$

such that:

$$M_{n,j}(x) = l(x) \frac{1}{\sum_{j=0}^n (x-x_j)}, \quad l(x) = \prod_{\substack{k=0 \\ k \neq j}}^n (x-x_k), \quad \text{and} \quad w_j = \frac{1}{\prod_{k \neq j} (x_j-x_k)}, \quad j = 0, \dots, n \quad (7)$$

Definition 4

Define BLP for $n+1$ data points $\{(x_0, u_0), (x_1, u_1), \dots, (x_n, u_n)\}$ as follows (22):

$$p_n(x) = \frac{\sum_{k=0}^n \frac{w_k}{x-x_k} u_k}{\sum_{k=0}^n \frac{w_k}{x-x_k}} = \sum_{j=0}^n u_j B_{n,j}(x) \quad (8)$$

such that:

$$B_{n,j}(x) = \frac{\frac{w_j}{x-x_j}}{\sum_{k=0}^n \frac{w_k}{x-x_k}}, \quad \text{and} \quad w_j = \frac{1}{\prod_{k \neq j} (x_j-x_k)}, \quad j = 0, \dots, n \quad (9)$$

Methodology

Solutions of Eq. (1) by using OLP, MLP and BLP will presented in this section

OLP Formula

To find the solution of Eqns. (1-2) using OLP, First substitute Eq. (4) in Eq. (1) to get:

$$\sum_{j=0}^n u_j L_{n,j}^\alpha(x) = f(x) + q(x) \sum_{j=0}^n u_j L_{n,j}(x) + \int_a^x k_1(x,t) \left(\sum_{j=0}^n u_j L_{n,j}(t) \right) dt + \int_a^b k_2(x,t) \left(\sum_{j=0}^n u_j L_{n,j}(t) \right) dt \quad (10)$$

therefore, in the sense of Caputo derivative definition, we get:

$$L_{n,j}^\alpha(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{L_{n,j}^{(m)}(t)}{(x-t)^{\alpha+1-m}} dt = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{\left(\prod_{\substack{k=0 \\ j \neq k}}^n \frac{(t-x_k)}{(x_j-x_k)}\right)^{(m)}}{(x-t)^{\alpha+1-m}} dt \quad (11)$$

where m is the m^{th} derivative, $m-1 < \alpha < m$ and $m \in \mathbb{Z}^+$. Using the properties of polynomial Eq. (11) can be determined exactly.

Therefore, after joining coefficients of $u_k, k = 0, 1, \dots, n$, we obtain:

$$f(x) = u_0 \left[L_{n,0}^\alpha(x) - q(x)L_{n,0}(x) - \int_a^x k_1(x,t) L_{n,0}(t) dt - \int_a^b k_2(x,t) L_{n,0}(t) dt \right] + u_1 \left[L_{n,1}^\alpha(x) - q(x)L_{n,1}(x) - \int_a^x k_1(x,t) L_{n,1}(t) dt - \int_a^b k_2(x,t) L_{n,1}(t) dt \right] + u_2 \left[L_{n,2}^\alpha(x) - q(x)L_{n,2}(x) - \int_a^x k_1(x,t) L_{n,2}(t) dt - \int_a^b k_2(x,t) L_{n,2}(t) dt \right] + \dots + u_n \left[L_{n,n}^\alpha(x) - q(x)L_{n,n}(x) - \int_a^x k_1(x,t) L_{n,n}(t) dt - \int_a^b k_2(x,t) L_{n,n}(t) dt \right] \quad (12)$$

Substitute $x = x_i$ in Eq. (12) for $i = 1, 2, \dots, n$, to obtain the following system of n linear equations:

$$D\vec{U} = \vec{C} \quad (13)$$

Where

$$D = d_{ij}, \vec{C} = c_i \text{ and } \vec{U} = [u_1 \ u_2 \ \dots \ u_n]^T$$

With

$$d_{ij} = L_{n,j}^\alpha(x_i) - q(x_i)L_{n,j}(x_i) - \int_a^{x_i} k_1(x_i,t) L_{n,j}(t) dt - \int_a^b k_2(x_i,t) L_{n,j}(t) dt \quad (14)$$

and

$$c_i = f(x_i) - u_0 \left[L_{n,1}^\alpha(x_i) - q(x_i)L_{n,1}(x_i) - \int_a^{x_i} k_1(x_i,t) L_{n,1}(t) dt - \int_a^b k_2(x_i,t) L_{n,1}(t) dt \right] \quad (15)$$

For all $i, j = 1, 2, \dots, n$.

