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A comparison among Different Methods for Estimating Regression Parameters with Autocorrelation Problem under Exponentially Distributed Error

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Abstract

Multiple linear regressions are concerned with studying and analyzing linear relationship between the dependent variable and a set of explanatory variables. From this relationship, the values of variables are predicted. In this paper the multiple linear regression model and three covariates were studied in the presence of the problem of auto-correlation of errors when the random error was distributed over the distribution of exponential. Three methods were compared (general least squares, M robust, and Laplace robust method). Simulation studies have been employed and calculated the statistical standard mean squares error with different sample sizes. Further the best method has been applied on real data representing the varieties of cigarettes according to the US Federal Trade Commission.

Key words: Autocorrelation, Exponential distribution, General least squares method, Laplace robust method, M robust method, Multiple linear regression.

Introduction

Multiple linear regressions are concerned with studying and analyzing the linear relationship between a dependent variable and two or more explanatory variables. Predicting the variable values of the phenomenon can be obtained. One of the problems of the linear model is an autocorrelation between the values of the random error variable. This affects the results of regression analysis. Many authors dealt with this problem in their researches like Ahmed (1), Eakambaram and Elangovan (2), Dietz (3), and Ebtisam (4). The purpose of this study is to compare the methods of the general least squares, M robust, and Laplace robust method using the simulation approach to the multi-linear regression model with three variables. A first order has been considered autocorrelation problem exists and a random error distributed with exponential distribution with two parameters. This distribution is an important statistical distribution particularly in applications and has extensive uses such as in determining the reliability of equipment, analysis of survival time, and other applications (5).

By assuming that the linear regression model with three variables is as follows:

$$Y_i=\gamma_0+\gamma_1 X_{1i}+\gamma_2 X_{2i}+\gamma_3 X_{3i}+e_i \quad , \quad i=1,2,\dots,n \qquad \textbf{(1)}$$

Where Y is the dependent variable, X_1, X_2, X_3 the explanatory variables, $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ the regression parameters, and e is a random error with first order autocorrelation $e_i=\phi e_{i-1}+\delta_i$ (6). Where ϕ is the simple autocorrelation coefficient $(-1\le\phi\le1)$, and δ is a random error distributed exponentially with probability density function:

$$f(\delta)=\alpha^{-1}e^{-((\delta-\mu)/\alpha)} \quad , \quad \delta\succ\mu \; , \; \alpha\succ0 \; , \; \mu\ge0 \qquad \textbf{(2)}$$

μ Location Parameter, α Scale Parameter and the cumulative density function of the distribution:

$$F(\delta)=1-e^{-((\delta-\mu)/\alpha)} \qquad \textbf{(3)}$$

the problem of autocorrelation can be solved by using the transformation method, so model (1) will be as follows:

$$Y_i - \phi Y_{i-1} = (1-\phi)\gamma_0 + \gamma_1(X_{1i} - \phi X_{1i-1})$$
$$+ \gamma_2(X_{2i} - \phi X_{2i-1}) + \gamma_3(X_{3i} - \phi X_{3i-1}) + \delta_i \quad (4)$$

$$Y_i^* = \gamma_0^* + \gamma_1 X_{1i}^* + \gamma_2 X_{2i}^* + \gamma_3 X_{3i}^* + \delta_i ,$$
$$\gamma_0^* = (1-\phi)\gamma_0 \quad (5)$$

General Least Squares Method (GLS): the formula for estimating the parameters of this method using matrices is as follows:

$$\tilde{\gamma} = \left(x^{*'} x^{*}\right)^{-1}\left(x^{*'} y^{*}\right) \quad (6)$$

Where X is a regular matrix according to the model in (4)

M –Robustness Method: the formula for estimating the parameters of this method is as follows:

$$\hat{\gamma} = \left(x^{*'} \lambda x^{*}\right)^{-1}\left(x^{*'} \lambda y^{*}\right) \quad (7)$$

λ: diagonal weight matrix has its components is as follows:

$$\lambda_i = \psi(\varepsilon)/\varepsilon \quad (8)$$

$$\varepsilon = \frac{\left(y_i^* - x_i^{*'} \gamma_d\right)}{\hat{\sigma}}$$

and (γ_d) is a vector of a primary parameter ($\hat{\sigma}$) is a scale parameter and is estimated as a follows (7):

$$\hat{\sigma} = 1.483[(median)|\delta - median(\delta)|] \quad (9)$$

And using Huber function as (8):

$$\rho(\delta) = \begin{bmatrix} \delta^2/2 \\ v|\delta| - v^2/2 \end{bmatrix} , \quad \psi(\delta) = \begin{bmatrix} \delta & |\delta| \le v \\ v\,sign(\delta) & |\delta| \succ v \end{bmatrix}$$

Whereas $(v = 1.345)$ and *sign* is a sign of (δ)

Laplace Robustness Method (Lp): this method depends on minimizing the absolute values of the sum of residuals as follows (9):

$$Min \sum_{i=1}^{n} |\delta_i|^k \qquad 1 \le k \le 2 \quad (10)$$

The value of k=1.8 and the parameters are represented by $(\gamma_0^L, \gamma_1^L, \gamma_2^L, \gamma_3^L)$.

