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# Some Types of Mappings in Bitopological Spaces

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**Abstract:**

This work, introduces some concepts in bitopological spaces, which are *nm-j-ω*-converges to a subset, *nm-j-ω*-directed toward a set, *nm-j-ω*-closed mappings, *nm-j-ω*-rigid set, and *nm-j-ω*-continuous mappings. The mainline idea in this paper is *nm-j-ω*-perfect mappings in bitopological spaces such that $n$ = 1,2 and $m$ =1,2 $n \neq m$. Characterizations concerning these concepts and several theorems are studied, where $j = \theta, \delta, \alpha$, *pre*, *b*, $\beta$.

**Key words:** Filter base, *nm-j-ω*-converges, *nm-j-ω*-closed mappings, *j-ω*-rigid a set, *nm-j-ω*-perfect mappings.

## Introduction and Preliminaries:

In 1963 Kelly J. C. (1) introduced the definition, a set $G$ with two topologies $\sigma_1$ and $\sigma_2$ is said to be bitopological space and denoted by $(G, \sigma_1, \sigma_2)$ and a subset $K \subseteq G$. The closure and interior of $K$ in $(G, \sigma_n)$ is denoted by $\sigma_n$–cl($K$) and $\sigma_n$–int($K$), where $n$ = 1, 2. A topological space $(G, \sigma)$ and a point $g$ in $G$ is said to be condensation point of $K \subseteq G$ if every open neighborhood $S$ in $\sigma$ with $g \in S$, the set $K \cap S$ is uncountable (2). In 1982 the $\omega$-closed set was first exhibited by H. Z. Hdeib in (3) defined it as a subset $K \subseteq G$ is called $\omega$-closed if it incorporates each its condensation points, and the $\omega$-open set is the complement of the $\omega$-closed set and the $\omega$-closed of the set $K \subseteq G$ denoted by $\mathrm{cl}^{\omega}(K)$. The $\omega$-interior of the set $K \subseteq G$ is defined as the union of all $\omega$-open sets content in $K$ and is denoted by $\mathrm{int}^{\omega}(K)$. In (4) a point $g \in G$ is said to $\theta$-cluster points of $K \subseteq G$ if $\mathrm{cl}(S) \cap K \neq \varphi$ for each open set $S$ of $G$ contained $g$.Al so in (4) the set of each $\theta$-cluster points of $K$ is called the $\theta$-closure of $K$ and is denoted by $\mathrm{cl}_{\theta}(K)$. A subset $K \subseteq G$ is called $\theta$-closed (4) if $K = \mathrm{cl}_{\theta}(K)$. The complement of $\theta$-closed set is said to be $\theta$-open. A point $g \in G$ is said to $\theta$-$\omega$-cluster points of $K \subseteq G$ if $\mathrm{cl}^{\omega}(S) \cap K \neq \varphi$ for each $\omega$-open set $S$ of $G$ containing $g$. The set of each $\theta$-$\omega$-cluster points of $K$ is called the $\theta$-$\omega$-closure of $K$ and is denoted by $\mathrm{cl}^{\omega}_{\theta}(K)$. A subset $K \subseteq G$ is called $\theta$-$\omega$-closed (4) if $K = \mathrm{cl}^{\omega}_{\theta}(K)$. The complement of $\theta$-$\omega$-closed set is said to be $\theta$-$\omega$-open. A subset $K \subseteq G$ is said to be $\delta$-closed (5) if $K = \mathrm{cl}_{\delta}(K) = \{g \in G : \mathrm{int}(\mathrm{cl}(S)) \cap K \neq \varphi, S \in \tau \text{ and } g \in S\}$. The complement of $\delta$-closed is called $\delta$-open set, and $K$ is $\delta$-$\omega$-closed if $K = \mathrm{cl}^{\omega}_{\delta}(K) = \{g \in G : \mathrm{int}^{\omega}(\mathrm{cl}(S)) \cap K \neq \varphi, S \in \tau \text{ and } g \in S\}$. For other notions or notations not defined here, R. Englking (6) should be followed closely. Several characterizations of $\omega$-closed sets were provided in (4, 5, 8, 9, and 10). Some of the results in (11), (12), (13), (14) and (15) will be bult.