MLP Formula

In the same way as in OLP we can get a system of n linear equation as in Eq. (13) with:

$$d_{ij} = w_j \left[M_{n,j}^\alpha(x_i) - q(x_i)M_{n,j}(x_i) - \int_a^{x_i} k_1(x_i,t) M_{n,j}(t) dt - \int_a^b k_2(x_i,t) M_{n,j}(t) dt \right] \quad (16)$$

and

$$c_i = f(x_i) - w_0 u_0 \left[M_{n,1}^\alpha(x_i) - q(x_i)M_{n,1}(x_i) - \int_a^{x_i} k_1(x_i,t) M_{n,1}(t) dt - \int_a^b k_2(x_i,t) M_{n,1}(t) dt \right] \quad (17)$$

For all $i, j = 1, 2, \dots, n$.

Where

$$M_{n,j}^\alpha(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{M_{n,j}^{(m)}(t)}{(x-t)^{\alpha+1-m}} dt = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{\left(\frac{l(t)}{\sum_{j=0}^n \frac{1}{(t-x_j)}}\right)^{(m)}}{(x-t)^{\alpha+1-m}} dt \quad (18)$$

BLP Formula

we can get a system of n linear equation as in Eq. (13) with:

$$d_{ij} = B_{n,j}^\alpha(x_i) - q(x_i)B_{n,j}(x_i) - \int_a^{x_i} k_1(x_i,t) B_{n,j}(t) dt - \int_a^b k_2(x_i,t) B_{n,j}(t) dt \quad (19)$$

and

$$c_i = f(x_i) - u_0 \left[B_{n,1}^\alpha(x_i) - q(x_i)B_{n,1}(x_i) - \int_a^{x_i} k_1(x_i,t) B_{n,1}(t) dt - \int_a^b k_2(x_i,t) B_{n,1}(t) dt \right] \quad (20)$$

For all $i, j = 1, 2, \dots, n$.

Where

$$B_{n,j}^\alpha(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{B_{n,j}^{(m)}(t)}{(x-t)^{\alpha+1-m}} dt = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{\left(\frac{w_j}{\frac{t-x_j}{t-x_k}}\right)^{(m)}}{(x-t)^{\alpha+1-m}} dt \quad (21)$$

General Algorithm for Methods

To evaluate numerical solutions of LFFVIDE using OLP, MLP and BLP, the following steps are introduced:

Step 1: assume $h = \frac{b-a}{n}, n \in \mathbb{N}, u(a) = u_0$ (the initial condition is given).

Step 2: put $x_i = a + ih$, with $x_0 = a$ and $x_n = b$, $i = 0, 1, \dots, n$.

Step 3: To find the values of a linear system $D\vec{U} = \vec{C}$, using step (1) and (2), three cases are considered:

Case (1) Using OLP: choose Eq. (14) and Eq. (15).

Case (2) Using MLP: choose Eq. (16) and Eq. (17).

Case (3) Using BLP: choose Eq. (19) and Eq. (20).

(Note that for the Caputo fractional derivative and integral in all equations, we use the exact value computed in MATLAB).

Step 4: Solve the system ($D\vec{U} = \vec{C}$) using step3 and Gauss elimination method with partial pivoting.

Note that, we can use another method to solve the system in step 4 like LU decomposition method, but the computational cost of computing a solution via Gaussian elimination or LU is the same.

Numerical Applications

In this section, four numerical examples are considered to confirm the efficiency of the above methods for solving LFVFIDE's. MATLAB\R2018a are used to apply the algorithms.

Example 1: Consider the FLVFIDE:

$$D^\alpha u(x) = q(x) u(x) + f(x) + \int_0^x \sin t u(t) dt + \int_0^1 (x+t) u(t) dt \tag{22}$$

with initial condition $u(0) = 0$.

for $f(x) = \frac{\sin(2x)}{4} - \frac{x}{2} + \cos(1) - \sin(1) + 1.1077321674324726030747001459531x^{2/3}$

$$\text{hypergeom}\left(1.0, \left[\frac{5}{6}, \frac{4}{3}\right], -0.25x^2\right) - \sin(x) (\cos(x) + e^x) + x(\cos(1) - 1), \quad q(x) = \cos x + e^x \text{ and } \alpha = \frac{1}{3}.$$

with the true solution $u(x) = \sin(x)$. where

hypergeom represent the generalized hyper geometric function in MATLAB.