Here k is a fitted value in which after many experiments the researchers found when the value of k=1.8, the results of the absolute values of the differences should be minimum. This means that the Lp method is a robust method with this value.

Simulation: by using matlab program the experiment has been repeated (10000) times, and for different sample sizes (n=15, 30, 60,100) with different values for correlation coefficients (ϕ =0.1, 0.5, 0.85, -0.1, -0.5, -0.85). The initial values have been taken of the parameters $(\gamma_0 = 0.02,\ \gamma_1 = 0.5, \gamma_2 = 2,\ \gamma_3 = 1)$. The supposed values of parameters distribution are $(\mu = 1, \alpha = 2,3)$. The values of variable (δ) are obtained from $(\alpha(-Ln(1-rand)) + \mu)$ and for the random variables are obtained from the following forms (10) (11):

$$X_1 \sim Rand , \qquad X_2 \sim Rand ,$$
$$X_3 \sim Rand , \qquad X_{10} \sim Rand/\sqrt{1-\phi^2} ,$$
$$X_{20} \sim Rand/\sqrt{1-\phi^2} ,$$
$$X_{30} \sim Rand/\sqrt{1-\phi^2} ,$$
$$Y_0 \sim \delta_0/\sqrt{1-\phi^2}$$

(Y_0) and (X_{10}, X_{20}, X_{30}) represent the first observations of variables that have been preserved. Tables 1,2,3, 4 show the values of mean squares error of parameter and for all the methods when the error is distributed as an exponential distribution.

Table 1: Values of Mean Square Error for Parameters and Sample Size (n=15) (μ=1)

			n=15					
α	Methods		ϕ					
			0.1	0.5	0.85	-0.1	-0.5	-0.85
2	GLS	γ_0	2.1041	0.6405	0.0667	3.6911	11.3688	30.2511
		γ_1	0.3097	0.3881	0.4362	0.3111	0.4627	0.6224
		γ_2	0.3012	0.3743	0.4197	0.3014	0.4406	0 .5597
		γ_3	0.3021	0.3793	0.4263	0.3023	0.4459	0 .5750
	M	γ_0	1.9626	0.6078	0.0648	3.4644	0.8972	29.3296
		γ_1	0.2884	0.3693	0.4175	0.2892	0.4410	0 .6024
		γ_2	0.2801	0.3548	0.4016	0.2796	0.4202	0 .5420
		γ_3	0.2803	0.3586	0.4074	0.2799	0.4241	0.5546
	Lp	γ_0	1.8942	0.5947	0.0647	3.3556	0.6865	28.7522
		γ_1	0.2794	0.3619	0.4096	0.2807	0.4349	0.5955
		γ_2	0.2724	0.3464	0.3935	0.2712	0.4167	0.5386
		γ_3	0.2711	0.3494	0.3991	0.2707	0.4180	0.5462
3	GLS	γ_0	4.5272	1.4203	0.1277	7.4823	20.3286	43.6605
		γ_1	0.6905	0.8480	0.9445	0.6919	0.9410	1.1440
		γ_2	0.6698	0.8170	0.9044	0.6702	0.9011	1.0607
		γ_3	0.6740	0.8288	0.9226	0.6727	0.9062	1.0719
	M	γ_0	4.2132	1.3405	0.1227	7.0013	9.6047	42.8187
		γ_1	0.6425	0.8042	0.8983	0.6433	0.9067	1.1272
		γ_2	0.6219	0.7728	0.8603	0.6216	0.8684	1.0420
		γ_3	0.6246	0.7812	0.8767	0.6230	0.8715	1.0492
	Lp	γ_0	4.0622	1.3040	0.1216	6.7802	9.3391	42.5377
		γ_1	0.6228	0.7859	0.8756	0.6241	0.8989	1.1275
		γ_2	0.6037	0.7533	0.8378	0.6031	0.8653	1.0452
		γ_3	0.6027	0.7588	0.8534	0.6025	0.8643	1.0468

