

## Experimental Evidence of Chaotic Resonance in Semiconductor Laser

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### Abstract:

In this paper, an experimental study has been conducted regarding the indication of resonance in chaotic semiconductor laser. Resonant perturbations are effective for harnessing nonlinear oscillators for various applications such as inducing chaos and controlling chaos. Interesting results have been obtained regarding to the effect of the chaotic resonance by adding the frequency on the systems. The frequency changes nonlinear dynamical system through a critical value, there is a transition from a periodic attractor to a strange attractor. The amplitude has a very relevant impact on the system, resulting in an optimal resonance response for appropriate values related to correlation time. The chaotic system becomes regular under a moderate frequencies or amplitudes. These dynamics of the laser output are analyzed by time series, FFT and bifurcation diagram as a result.

**Keywords:** Chaos, Control, Frequency, Optoelectronic feedback, Resonance.

### Introduction:

Chaos is "a term used to denote the irregular behavior of dynamical systems arising from a strictly deterministic time evolution without any source of noise or external stochasticity (1). Dynamic chaos can be defined as a remarkable non-linear phenomenon that was thoroughly examined throughout the past forty years. The chaotic behavior could be identified in many systems studied in various areas, such as engineering, neural sciences, and physics (2). Control theory is a somehow new topic, yet it is extremely an advanced branch of non-linear sciences that deal with studying input-feedback processes. It has the ability of impacting the behavior of dynamical systems. To generate chaos in semiconductor lasers, non-linearity is considered very important (3). The solitary laser dynamics are ruled by two coupled variables (intensity and population inversion) evolving with two very different characteristic timescales. The introduction of a third degree of freedom (and a third timescale) describing the ac-feedback loop, leads to a three-dimensional slow-fast system displaying a transition from a stable steady state to periodic spiking sequences as the dc-pumping current is varied. For intermediate values

of the current, a regime is found where regular large pulses are separated by fluctuating time intervals in a scenario of resembling chaos (4, 5). Through changing the intensity of laser diode, the optical chaotic carrier could be created. The dynamic operating state related to the laser could be accomplished via many approaches such as optoelectronic feedback and optical feedback (6). Concerning the optoelectronic feedback, a photo detector responds just to the intensity of laser output. Optoelectronic feedback is an efficient technique producing and controlling nonlinear dynamics in semiconductor laser (7).

The problem of controlling chaos in a physical system, that is to vary the state of the system from the chaotic behavior to a periodic time dependence which is predictable, has attracted attention. Control of chaos indicates a process in which a tiny perturbation is applied to a chaotic system, in order to sense a desirable (chaotic, periodic, or stationary) behavior. Control of non-linear dynamical system includes significant ideas such as stability, optimality or uncertainty. A distinct connection of optimization and control was developed with the idea of chaos. Many researches

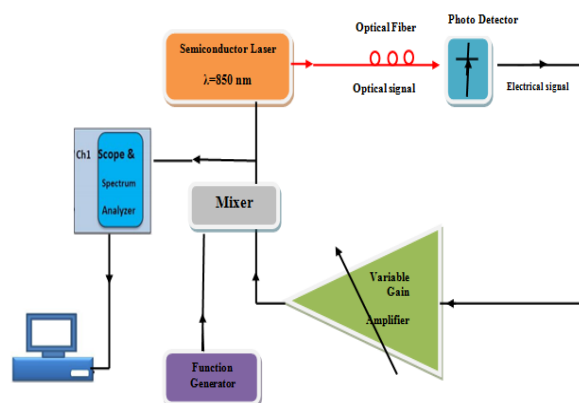
have been being published in the past years on control chaotic systems; i.e., to stabilize certain periodic motion from the chaos (8). It was proved that, for periodically forced non-linear oscillator, a resonant perturbation with suitable strength might result in that a periodic attractor will replace the originally chaotic attractor (9). Resonant perturbations can be considered as a proven and efficient approach to control dynamics related to non-linear oscillators in many applications (10). Chaotic Resonance is a recognized function of chaos in neural systems, where systems respond to weak signals via the impact of chaotic activities (11). Regarding stochastic resonance (SR), the existence of noise helps the non-linear system in amplifying weak (sub-threshold) signals. Chaotic Resonance might be considered comparable with SR but with no stochastic noise, that was identified in neural systems (12). Resonant perturbations are effective for harnessing nonlinear oscillators. Of physical interest is the effect of small frequency mismatch on the attractors of the underlying dynamical systems. Xingang Wang et al. examined the impact of small frequency mismatch on attractors related to underlying dynamical system. He used prototype model of non-linear oscillators and the periodically forced Duffing oscillator (9). Sora et al. examined the changing of dynamical state of a nonlinear system (semi-conductor laser) by adding noise. These changes depend on the noise amplitude (2). Sou Nobukawa et al. proposed an approach to control signal response of Chaotic Resonance through modifying the strength related to external feedback control (12). Pisarchik indicated that a small mismatch between natural frequencies of the unidirectionally coupled with chaotic oscillators could induce CR in the slave oscillator for a particular coupling strength (13). The future of chaotic evolutions cannot be predicted, because of a small deviation of the initial condition in a nonlinear system that results in a completely different solution of the system output.

