

New Types of Pseudo Ideals in Pseudo Q-algebra

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Abstract:

In this paper, we introduce the notions of Complete Pseudo Ideal, K-pseudo Ideal, Complete K-pseudo Ideal in pseudo Q-algebra. Also, we give some theorems and relationships among them are debated.

Key words: Bounded pseudo Q-algebra, pseudo ideal, pseudo Q-algebra.

Introduction:

Negggers J., Ahn. S. S., Kim H. S., at 2001 introduced the class of Q-algebras, (1). Georgescu G. and Iorgulescu A., at the same time introduced pseudo BCK-algebras as an extension of BCK-algebras, (2). Dudek. W. A. and Jun. Y. B., at 2008 introduced pseudo BCI-algebras as a natural generalization of BCI-algebras and of pseudo BCK-algebras, (3). Walendziak A., at 2015 defined pseudo BCH-algebras and considered ideals in such algebras, (4). Jun. Y. B., Kim. H. S. and Ahn. S.S., at 2016 introduced pseudo Q-algebra as a generalization of Q-algebra, and structures of pseudo ideal and pseudo atom in a pseudo Q-algebra, (5). The aim of this paper is to introduce new types of pseudo ideals and also some of theorems and we make clear it relationships with pseudo ideal in pseudo Q-algebra.

Preliminaries of Pseudo Q-algebra

In this part, definitions of pseudo Q-algebra, bounded pseudo Q-algebra, pseudo ideal, and some of their properties are presented.

Definition (2.1) (5):

A pseudo Q-algebra U is a nonempty set with a constant 0 and two binary operations $*$ and $\#$ satisfying the following axioms : for any $u, v, w \in U$

1. $u * u = u \# u = 0$
2. $u * 0 = u \# 0 = u$
3. $(u * v) \# w = (u \# w) * v, \forall u, v, w \in U$.

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Remark (2.2) (5):

In pseudo Q-algebra U we can define a binary operation \leq by $u \leq v$ if and only if $u * v = 0$ and $u \# v = 0$, for all $u, v \in U$.

Proposition (2.3) (5),(6):

In any pseudo Q-algebra $(U, *, \#, 0)$, the following hold : for all $u, v \in U$

1. $(u * (u \# v)) \# v = (u \# (u * v)) * v = 0$.
2. $u \leq 0 \Rightarrow u = 0$.
3. $0 * (u * v) = (0 \# u) \# (0 * v)$.
4. $0 \# (u \# v) = (0 * u) * (0 \# v)$.
5. $0 * u = 0 \# u$.

Theorem (2.4) (6):

Let $(U, *, \#, 0)$ be a pseudo Q-algebra. The following statements are equivalent :

1. $u * (v * w) = (u * v) * w$, for all $u, v, w \in U$.
2. $0 * u = u = 0 \# u$, for every $u \in U$.
3. $u * v = u \# v = v * u$, for all $u, v \in U$.
4. $u \# (v \# w) = (u \# v) \# w$, for all $u, v, w \in U$.

Theorem (2.5) (6):

Every pseudo Q-algebra $(U, *, \#, 0)$ satisfying the associative law is a group under each operation $*$ and $\#$.

Definition (2.6):

Let $(U, *, \#, 0)$ be a pseudo Q-algebra, we call U is bounded if there is an element $e \in U$ satisfying $u \leq e$ for all $u \in U$, then e is called an unit of U .

In bounded pseudo Q-algebra U , we denoted $e * u$ and $e \# u$ by u^* and $u^\#$, respectively, for every $u \in U$.

Example (2.7):

Let $U = \{0, a, b, c\}$ be a set in Table 1:

Table 1. bounded pseudo Q-algebra

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	a
c	c	c	0	0

#	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	c	0	c
c	c	c	0	0

It is easy to check that $(U, *, \#, 0)$ is a pseudo Q-algebra, (5). Notice that U is bounded pseudo Q-algebra with unit b .