Table 1 shows the absolute error by using OLP, MLP, and BLP with n=5. Table 2 contains the maximum error by using OLP, MLP, and BLP with n=4,5,8, 10. Such that $\|err\|_\infty$ represents the maximum absolute error and R.T. represents running time.

Table 1. The Absolute Error of Example (1) by using OLP, MLP, and BLP with n=5.

X	OLP	MLP	BLP
0.2000	5.77313732e-07	5.772650612e-07	5.772650612e-07
0.4000	2.408428215e-08	2.411134951e-08	2.411134951e-08
0.6000	4.25561146e-08	4.244503536e-08	4.244503536e-08
0.8000	6.215314534e-07	6.213904772e-07	6.213904772e-07
1.0000	3.33474767644e-06	3.334822103e-06	3.334822103e-06
$\ err\ _\infty$	3.33474767644e-06	3.334822103e-06	3.334822103e-06
R.T.	39.7608	24.8563	26.1071

Table 2. The Max. Error of Example (1) by using OLP, MLP, and BLP for n=4,5,8,10.

N	OLP	MLP	BLP
4	5.50151947949e-04	5.50151947949e-04	7.3189e-05
5	3.33474767644e-06	3.334822103e-06	3.334822103e-06
8	5.06372719754e-10	5.06372275665e-10	
10	4.52084592472e-13	4.4764370037e-13	

Example 2: Consider the FLFIDE (23):

$$D^\alpha u(x) = q(x) u(x) + f(x) + \int_0^1 x e^t u(t) dt \tag{23}$$

with the initial condition $u(0) = 0$.

Where

$$f(x) = \frac{-3x^{1/6} \Gamma(5/6)(-91+216x^2)^{91\pi}}{6} + (5 - 2e)x, \quad q(x) = 0 \text{ and } \alpha = \frac{5}{6}.$$

With the true solution $u(x) = x - x^3$. (Note that in this case $k_f(x,t)=0$). where Γ represent the gamma function.

Table 3 represents the absolute error by using OLP, MLP, and BLP with n=5. Table 4 contains the maximum error by using OLP, MLP, and BLP. with n=4,5,8,10, with the best results obtain in (23) using Laguerre polynomials.

Table 4. The Absolute Error of Example (2) by using OLP, MLP, and BLP with n=5.

x	OLP	MLP	BLP
0.2000	0	0	0
0.4000	0	0	0
0.6000	0	0	0
0.8000	0	0	0
1.0000	5.1000212345e-16	5.1000212345e-16	5.100021235e-16
$\ err\ _\infty$	5.1000212345e-16	5.1000212345e-16	5.100021235e-16
R.T.	28.5014	16.9414	25.9871

Table 4. The Maximum Error of Example (2) by using OLP, MLP, and BLP with n=4,5,8,10.

N	OLP	MLP	BLP	OLP
4	3.2477452607e-16	3.2477452607e-16	1.293181118e-10	8.5722×10^{-7}
5	5.1000212345e-16	5.1000212345e-16	5.100021235e-16	
8	4.1440180104e-15	2.6172745328e-15		
10	3.5980141117e-14	6.7699476050e-14		

Example 3: Consider the FLVIDE (24):

$$D^\alpha u(x) = q(x)u(x) + f(x) + \int_0^x e^x t u(t) dt \quad (24)$$

With the initial condition $u(0) = 0$.

(Note that in this case $k_2(x,t)=0$).

Where

$$f(x) = \frac{6x^{2.25}}{\Gamma(3.25)}, \quad q(x) = -\frac{x^2 e^x}{5} \quad \text{and} \quad \alpha = 0.75$$

.With the exact solution $u(x) = x^3$.

Table 5 represents the absolute error by using OLP, MLP, and BLP with n=5. Table 6 contains the maximum error by using OLP, MLP, and BLP with n=4,5,8,10.

Table 5. The Absolute Error of Example (3) by using OLP, MLP, and BLP with n=5.

X	OLP	MLP	BLP
0.2000	0	0	0
0.4000	0	0	0
0.6000	0	0	0
0.8000	0	0	0
1.0000	0	0	0
$\ err\ _\infty$	0	0	0
R.T.	19.7803	20.6366	36.8587

Table 6. The Maximum Error of Example (3) by using OLP, MLP, and BLP with n=4,5,8,10.