**Definition 1.** (1) A nonempty family $\Im$ of nonempty subsets of $G$ is called filter base if $M_1, M_2 \in \Im$ then $M_3 \subseteq M_1 \cap M_2$ for some $M_3 \in \Im$.

The filter generated by a filter base $\Im$ consists of all supersets of elements of $\Im$. An open filter base on a space $G$ is a filter base with open members.

The set $\aleph_g$ of all neighborhoods (nbds) of $g \in G$ is a filter on $G$, and any nbd base at $g$ is a filter base for $\aleph_g$. This filter called the nbd filter at $g$.

**Definition 2.** (1) Let $\Im$ and $\wp$ be filter bases on $G$. Then $\wp$ is called finer than $\Im$ (written as $\Im < \wp$) if for all $M \in \Im$, there is $\mathcal{G} \in \wp$, $\mathcal{G} \subseteq M$ also, that $\Im$ meets $\mathcal{G}$ if $M \cap \mathcal{G} \neq \phi$ for all $M \in \Im$ also, $\mathcal{G} \in \wp$. Notice, $\Im \to g$ iff $\aleph_g < \Im$.

**Definition 3.** (7) A subset $K$ of a space $G$ is called:

(a) *α-ω*-open if $K \subseteq \mathrm{int}^{\omega}(\mathrm{cl}(\mathrm{int}^{\omega}(K)))$.
(b) *pre-ω*-open if $K \subseteq \mathrm{int}^{\omega}(\mathrm{cl}(K))$.
(c) *b-ω*-open if $K \subseteq \mathrm{cl}(\mathrm{int}^{\omega}(K)) \cup \mathrm{int}^{\omega}(\mathrm{cl}(K))$.
(d) *β-ω*-open if $K \subseteq \mathrm{cl}(\mathrm{int}^{\omega}(\mathrm{cl}(K)))$.
The complement of an *α-ω*-open (resp., *pre-ω*-open, *b-ω*-open, *β-ω*-open) is called (resp. *α-ω*-closed (resp., *pre-ω*-closed, *b-ω*-closed, *β-ω*-closed).
The *j-ω*-closure of $K \subseteq G$ is denoted by $\mathrm{cl}_j^{\omega}(K)$ and defined by $\mathrm{cl}_j^{\omega}(K) = \cap\{M \subseteq G;\ M$ is *j-ω*-closed and $K \subseteq M\}$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Filter Bases and *nm-j-ω*-Perfect Mappings in Bitopological Spaces**

This section, defines filter bases and *nm-j-ω*-converges to a subset, *nm-j-ω*-directed toward a set, *nm-j-ω*-closed mapping, *j-ω*-continuous mappings, *j-ω*-rigid a set, and used to obtain characterization theorem for an *nm-j-ω*-perfect mappings in bitopological spaces.