In the present study, chaos can be made controllable by adding an external perturbation which achieves the chaotic resonance in semiconductor laser. The experimental system is built to examine the effect of frequency and amplitude on a chaotic semiconductor laser with optoelectronic feedback. The first condition is, when the frequency of the external perturbation is varied, secondly, when the amplitude of this perturbation is changed.

### Materials and Methods:

The experimental setup for semiconductor laser with optoelectronic feedback in order to study

the nonlinear dynamics of the laser is schematically shown in Fig.1. It is an optical system with closed loop, including single semi-conductor laser (hp / Agilent model 8150A optical signal source) with ac-coupled optoelectronic feedback. The laser provides an emission with a wavelength of 850 nm and continuous output power of 2mW. The output laser light is sent to a photo detector through an optical fiber producing an electrical signal. The generated electrical current is proportional to optical intensity. The generated electrical signal is subsequently amplified by a variable amplifier, and then added to the bias current of SL by using a mixer. After that, the electrical signal is feedback to the semiconductor laser. The laser output is modulated via external signal using a function generator.



**Figure 1. Experimental setup with optoelectronic feedback (OEFB) for chaos modulation in chaotic dynamics.**

The output signal from the variable gain amplifier was modulated by an external perturbation through a periodic signal with two control parameters frequencies. The modulated frequency could be seen from the FFT. Different time series could be obtained by changing either the frequency or amplitude of perturbation. The bifurcation diagram was drawn for each parameter to identify the differences between them.

The experimental part included the following procedure: the net amplifier gain of the entire feedback loop and the dc-pumping current have been fixed. The output signal is a sinusoidal signal that has two control parameters: amplitude and frequency.

To generate chaos in semiconductor lasers, non-linearity and three fold dimensionality are considered as conditions of high significance. Nonlinear dynamic is an interdisciplinary area of science, which deals with the study of the systems described by mathematical equations.

The dynamics regarding carrier density  $N$  and field density  $S$  is defined via rate equations of single-mode semi-conductor laser (4):

$$S' = [g(N - N_t) - \gamma_0]S \quad 1$$

$$N' = \frac{I_0}{eV} + \frac{f_F(I)}{eV} - \gamma_c N - g(N - N_t)S \quad 2$$

$$I' = -\gamma_f I + kS' \quad 3$$

Where  $S'$  = the photon density or output laser ray intensity

$N'$  = the population inversion,  $I'$  = The optoelectronic feedback.

$I$  = high-pass filtered feedback before the nonlinear amplifier,

$V$  = volume of active layer,  $N_t$  = carrier density at transparency,

$I_0$  = bias current,  $e$  = electron charge,

$f_F(I) \equiv AI/(1+S'I)$  = feedback amplifier function,  $\gamma_c$  = population relaxation rate,  $k$  = a proportional coefficient to the responsivity of the photo detector,  $g$  = differential gain,  $\gamma_0$  = photon damping,  $\gamma_f$  = the high-pass filter cut off frequency.

The field density and carrier density are two linked variables used to describe the total dynamics in the system. These variables have two extremely distinct characteristic time-scales. Application of optoelectronic delay feedback shows two benefits: firstly, adds a third degree of freedom in our system, secondly, adds a third slower time-scale.