N	OLP	MLP	BLP
4	0	0	0
5	0	0	0
8	5.40366362766e-16	5.566293953540e-16	
10	9.32368418582e-15	1.193253308662e-14	

Example 4: Consider the LFDE (24):

$$D^\alpha u(x) = q(x)u(x) + f(x) \quad (25)$$

With the initial condition $u(0) = 0$

Where $f(x) = \frac{2}{\Gamma(3-\alpha)} x^{2-\alpha} - \frac{1}{\Gamma(2-\alpha)} x^{1-\alpha} + x^2 - x$

, $q(x) = -1$, and $\alpha = 0.5$.

With the exact solution $u(x) = x^2 - x$.

(Note that in this case $k_1(x,t)=0$ and $k_2(x,t)=0$).

Table 7 represents the absolute error by using OLP, MLP, and BLP with n=5. Table 8 contains the maximum error by using OLP, MLP, and BLP with n=4,5,8,10, with the best results obtain in (24) using fractional Euler's method.

Table 7. The Absolute Error of Example (4) by using OLP, MLP, and BLP with n=5.

X	OLP	MLP	BLP
0.2000	0	0	0
0.4000	0	0	0
0.6000	0	0	0
0.8000	0	0	0
1.0000	2.1527833558 E-17	2.1527833558 E-17	2.152783356 E-17
$\ err\ _\infty$	2.1527833558 E-17	2.1527833558 E-17	2.152783356 E-17
R.T.	30.1743	13.8447	21.4953

Table 8. The Maximum Error of Example (4) by using OLP, MLP, and BLP with n=4,5,8,10.

n	OLP	MLP	BLP	Result in (24)(min error)
4	0	0	0	4.1574e-05
5	2.1527833558 E-17	2.1527833558 E-17	2.152783356 E-17	
8	3.3036527958e-17	1.6518263979e-17		
10	1.98693656252e-16	3.3777921562e-16		

Conclusion:

In this study, Lagrange polynomials including: (OLP), (MLP), (BLP) are applied for solving the LFVFIDE's.

According to results from examples, we conclude that:

- The three methods offer the force and capability of the introduced algorithms.
- The faster method is MLP due to the point that computing the integral part appearing in Eq. (18) is easier than that in Eq. (11) and Eq. (21), and this result agrees with references (14) and (15).
- As n (the degree of polynomials) increases, the error term is decreased in all methods except when the exact solution is a polynomial with low degree (we can satisfy with low degree).
- Also from Tables (2, 4 and 6) one can note that the BLP method has no results for $n=8$ and $n=10$ because of difficulty for finding the fractional derivatives which are so hard to compute exactly by hand or by using MATLAB within reason of shape of BLP appear in Eq.(18) and this result does not agree with the other papers since the fractional derivative appears here only not in the other .
- Due to the same reason, we suggest using numerical integration instead of exact value to avoid the difficulty of finding the integration in Eq. (21).
- Methods can be extended and applied to nonlinear FVFIDE, in this case the problem transforms to nonlinear system of equation which can be solved by using Newton's method.

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Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

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الحل العددي لمعادلة فولتيرا – فريدهولم التكاملية - التفاضلية الكسورية باستخدام متعددات حدود لاكرانج

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الخلاصة:

في هذا البحث، ستراتيجيات جديدة لإيجاد الحل العددي للمعادلات الخطية الكسورية التفاضلية - التكاملية فولتيرا- فريدهولم (LFVFIDE) تم دراستها. الطرق المتبعة على ثلاث انواع من متعددات الحدود لاكرانج وهي: متعددة حدود لاكرانج الأصلية (OLP) ، متعددة حدود لاكرانج ذات الدعامة المركزية (BLP) و متعددة حدود لاكرانج المعدلة (MLP). كما تم اقتراح خوارزمية عامة واعطاء أمثلة لبرهنة فعالية الطرق وتنفيذها. وأخيراً، تم استخدام مقارنة بين الطرق المقترحة والطرق الأخرى لحل هذا النوع من المعادلات.

الكلمات المفتاحية: المعادلات الكسورية التكاملية - التفاضلية فولتيرا-فريدهولم، متعددات حدود لاكرانج.