Proposition (2.8):

In a bounded pseudo Q-algebra U , for any $u, v \in U$ the following are hold :

1. $e^* = 0 = e^\#$
2. $0^* = e = 0^\#$
3. $u^* \# v = v^\# * u$
4. $u^* \# v^* = (v^*)^\# * u$
5. $u^\# * v^\# = (v^\#)^* \# u$
6. $0 * v = 0 = 0 \# v$
7. $e^* \# u = 0 = e^\# * u$
8. $(u^*)^\# \leq u, (u^\#)^* \leq u$

Proof :

1. $e^* = e * e = 0 = e \# e = e^\#$
2. $0^* = e * 0 = e = e \# 0 = 0^\#$
3. $u^* \# v = (e * u) \# v = (e \# v) * u = v^\# * u$
4. $u^* \# v^* = (e * u) \# v^* = (e \# v^*) * u = (v^*)^\# * u$
5. $u^\# * v^\# = (e \# u) * v^\# = (e * v^\#) \# u = (v^\#)^* \# u$

6. let $v \in U$, then

$$0 = (0 * v) \# e \quad (\text{since } U \text{ is bounded}) \\ = (0 \# e) * v = 0 * v$$

Also, $0 = (0 \# v) * e$ (since U is bounded)

$$= (0 * e) \# v = 0 \# v$$

7. $e^* \# u = 0 \# u$ (by 1)

$$= 0 \quad (\text{by 6})$$

$$= 0 * u \quad (\text{by 6})$$

$$= e^\# * u \quad (\text{by 1})$$

8. $(u^*)^\# * u = (e \# u^*) * u = (e \# (e * u)) * u = 0$

Also, $(u^\#)^* \# u = (e * u^\#) \# u = (e * (e \# u)) \# u = 0$

Definition (2.9) (5):

Let $(U, *, \#, 0)$ be a pseudo Q-algebra and L be a nonempty subset of U . Then L is called a pseudo ideal of U if for any $u, v \in U$,

1. $0 \in L$

2. $u * v, u \# v \in L$ and $v \in L$ imply $u \in L$.

Proposition (2.10) (5):

Let L be a pseudo ideal of a pseudo Q-algebra U . If $u \in L$ and $v \leq u$, then $v \in L$.

The Main Results

In this part , we provide definitions of complete pseudo ideal , K-pseudo ideal , complete K-pseudo ideal and study its relationships with pseudo ideal in bounded pseudo Q-algebra .

Definition (3.1):

A nonempty subset L of a bounded pseudo Q-algebra U is called complete pseudo ideal (briefly, c-pseudo ideal), if

1. $0 \in L$
2. $u * v, u \# v \in L, \forall v \in L$ such that $v \neq 0$ implies $u \in L$.

Example (3.2):

Let $U = \{0, a, b, c\}$ and two binary operations $*$ and $\#$ defined by Table 2 :

Table 2. bounded pseudo Q-algebra

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	0	0	c
c	c	c	0	0

#	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	a
c	c	c	0	0

Then $(U, *, \#, 0)$ is a bounded pseudo Q-algebra with unit b . A subset $L = \{0, a, c\}$ is c-pseudo ideal of U . While $L = \{0, b\}$ is not c-pseudo ideal, because $a * b = a \# b = 0 \in M, b \in M$ and $b \neq 0$ but $a \notin M$.

Remark (3.3):

In general , $\{0\}$ and U are called trivial c-pseudo ideals .

Proposition (3.4):

Every pseudo ideal of bounded pseudo Q-algebra is c-pseudo ideal .

Proof :

Suppose that L is a pseudo ideal from a bounded pseudo Q-algebra U and $u * v, u \# v \in L, \forall v \in L$ such that $v \neq 0$

1. If $L = \{0\}$, then L is c-pseudo ideal .
2. If $L \neq \{0\}$, thereafter $\exists v \in L$ such that $v \neq 0$. Thus $u \in L$ (Since L is pseudo ideal). Hence L is c-pseudo ideal.

Note that, in general, the converse of this proposition is not correct and the following example shows that .

Example (3.5):

In Example (3.2), the subset $L = \{0, a, c\}$ is a c -pseudo ideal, while L is not pseudo ideal, because $b * c = c \in L$, $b \# c = a \in L$ and $c \in L$ but $b \notin L$.

Definition (3.6):

Let $(U, *, \#, 0)$ be a bounded pseudo Q-algebra and L be a nonempty subset of U . Then L is called K^* -pseudo ideal of U (resp. $K^\#$ -pseudo ideal of U), if

1. $0 \in L$
2. $u^* * v, v^\# * u \in L$ (resp. $u^\# \# v, v^* \# u \in L$) and $v \in L$ implies $u^* \in L$ (resp. $u^\# \in L$), for all $u \in U$.

Then L is called K -pseudo ideal of U , if L is K^* -pseudo ideal and $K^\#$ -pseudo ideal of U .