There is a need to rewrite Eq.'s 1, 2 and 3 in dimensionless form for analytical and numerical purposes; therefore, new variables are inserted:

$$x = \frac{g}{\gamma_c} S, y = \frac{g}{\gamma_0} (N - N_t), w = \frac{g}{k\gamma_c} I - x \text{ and the time scale } t' = \gamma_0 t$$

Then, the rate equations become:

$$x' = x(y - 1) \quad 4a$$

$$y' = \gamma(\delta_0 - y + f(w + x) - xy) \quad 4b$$

$$w' = -\varepsilon(w + x) \quad 4c$$

Where  $S = \gamma_c S/k/g$  is the saturation coefficient,  $\delta_0 = (I_0 - I_t)/(I_{th} - I_t)$  is the bias current,

$f(w + x) \equiv \alpha \frac{w+x}{1+S(w+x)}$ , ( $I_{th} = eV\gamma_c (\frac{\gamma_0}{g} + N_t)$ ) is the current of laser

$\alpha = Ak/(eV\gamma_0)$  is the strength of feedback,

$\varepsilon = \frac{\gamma_0}{\gamma_0}$  the bandwidth at resonant frequency  $w_0$ ,  $\gamma = \frac{\gamma_c}{\gamma_0}$

For more simplifications of dimensionless equations, let  $z = w + x$ .

The nonlinear dynamics of SLs with optoelectronic feedback (OEFD) are represented in Eq.'s 4 (a, b, and c), where equation 4a represents the photon density or output laser ray intensity and equation 4b represents the population inversion. The feedback needed to generate chaos is represented by eq. 4c; this feedback is composed of the bias current and the intensity of laser output.

A new term  $(1+K)$  for chaos modulation is added to eq. 4c, where  $K$  is an external perturbation. Then, the equations become:

$$x' = x(y - 1) \quad 5a$$

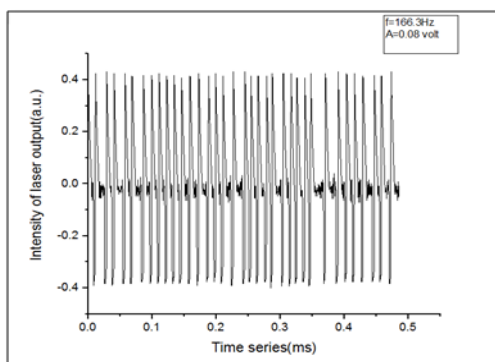
$$y' = \gamma \left( \delta_0 - y + \alpha \frac{z}{1+Sz} - xy \right) \quad 5b$$

$$w' = -\varepsilon(w + x)(1 + K) \quad 5c$$

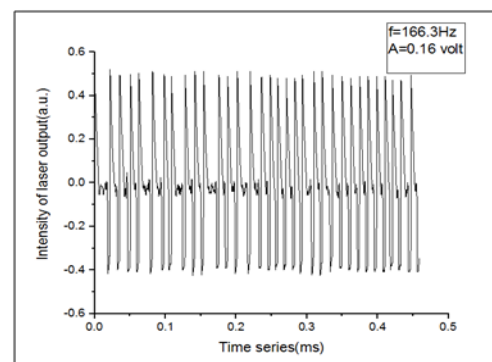
Where  $K = A \sin(2\pi ft)$ ,  $A$  is the amplitude of perturbation and  $f$  is the frequency of perturbation.