Table 2: Values of Mean Square Error for Parameters and Sample Size (n=30) (μ=1)

			n=30					
α	Methods		ϕ					
			0.1	0.5	0.85	-0.1	-0.5	-0.85
2	GLS	γ_0	0.5384	0.1411	0.0208	1.0621	3.2537	8.4638
		γ_1	0.0640	0.0836	0.0968	0.0636	0.0886	0.1117
		γ_2	0.0611	0.0814	0.1030	0.0613	0.0887	0.1112
		γ_3	0.0643	0.0839	0.0987	0.0641	0.0886	0.1085
	M	γ_0	0.4861	0.1281	0.0201	0.9769	3.0918	8.0629
		γ_1	0.0575	0.0753	0.0876	0.0572	0.0826	0.1057
		γ_2	0.0552	0.0741	0.0946	0.0551	0.0825	0.1060
		γ_3	0.0580	0.0757	0.0894	0.0577	0.0828	0.1035
	Lp	γ_0	0.4802	0.1250	0.0200	0.9608	3.0617	7.9320
		γ_1	0.0575	0.0730	0.0846	0.0572	0.0823	0.1051
		γ_2	0.0552	0.0724	0.0920	0.0550	0.0822	0.1055
		γ_3	0.0578	0.0737	0.0867	0.0577	0.0826	0.1037
3	GLS	γ_0	1.1440	0.3143	0.0340	2.0494	5.6284	12.2805
		γ_1	0.1432	0.1834	0.2091	0.1424	0.1891	0.2222
		γ_2	0.1366	0.1767	0.2130	0.1372	0.1875	0.2236
		γ_3	0.1439	0.1844	0.2124	0.1432	0.1890	0.2189
	M	γ_0	1.0284	0.2803	0.0315	1.8720	5.3502	11.8449
		γ_1	0.1284	0.1645	0.1869	0.1281	0.1777	0.2131
		γ_2	0.1232	0.1598	0.1924	0.1234	0.1757	0.2146
		γ_3	0.1295	0.1657	0.1901	0.1291	0.1782	0.2111
	Lp	γ_0	1.0148	0.2709	0.0308	1.8437	5.3161	11.7658
		γ_1	0.1284	0.1587	0.1789	0.1283	0.1776	0.2131
		γ_2	0.1229	0.1555	0.1853	0.1231	0.1758	0.2145
		γ_3	0.1291	0.1609	0.1830	0.1290	0.1788	0.2121

Table 3: Values of Mean Square Error for Parameters and Sample Size (n=60) (μ=1)

			n=60					
α	**Methods**		ϕ					
			0.1	**0.5**	**0.85**	**-0.1**	**-0.5**	**-0.85**
2	**GLS**	γ_0	**0.1637**	**0.0334**	**0.0076**	**0.3704**	**2.8006**	**2.8006**
		γ_1	**0.0145**	**0.0191**	**0.0224**	**0.0146**	**0.0241**	**0.0241**
		γ_2	**0.0148**	**0.0210**	**0.0303**	**0.0148**	**0.0299**	**0.0299**
		γ_3	**0.0143**	**0.0193**	**0.0238**	**0.0142**	**0.0239**	**0.0239**
	M	γ_0	**0.1459**	**0.0303**	**0.0076**	**0.3407**	**2.6583**	**2.6583**
		γ_1	**0.0129**	**0.0168**	**0.0197**	**0.0130**	**0.0226**	**0.0226**
		γ_2	**0.0132**	**0.0189**	**0.0278**	**0.0132**	**0.0288**	**0.0288**
		γ_3	**0.0128**	**0.0171**	**0.0213**	**0.0127**	**0.0226**	**0.0226**
	Lp	γ_0	**0.1468**	**0.0303**	**0.0077**	**0.3400**	**2.6503**	**2.6503**
		γ_1	**0.0134**	**0.0166**	**0.0195**	**0.0133**	**0.0229**	**0.0229**
		γ_2	**0.0136**	**0.0188**	**0.0276**	**0.0136**	**0.0291**	**0.0291**
		γ_3	**0.0132**	**0.0170**	**0.0211**	**0.0130**	**0.0228**	**0.0228**
3	**GLS**	γ_0	**0.3423**	**0.0727**	**0.0096**	**0.6783**	**1.8990**	**4.0453**
		γ_1	**0.0325**	**0.0421**	**0.0484**	**0.0326**	**0.0434**	**0.0503**
		γ_2	**0.0331**	**0.0443**	**0.0571**	**0.0332**	**0.0456**	**0.0572**
		γ_3	**0.0321**	**0.0423**	**0.0500**	**0.0319**	**0.0425**	**0.0494**
	M	γ_0	**0.3036**	**0.0630**	**0.0092**	**0.6176**	**1.7984**	**3.8664**
		γ_1	**0.0290**	**0.0368**	**0.0419**	**0.0291**	**0.0404**	**0.0477**
		γ_2	**0.0296**	**0.0394**	**0.0510**	**0.0297**	**0.0426**	**0.0547**
		γ_3	**0.0287**	**0.0373**	**0.0438**	**0.0284**	**0.0395**	**0.0469**
	Lp	γ_0	**0.3057**	**0.0627**	**0.0092**	**0.6168**	**1.7974**	**3.8574**
		γ_1	**0.0299**	**0.0365**	**0.0413**	**0.0299**	**0.0411**	**0.0483**
		γ_2	**0.0303**	**0.0392**	**0.0503**	**0.0305**	**0.0435**	**0.0554**
		γ_3	**0.0295**	**0.0370**	**0.0431**	**0.0292**	**0.0402**	**0.0473**