Example (3.7):

Let $U = \{0, a, b, c, d\}$. Define the binary operations $*$ and $\#$ on U by Table 3 :

Table 3. bounded pseudo Q-algebra

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	a
b	b	0	0	b	b
c	c	0	c	0	b
d	d	0	d	0	0

#	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	a	a
b	b	0	0	0	0
c	c	0	d	0	d
d	d	0	d	0	0

Then $(U, *, \#, 0)$ is a bounded pseudo Q-algebra with unit a .

A subset $L = \{0, a\}$ is K -pseudo ideal of U . While $M = \{0, b\}$ is not K -pseudo ideal, because $0^* * b = b^\# * 0 = 0 \in M$, $b \in M$ however $0^* = a \notin M$.

Remark (3.8):

In bounded pseudo Q-algebra, a K^* -pseudo ideal needs not be $K^\#$ -pseudo ideal, as well a $K^\#$ -pseudo ideal needs not be K^* -pseudo ideal as showing in the following example.

Example (3.9):

Let $U = \{0, a, b, c\}$ and two binary operations $*$ and $\#$ defined by Table 4 :

Table 4. bounded pseudo Q-algebra

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	c	0	b
c	c	c	0	0

#	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	a
c	c	c	0	0

Then $(U, *, \#, 0)$ is a bounded pseudo Q-algebra with unit b . A subset $L = \{0, b, c\}$ is a K^* -pseudo ideal of U , but it's not $K^\#$ -pseudo ideal, because $c^\# \# b = b^* \# c = 0 \in L$, $b \in L$ however $c^\# = a \notin L$.

Also $M = \{0, a, b\}$ is a $K^\#$ -pseudo ideal of U , but it's not K^* -pseudo ideal, because $a^* * b = b^\# * a = 0 \in M$, $b \in M$ however $a^* = c \notin M$.

Definition (3.10):

Let U be bounded pseudo Q-algebra. An element $u \in U$ satisfies $u^{**} = u = u^{\#\#}$, then u is called an involution (i.e u is $*$ -involution and $\#$ -involution). If every element $u \in U$ is an involution, we call U is an involutory pseudo Q-algebra.

Example (3.11) :

Let $U = \{0, a, b, c, d\}$. Define the binary operations $*$ and $\#$ on U by Table 5 :

Table 5. involutory pseudo Q-algebra

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	d	0	0	0
c	c	a	d	0	b
d	d	0	0	0	0

#	0	a	b	c	d
0	0	0	0	0	0
a	a	0	d	0	0
b	b	0	0	0	0
c	c	d	b	0	a
d	d	0	0	0	0

Then $(U, *, \#, 0)$ is a bounded pseudo Q-algebra with unit c . Notice that U is an involutory.

Theorem (3.12):

Let L be a subset of an involutory pseudo Q-algebra U . Then the following statements are equivalent :

1. L is a K^* -pseudo ideal
2. L is a $K^\#$ -pseudo ideal

Proof :

$1 \Rightarrow 2$: let L be K^* -pseudo ideal of U and $u^\# \# v, v^* \# u \in L$ & $v \in L$

Since U is $*$ -involutory, then $u^\# \# v = (u^\#)^{**} \# v = v^\# * (u^\#)^*$ and $v^* \# u = u^\# * v = (u^\#)^{**} * v$. Thus $(u^\#)^{**} * v, v^\# * (u^\#)^* \in L$.

Since L is K^* -pseudo ideal and $v \in L$, then $(u^\#)^{**} \in L$ i.e $u^\# \in L$

Hence L is a $K^\#$ -pseudo ideal of U .

$2 \Rightarrow 1$: let L be $K^\#$ -pseudo ideal of U and $u^* * v, v^\# * u \in L$ & $v \in L$

Since U is $\#$ -involutory, then $u^* * v = (u^*)^{\#\#} * v = v^* \# (u^*)^\#$ and

$v^\# * u = u^* \# v = (u^*)^{\#\#} \# v$. Thus $(u^*)^{\#\#} \# v, v^* \# (u^*)^\# \in L$. Since L is K^* -pseudo ideal and $v \in L$, then $(u^*)^{\#\#} \in L$ i.e $u^* \in L$.

Hence L is a K^* -pseudo ideal of U .

Proposition (3.13):

Every pseudo ideal of bounded pseudo Q-algebra is K -pseudo ideal.