## Results and discussion

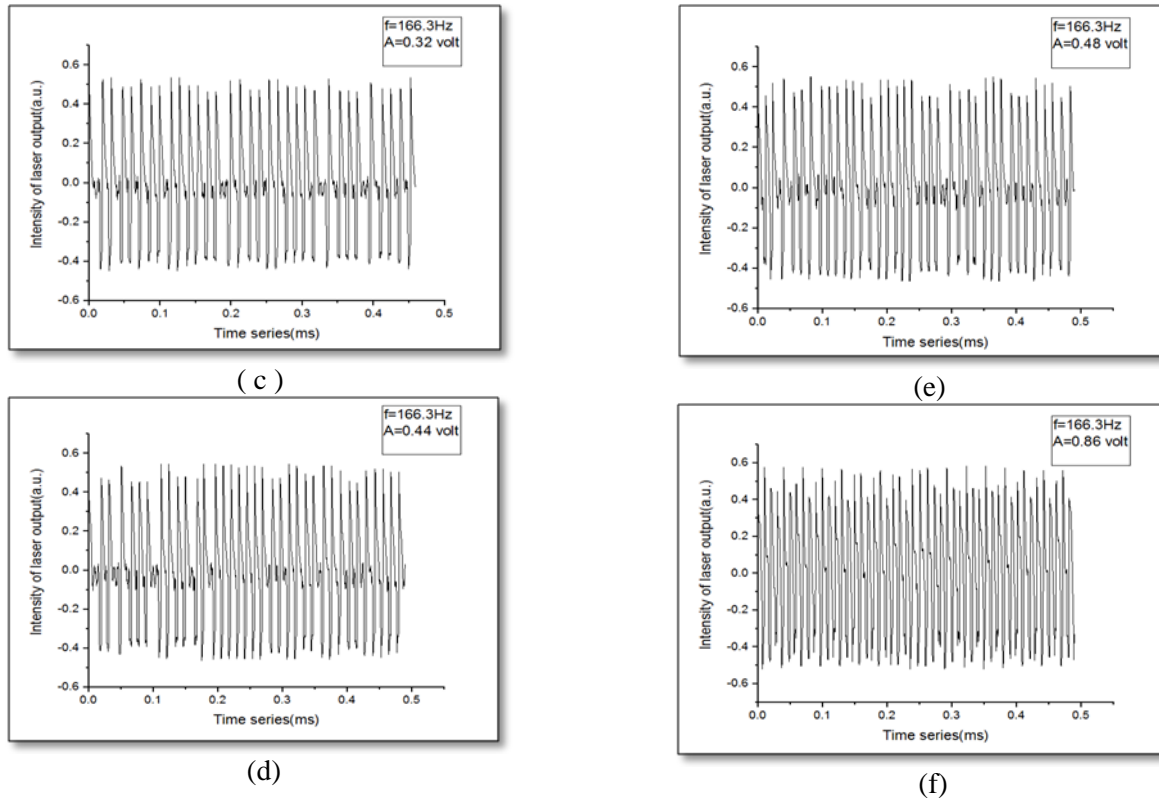
External chaotic modulation is an important method to control chaos. In our study, the modulation is applied on the source by a function generator as mentioned in the experimental part. Figure 2 includes the time series of different amplitude where the modulation frequency has been fixed at 167Hz which represents the minimum values of the frequency added which is convert the behavior of the system while the amplitude is changing.



(a)



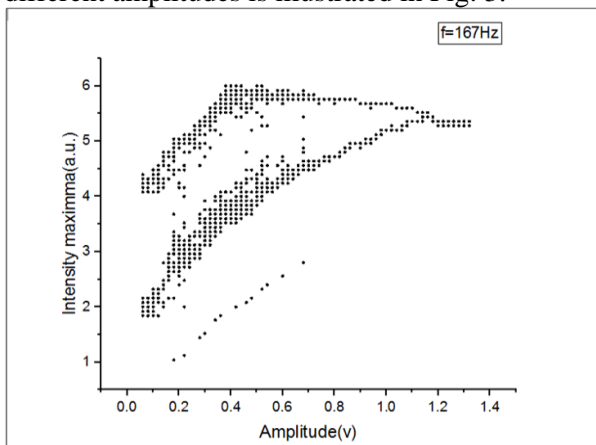
(b)



**Figure 2. Time series at fixed frequency 167Hz and amplitude values (a) 0.08 V, (b) 0.16 V, (c) 0.32 V, (d) 0.44 V (e) 0.48 V, (f) 0.86 V.**

Figure 2 shows that the system transition from chaotic to regular state. The regularity of nonlinear dynamic system is related to the optimal amount of frequency and increment of the amplitude .

The bifurcation diagram is utilized for checking the chaotic routes and evolutions of output in non-linear systems for variations of control parameter; therefore bifurcation diagram for different amplitudes is illustrated in Fig. 3.