Table 4: Values of Mean Square Error for Parameters and Sample Size (n=100 (μ=1)

			n=100					
α	**Methods**		ϕ					
			0.1	**0.5**	**0.85**	**-0.1**	**-0.5**	**-0.85**
2	**GLS**	γ_0	**0.0777**	**0.0122**	**0.0040**	**0.1929**	**0.6107**	**1.4025**
		γ_1	**0.0051**	**0.0067**	**0.0080**	**0.0051**	**0.0069**	**0.0081**
		γ_2	**0.0051**	**0.0079**	**0.0132**	**0.0051**	**0.0081**	**0.0126**
		γ_3	**0.0051**	**0.0071**	**0.0092**	**0.0051**	**0.0072**	**0.0089**
	M	γ_0	**0.0694**	**0.0113**	**0.0041**	**0.1787**	**0.5852**	**1.3391**
		γ_1	**0.0046**	**0.0059**	**0.0070**	**0.0046**	**0.0064**	**0.0077**
		γ_2	**0.0046**	**0.0071**	**0.0122**	**0.0046**	**0.0076**	**0.0124**
		γ_3	**0.0046**	**0.0062**	**0.0082**	**0.0046**	**0.0067**	**0.0085**
	Lp	γ_0	**0.0704**	**0.0114**	**0.0041**	**0.1800**	**0.5889**	**1.3456**
		γ_1	**0.0048**	**0.0060**	**0.0070**	**0.0047**	**0.0065**	**0.0078**
		γ_2	**0.0048**	**0.0071**	**0.0123**	**0.0048**	**0.0078**	**0.0125**
		γ_3	**0.0048**	**0.0063**	**0.0082**	**0.0048**	**0.0069**	**0.0087**
3	**GLS**	γ_0	**0.1609**	**0.0257**	**0.0041**	**0.3432**	**0.9745**	**2.0230**
		γ_1	**0.0115**	**0.0149**	**0.0172**	**0.0115**	**0.0151**	**0.0173**
		γ_2	**0.0115**	**0.0160**	**0.0225**	**0.0115**	**0.0163**	**0.0222**
		γ_3	**0.0116**	**0.0153**	**0.0186**	**0.0116**	**0.0154**	**0.0183**
	M	γ_0	**0.1429**	**0.0220**	**0.0041**	**0.3148**	**0.9280**	**1.9390**
		γ_1	**0.0103**	**0.0130**	**0.0148**	**0.0102**	**0.0140**	**0.0164**
		γ_2	**0.0103**	**0.0141**	**0.0201**	**0.0103**	**0.0154**	**0.0215**
		γ_3	**0.0104**	**0.0134**	**0.0161**	**0.0103**	**0.0144**	**0.0175**
	Lp	γ_0	**0.1452**	**0.0222**	**0.0041**	**0.3187**	**0.9354**	**1.9508**
		γ_1	**0.0107**	**0.0131**	**0.0148**	**0.0106**	**0.0144**	**0.0167**
		γ_2	**0.0107**	**0.0142**	**0.0202**	**0.0107**	**0.0158**	**0.0218**
		γ_3	**0.0109**	**0.0135**	**0.0161**	**0.0108**	**0.0149**	**0.0178**

All through these Tables, it can be noticed that when the sample sizes (15, 30, 60) and with the expansion in the estimation of auto connection coefficients and the estimation of the scale parameter it's inferred that the Lp method is the best. The M method follows it and after that comes GLS. At the point when the extent of the sample is 100 it is seen that M method is generally useful. At that point comes the Lp lastly GLS. All these comparisons depend on the MSE value and when the error distributed exponentially.