Proof :

Assume that L be a pseudo ideal in a bounded pseudo Q-algebra U such that $u^* * v, v^\# * u \in L$ and $v \in L$. Since $v^\# * u = u^* \# v$. Thus $u^* * v, u^* \# v \in L$ and $v \in L$. Since L is pseudo ideal, then $u^* \in L$. Thus L is a K^* -pseudo ideal of U .

Similarly, if $u^\# \# v, v^* \# u \in L, v \in L$ we get $u^\# \in L$. Thus L is $K^\#$ -pseudo ideal of U . Hence L is K -pseudo ideal of U .

Note that the converse of this proposition needs not be true and we can show that in the following example.

Example (3.14):

In Example (3.7), a subset $L = \{0, a\}$ is K -pseudo ideal of U , but it's not pseudo ideal, since $b * a = b \# a = 0 \in L$ and $a \in L$ but $b \notin L$.

Proposition (3.15):

If L is K -pseudo ideal of an involutory pseudo Q-algebra U . Then L is a pseudo ideal.

Proof :

Assume that L be K -pseudo ideal of U , then L is K^* -pseudo ideal and $K^\#$ -pseudo ideal of U . Thus L is K^* -pseudo ideal and L is $K^\#$ -pseudo ideal are equivalent (since U is involutory pseudo Q-algebra). To prove L is pseudo ideal of U

Let $u * v, u \# v \in L$ and $v \in L$

Since U is $*$ -involutory, then $u * v = u^{**} * v$ and $u \# v = u^{**} \# v = v^\# * u^*$. Thus $u^{**} * v, v^\# * u^* \in L$ and $v \in L$. But L is K^* -pseudo ideal, then $u^{**} \in L$ i.e $u \in L$. Hence L is pseudo ideal of U .

Proposition (3.16):

Let L be K -pseudo ideal of a bounded pseudo Q-algebra U . Then

1. If $u^* \leq v$ and $v \in L$ implies $u^* \in L$.
2. If $u^\# \leq v$ and $v \in L$ implies $u^\# \in L$.

Proof :

Let L be K -pseudo ideal of U , then L is K^* -pseudo ideal and $K^\#$ -pseudo ideal of U

1. If $u^* \leq v$. Then $u^* * v = 0$ and $u^* \# v = 0$ But $u^* \# v = v^\# * u$, thus $u^* * v, v^\# * u \in L$.

Since L is K^* -pseudo ideal & $v \in L$, then $u^* \in L$.

2. Similarly, if $u^\# \leq v$ and $v \in L$ we get $u^\# \in L$.

Proposition (3.17):

Let L be K -pseudo ideal of a bounded pseudo Q-algebra U ,

1. If $e \in L$, then $u^*, u^\# \in L$, for all $u \in U$.
2. If $u \in L$ and $u^* = 0 = u^\#$, then $e \in L$.

Proof :

1. Since L is K^* -pseudo ideal of U and $e \in L$, then for all $u \in U$

$u^* * e = 0 \in L$ and $e^\# * u = 0 * u = 0 \in L$. Thus $u^* \in L$.

Similarly, if L is $K^\#$ -pseudo ideal of U and $e \in L$, we get $u^\# \in L$.

2. Since L is K^* -pseudo ideal of U and $u \in L$ such that $u^* = 0 = u^\#$, then $0^* * u = u^* = 0 \in L$ and $u^\# * 0 = u^\# = 0 \in L$. Thus $e = 0^* \in L$.

Definition (3.18):

A nonempty subset L of a bounded pseudo Q-algebra U is called complete K^* -pseudo ideal of U (resp. complete $K^\#$ -pseudo ideal of U), briefly, $c-K^*$ -pseudo ideal (resp. $c-K^\#$ -pseudo ideal), if

1. $0 \in L$
2. $u^* * v, v^\# * u \in L$ (resp. $u^\# \# v, v^* \# u \in L$) $\forall v \in L$ such that $v \neq 0$ implies $u^* \in L$ (resp. $u^\# \in L$), $\forall u \in U$.

Then L is called $c-K$ -pseudo ideal of U , if L is $c-K^*$ -pseudo ideal and $c-K^\#$ -pseudo ideal.

Note that in bounded pseudo Q-algebra, there are trivial $c-K$ -pseudo ideals, $\{0\}$ and U .

Example (3.19):

In Example (2.7), a subset $L = \{0, a, c\}$ is a $c-K$ -pseudo ideal of U .

Remark (3.20):

In bounded pseudo Q-algebra, a $c-K^*$ -pseudo ideal needs not be $c-K^\#$ -pseudo ideal, as well a $c-K^\#$ -pseudo ideal needs not be $c-K^*$ -pseudo ideal as showing in the following examples.