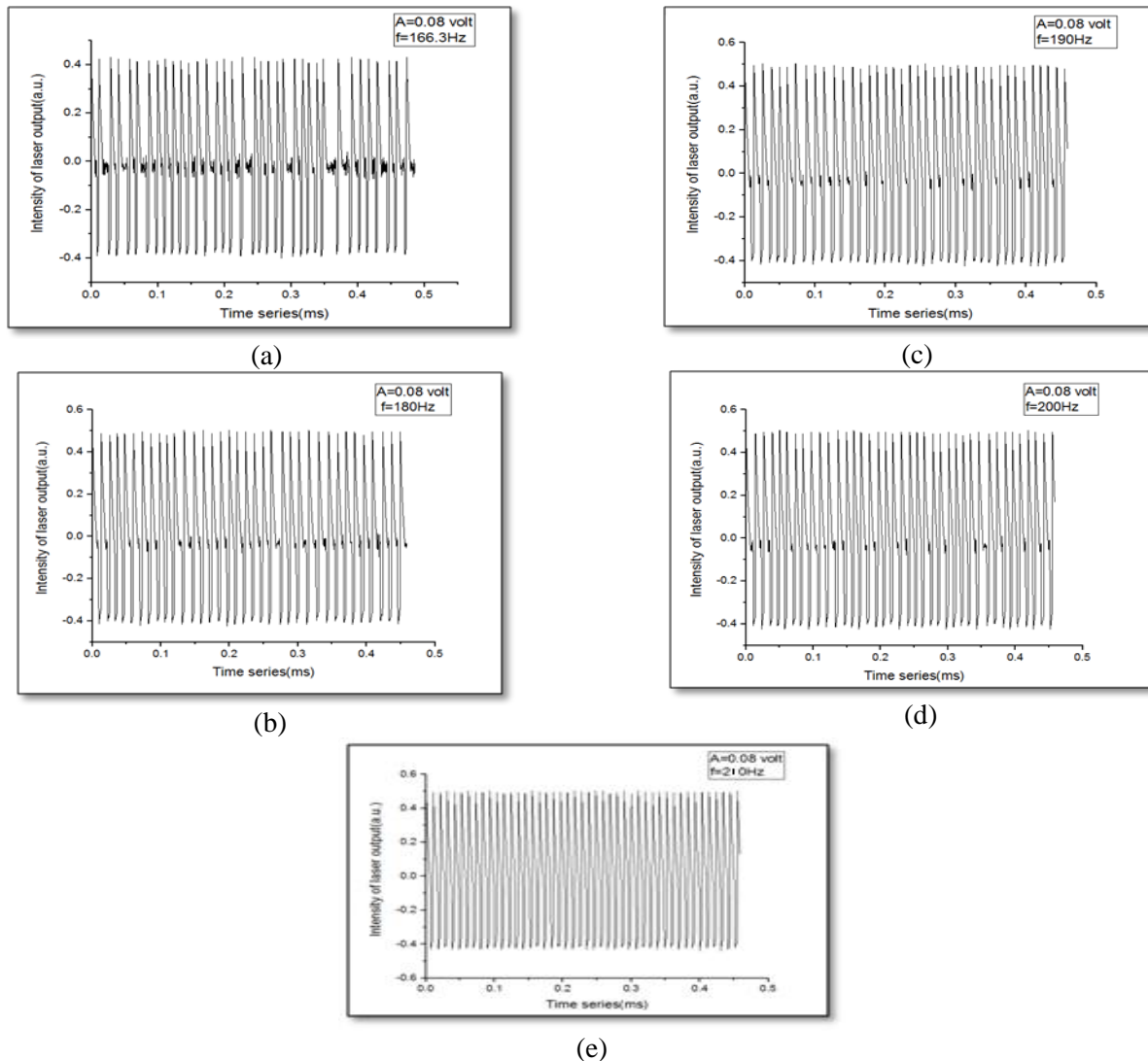
**Figure 3. Bifurcation diagram with different modulation Amplitudes.**

This Figure shows that these amplitudes lead the system to change from chaos(0.08-0.65)

volt to quasi periodic from(0.66-1.2) Volt and then to periodic from(1.3-1.4) Volt

The results indicate that the amplitude could be considered as a parameter at optimal amount of frequency (167 Hz) which controls the system's collective dynamics, the different amplitudes have controlled the chaotic system from chaos to quasi periodic and then to periodic. So, very interesting results have been obtained regarding to the effect of the chaotic resonance by adding the amplitude on the chaotic systems. This phenomenon was named as resonance in excitable nonlinear dynamical systems (14).