Application: the data represent cigarette varieties according to US Federal Trade Commission classification for a sample of 25 brands [9]. Y represents the first carbon oxide, X_1 represents the Tar, X_2 represents nicotine, and X_3 is the carbon and the data as follows:

Y = 13.6, 16.6, 23.5, 10.2, 5.4, 15, 9, 12.3, 16.3, 15.4, 13, 14.4, 10, 10.2, 9.5, 1.5, 18.5, 12.6, 17.5, 4.9, 15.9, 8.5, 10.6, 13.9, 14.9.

X_1 = 14.1, 16, 29.8, 8, 4.1, 15, 8.8, 12.4, 16.6, 14.9, 13.7, 15.1, 7.8, 11.4, 9, 1, 17, 12.8, 15.8, 4.5, 14.5, 7.3, 8.6, 15.2, 12.

X_2 = 0.86, 1.06, 2.03, 0.67, 0.40, 1.04, 0.76, 0.95, 1.12, 1.02, 1.01, 0.90, 0.57, 0.78, 0.74, 0.13, 1.26, 1.08, 0.96, 0.42, 1.01, 0.61, 0.69, 1.02, 0.82 .

X_3 = 0.9853, 1.0938, 1.1650, 0.9280, 0.9462, 0.8885, 1.0267, 0.9225, 0.9372, 0.8858, 0.9643, 0.9316, 0.9705, 1.1240, 0.8517, 0.7851, 0.9186, 1.0395, 0.9573, 0.9106, 1.0070, 0.9806, 0.9693, 0.9496, 1.1184 .

The data has been tested using Kolmogorov-Smirnov test when (μ=1, α=1, 2). It was noticed that the data is distributed exponentially where the value of D= 0.2457. The tabulated value with 0.05 significance level = 0.272 after applying the standard degree (Z-score) for the dependent variable Y and for the variables X_1, X_2 and X_3. From the Durbin Watson Tables with sample size = 25 and under 0.05 significant level it is found that the lower value of the test d_l = 1.12 and d_u=1.66. The value of D = 2.860 is included in $\left(4-d_u \prec D.W \prec 4-d_l\right)$. This shows that the test is unclear. So the authors estimated the auto correlation coefficient and it was equal 0.958. As it was given that the value of $\left(Y_0\right)$, $\left(X_{30}, X_{20}, X_{10}\right)$ are zero. In Laplace method the value of the linear regression parameters with auto correlation $\hat{\gamma}_0 = 0.0068$, $\hat{\gamma}_1 = 1.1597$, $\hat{\gamma}_2 = -0.1518$ and $\hat{\gamma}_3 = -0.0833$.

Conclusion

Tables (1), (2), (3) and (4) show that the Laplace method is the best when the auto correlation coefficients increase with different sample sizes (15, 30, 60). When the sample size =100 the M method is the best. And the value of MSE is less when increasing the sample sizes. The value of MSE of parameters is increased when the positive value of auto correlation coefficient increase $(\phi = 0.1, 0.5, 0.85)$. While it is decreased with the negative value of auto correlation coefficients $(\phi = -0.1, -0.5, -0.85)$ for different sample sizes and for all values of (α).

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- The author has signed an animal welfare statement.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

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مقارنة بين الطرائق المختلفه لتقدير معالم الانحدار بوجود مشكلة الارتباط الذاتي عندما يتوزع الخطأ توزيعا اسيا

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الخلاصه:

يهتم الانحدار الخطي المتعدد بدراسة وتحليل العلاقة بين المتغير المعتمد ومجموعة من المتغيرات المستقلة ومن هذه العلاقة يتم التنبؤ بقيم المتغيرات, وفي هذا البحث تم دراسة انموذج الانحدار الخطي المتعدد ولثلاثة متغيرات بوجود مشكلة الارتباط الذاتي للاخطاء عندما يتوزع الخطأ العشوائي توزيع اسيا, وتمت المقارنة بين ثلاثة طرائق (طريقة المربعات الصغرى العامة, طريقة (M) الحصينة وطريقة لابلاس الحصينة) باستعمال المحاكاة ومن خلال المقياس الاحصائي متوسط مربعات الخطأ ولاحجام العينات (15, 30, 60, 100) وتطبيق افضل طريقة على بيانات تجربة حقيقية تمثل اصناف السجائر وفقا لتصنيف لجنة التجارة الفيدرالية الامريكية.

الكلمات المفتاحيه: الانحدار الخطي المتعدد, الارتباط الذاتي , التوزيع الاسي , طريقة المربعات الصغرى العامة, طريقة (M) الحصينه, طريقة لابلاس الحصينة.