Example (3.21):

Let $U = \{0, a, b, c\}$ be a set in Table 6 :

Table 6. bounded pseudo Q-algebra

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	0
c	c	b	b	0
#	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	0	0	0
c	c	a	a	0

Then $(U, *, \#, 0)$ is a bounded pseudo Q-algebra with unit c . If $L = \{0, b, c\}$, then L is a $c\text{-K}^*$ -pseudo ideal of U . But L is not $c\text{-K}^\#$ -pseudo ideal of U , because $a^\# \# v = v^* \# a = 0 \in L, \forall v \in L$ and $v \neq 0$ however $a^\# = a \notin L$.

Example (3.22):

In Example (2.7), A subset $M = \{0, b, c\}$ is a $c\text{-K}^\#$ -pseudo ideal of U .

But M is not $c\text{-K}^*$ -pseudo ideal of U , because $c^* * v = v^\# * c = 0 \in M, \forall v \in M$ such that $v \neq 0$ however $c^* = a \notin M$.

Theorem (3.23):

Let L be a subset of an involutory pseudo Q-algebra U . Then the following statements are equivalent :

1. L is a $c\text{-K}^*$ -pseudo ideal
2. L is a $c\text{-K}^\#$ -pseudo ideal

Proof :

Similarly with equivalence K^* -pseudo ideal and $\text{K}^\#$ -pseudo ideal.

Proposition (3.24):

Every K -pseudo ideal of bounded pseudo Q-algebra is $c\text{-K}$ -pseudo ideal.

Proof :

Let L be K -pseudo ideal in bounded pseudo Q-algebra U , then L is K^* -pseudo ideal and $\text{K}^\#$ -pseudo ideal.

1. If $L = \{0\}$, thus L is a $c\text{-K}$ -pseudo ideal .
2. If $L \neq \{0\}$,

(i) Let $u^* * v, v^\# * u \in L, \forall v \in L$ and $v \neq 0$, thereafter $\exists v \in L \ \& \ v \neq 0$. Since L is K^* -pseudo ideal & $u^* * v, v^\# * u \in L$, then $u^* \in L$. Thus L is a $c\text{-K}^*$ -pseudo ideal.

(ii) Similarly, if $u^\# \# v, v^* \# u \in L, \forall v \in L$ such that $v \neq 0$, we get $u^\# \in L$.

Thus L is a $c\text{-K}^\#$ -pseudo ideal. Hence L is $c\text{-K}$ -pseudo ideal of U .

Note that the converse of this proposition needs not be true in general as shown in the following example .

Example (3.25):

In Example (3.19), a subset $L = \{0, a, c\}$ is $c\text{-K}$ -pseudo ideal , but it's not K -pseudo ideal, because $0^* * c = a \in L, c^\# * 0 = c \in L$ and $c \in L$ however $0^* = b \notin L$.

Proposition (3.26):

Let L be $c\text{-K}$ -pseudo ideal in a bounded pseudo Q-algebra U . If $u^* \leq v$ and $u^\# \leq v, \forall v \in L$ such that $v \neq 0$. Then L is K -pseudo ideal of U .

Proof :

Assume that L be $c\text{-K}$ -pseudo ideal in U , then L is $c\text{-K}^*$ -pseudo ideal and $c\text{-K}^\#$ -pseudo ideal in U .

1. Let $u^* * v, v^\# * u \in L, v \in L$ such that $v \neq 0$

Since $u^* \leq v$, then $u^* * v = 0$ and $u^* \# v = 0$

But $u^* \# v = v^\# * u$. Thus $u^* * v = v^\# * u = 0 \in L, \forall v \in L$ such that $v \neq 0$. Since L is $c\text{-K}^*$ -pseudo ideal , thereafter $u^* \in L$. Thus L is K^* -pseudo ideal of U .

2. Similarly , if $u^\# \# v, v^* \# u \in L, v \in L$ such that $v \neq 0$, we get $u^\# \in L$.

Thus L is $\text{K}^\#$ -pseudo ideal of U . Hence L is a K -pseudo ideal of U .

Corollary (3.27):

Every pseudo ideal of bounded pseudo Q-algebra is a $c\text{-K}$ -pseudo ideal .

Proposition (3.28):

Any c -pseudo ideal from bounded pseudo Q-algebra is a $c\text{-K}$ -pseudo ideal.