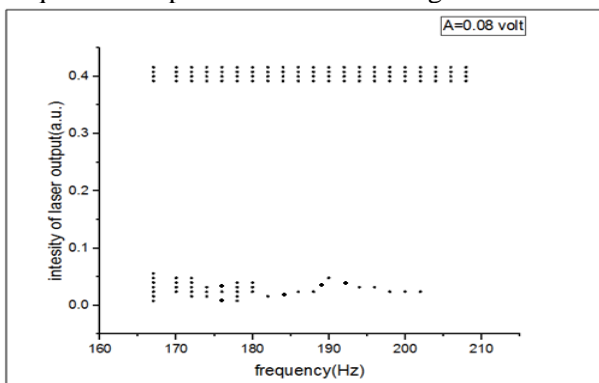
The second step, amplitude is fixed at 0.06V with gradually increasing the frequency (166, 170, 174,...)Hz until the output dynamics of the semiconductor laser become regular, it shows no effect on the chaotic behavior of output semiconductor laser (still chaotic). Then, the amplitude of perturbation has been fixed at 0.08 volt with gradually increasing the frequency (166, 170, 174,...210) Hz, the output dynamics of the semiconductor laser become regular as illustrated in Fig.4. It has been observed that the dynamical sequence contains the time series of different frequencies. These frequencies cause decreasing in chaotic behavior and convert the behavior of the system.



**Figure 4. Time series at fixed amplitude 0.08V and frequency values (a) 166Hz, (b) 180Hz, (c) 190Hz,(d) 200Hz,(e)210Hz.**

In Fig.4, the first region at frequency 166Hz shows the chaotic behavior and then any increase in the frequencies (170,173...210) Hz , causes a decrease in chaotic behavior and convert the system to periodic state.

The bifurcation diagram for different frequencies is plotted as shown in Fig.5.



**Figure 5. Bifurcation diagram with different Frequencies.**

The variation of frequencies lead to change the bifurcation diagram, these frequencies at (160-180) Hz will cause the system to change from chaos to a quasi periodic state (182-202) Hz and after that to periodic state (204-210) Hz. This means, that amplitude can be used as a control parameter. Therefore, the two control parameters (amplitude and frequency of external perturbation) are used to control chaotic system. The heavy dependence of such control on system's symmetries is emphasized, and the chaotic system is heavily impacted through frequency and amplitude of harmonic perturbation.

### Conclusions:

Chaotic behavior in a semiconductor laser with optoelectronic feedback can be controlled by adding perturbations. The impact of amplitude and frequency on chaotic spiking systems have been examined experimentally. The results indicate that the frequency and amplitude could be considered as parameters that control the system's collective

dynamics. Very interesting results have been obtained regarding to the effect of the chaotic resonance by adding optimal value of frequency or amplitude to the chaotic systems.

#### Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Al-Mustansiryah University.

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## الدليل التجريبي لعتبة التردد في ليزر أشباه الموصلات الشواشي

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#### الخلاصة:

تم في هذا البحث تقديم دراسة تجريبية بشأن إشارة الرنين في ليزر أشباه الموصلات الشواشي. تعتبر اضطرابات الرنين فعالة في تسخير مؤشرات التذبذب غير الخطية لتطبيقات مختلفة مثل إحداث الشواش والسيطرة على الشواش. تم الحصول على نتائج مثيرة للاهتمام فيما يتعلق بتأثير الرنين الشواشي عن طريق إضافة التردد على الأنظمة. يغير التردد القسري النظام الديناميكي غير الخطي من خلال قيمة حرجة، وهناك انتقال من جاذب دوري إلى جاذب غريب. كما ان السعة لها تأثير وثيق الصلة للغاية بالنظام، مما أدى إلى استجابة الرنين الأمثل للقيم المناسبة المتعلقة بزمان الارتباط. فيصبح النظام الشواشي منتظمًا تحت ترددات أو ساعات معتدلة. كما تم تحليل هذه الديناميكيات لمخرجات الليزر من خلال السلاسل الزمنية واطياف القدرة المستخرجة (FFT) وقد تعززت بواسطة مخطط التشعب.

**الكلمات المفتاحية:** شواش، تغذية استرجاعية، سيطرة، رنين، تردد.