Proof :

Let L be c -pseudo ideal from a bounded pseudo Q-algebra U and $u^* * v, v^\# * u \in L, \forall v \in L$ such that $v \neq 0$. Since $u^* \# v = v^\# * u$, implies $u^* * v, u^* \# v \in L, \forall v \in L$ such that $v \neq 0$. Since L is c -pseudo ideal from U , then $u^* \in L$. Thus L is $c\text{-K}^*$ -pseudo ideal.

Similarly , if $u^\# \# v, v^* \# u \in L, \forall v \in L$ such that $v \neq 0$, we get $u^\# \in L$.

Thus L is $c\text{-K}^\#$ -pseudo ideal. Hence L is $c\text{-K}$ -pseudo ideal of U .

In general, the inverse of this proposition needs not be true as shown in the following example.

Example (3.29):

In Example (3.7), if $L = \{0, a, c\}$, then L is a $c\text{-K}$ -pseudo ideal of U , while L is not c -pseudo ideal , because $d * w = d \# w = 0 \in L, \forall w \in L$ and $w \neq 0$ but $d \notin L$.

Proposition (3.30):

If L be $c\text{-K}$ -pseudo ideal in an involutory pseudo Q-algebra U , then L is c -pseudo ideal .

Proof :

Suppose that L be $c\text{-K}$ -pseudo ideal of U , then L is $c\text{-K}^*$ -pseudo ideal and $c\text{-K}^\#$ -pseudo ideal of U . Thus L is $c\text{-K}^*$ -pseudo ideal and L is $c\text{-K}^\#$ -pseudo ideal are equivalent (since U is an involutory pseudo Q-algebra). To prove L is c -pseudo ideal of U . Let $u * v, u \# v \in L, \forall v \in L$ such that $v \neq 0$. Since U is $*$ -involutory, then $u * v = u^{**} * v$ and $v^\# * u^* = u^{**} \# v = u \# v$. Thus $u^{**} * v, v^\# * u^* \in L, \forall v \in L$ such that $v \neq 0$. Since L is $c\text{-K}^*$ -pseudo ideal, then $u^{**} \in L$, i.e , $u \in L$. Hence L is a c -pseudo ideal of U .

Remark (3.31):

The diagram in (Fig. 1) shows the relations among pseudo ideal, K -pseudo ideal, c -pseudo ideal, and $c\text{-K}$ -pseudo ideal in bounded pseudo Q-algebra :

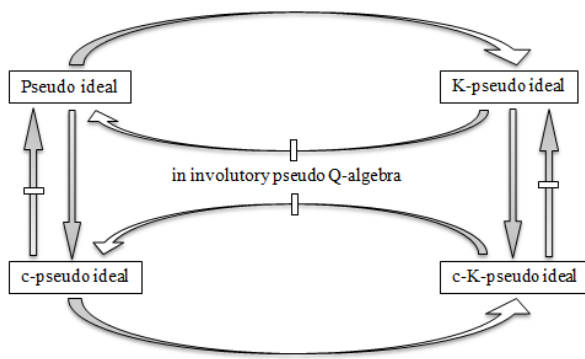


Figure 1. The relations among pseudo ideals in bounded pseudo Q-algebra.

Future Works

There are many avenues that one could explore. In this section, we state some of these open problems and conjectures.

1. Studying the Fuzzy pseudo ideal of pseudo Q-algebra.
2. Studying the Fuzzy complete pseudo ideal (as a generalization of fuzzy pseudo ideal) of pseudo Q-algebra.
3. Studying the Fuzzy K-pseudo ideal (as a generalization of fuzzy pseudo ideal) of pseudo Q-algebra.

4. Studying the Fuzzy complete K-pseudo ideal (as a generalization of fuzzy K-pseudo ideal and fuzzy complete pseudo ideal) of pseudo Q-algebra.
5. Trying to study these pseudo ideals on other algebras in the two cases (Ordinary and Fuzzy).

Conflicts of Interest: None.

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انواع جديدة من المثاليات الكاذبة في جبر-Q الكاذب

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الخلاصة :

في هذا البحث قدمنا المفاهيم لمكمل المثالي الكاذب ، مثالي-K الكاذب ، مكمل مثالي-K الكاذب في جبر-Q الكاذب. كذلك نعطي بعض الخواص و العلاقات التي تناقش فيما بينها .

الكلمات المفتاحية : جبر-Q الكاذب ، جبر-Q الكاذب المقيد ، المثالي الكاذب .