**Definition 4.** A point $g$ in bitopological space $(G, \sigma_1, \sigma_2)$ is said to be *nm-j-ω*-condensation point of a subset $K$ of $G$ iff for any $\sigma_n$–open nbd $S$ of $g$, $(\mathrm{cl}_j^{\omega}(S)) \cap K \neq \varphi$. The set of all *nm-j-ω*-condensation point of $K$ is called *nm-j-ω*-closure of $K$ and means by $nm\ \omega\text{-}\mathrm{cl}_j^{\omega}(K)$. A set $K \subseteq G$ is said to be *nm-j-ω*-closed if $K = nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Definition 5.** A point $g$ in a bitopological space $(G, \sigma_1, \sigma_2)$ is said to be *nm-j-ω*-condensation point of a filter base $\Im$ on $K$ if it is an *nm-j-ω*-condensation point of every number of $\Im$. The set of all *nm-j-ω*-condensation point of $\Im$ is called *nm-j-ω*-condensed of $\Im$ and means by $nm\text{-}j\text{-}\omega\text{-}cod\Im$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Definition 6.** A filter base $\Im$ on a bitopological space $(G, \sigma_1, \sigma_2)$ is called *nm-j-ω*-converges to a subset $K \subseteq G$ (written as $\Im nm\text{-}j\text{-}\omega \rightarrow K$) if for each $\sigma_n$–open cover $\mathcal{K}$ of $K$, yond is a finite subfamily $\mathcal{L} \subseteq \mathcal{K}$ and $M \in \Im$ such that $M \subseteq \cup\{\ \sigma_n\text{–}\mathrm{cl}_j^{\omega}(L) : L \in \mathcal{L}\ \}$. $\Im$ *nm-j-ω*-converges to a point $g \in G$ (written as $\Im nm\text{-}\ j\text{-}\omega \rightarrow\ g$) iff $\Im nm\text{-}\ _j\text{-}\omega\ \rightarrow \{g\}$, or equivalently, $\sigma_n\text{-}\ \mathrm{cl}_j^{\omega}(S)$ of every $\sigma_m$–open nbd $S$ of $g$ contains some member of $\Im$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 1.** In a bitopological space $(G, \sigma_1, \sigma_2)$ a point $g$ is an *nm-j-ω*-condensation of a filter base $\Im$ on $G$ if there subsistent a filter base $\Im^*$ finer than $\Im$ such that $\Im^* nm\text{-}\ _j\text{-}\omega \rightarrow g$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Proof:** ($\Rightarrow$) Let $g$ be an *nm-j-ω*-condensation point of a filter base $\Im$ on $G$, then every $\sigma_n$ –open nbd $S$ of $g$, the *j-ω*-closure of $S$ contains a member of $\Im$ and thus contains a member of any filter base $\Im^*$ minutes than $\Im$, so that $\Im^*\ nm\text{-}\ _j\text{-}\omega \rightarrow g$.
($\Leftarrow$) Assume that $g$ is not an *nm-j-ω*-condensation point of a filter base $\Im$ on $G$, then there subsistent an $\sigma_n$–open nbd $S$ of $g$, such that *j-ω*-closure of $S$ contains no member of $\Im$, denote by $\Im^*$ the family of sets $M^* = M\ \cap (G - (\mathrm{cl}_j^{\omega}(S))$ for $M \in \Im$, then the sets $M^*$ are nonempty. And $\Im^*$ is a filter base and indeed it is minute than $\Im$, since $M_1^* = M_1 \cap (G - \mathrm{cl}_j^{\omega}(S))$ and $M_2^* = M_2 \cap (G - \mathrm{cl}_j^{\omega}(S))$, so there is an $M_3 \subseteq M_1 \cap M_2$ and this lead to:
$M_3^* = M_3 \cap (G - (\mathrm{cl}_j^{\omega}(S)) \subseteq M_1 \cap M_2 \cap (G - (\mathrm{cl}_j^{\omega}(S))$
$= M_1 \cap (G - (\mathrm{cl}_j^{\omega}(S)) \cap M_2 \cap (G - (\mathrm{cl}_j^{\omega}(S))$.
By construction $\Im^*$ not *nm-j-ω*-convergent to $g$.
This contradiction, and thus $g$ is an *nm-j-ω*-condensation point of a filter base $\Im$ on $G$.

**Definition 7.** A filter base $\Im$ on a bitopological space $(G, \sigma_1, \sigma_2)$ is said to be *nm-j-ω*- directed toward to a set $K \subseteq G$ (written as $\Im nm\text{-}_{j\text{-}\omega}\text{-}dir\text{-}tow \rightarrow K$) if for each filter base $\wp$ finer $\Im$ has an *nm-j-ω*-condensation point in $K$. i.e $(nm\text{-}\ j\text{-}\omega\text{-}cod\,\wp) \cap K \neq \varphi$. $\Im nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \rightarrow g$ used to mean $\Im nm\text{-}_{j\text{-}\omega}\text{-}dir\text{-}tow \rightarrow \{g\}$, where $g \in G$, and $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 2.** Let $\Im$ be a filter base on a bitopological space $(G, \sigma_1, \sigma_2)$ and point $g \in G$, then $\Im\ nm\text{-}\ _j\text{-}\omega \rightarrow g$ if and only if $\Im nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \rightarrow g$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Proof:** ($\Rightarrow$) Clear.
($\Leftarrow$) Assume that $\Im$ is not an *nm-j-ω*-converge to $g$, there exists an $\sigma_n$–open nbd $S$ of $g$, such that $M \not\subset \mathrm{cl}_j^{\omega}(S)$, for all $M \in \Im$. Then $\wp = \{(M \cap (G - (\sigma_n - \mathrm{cl}_j^{\omega}(S)):\ M \in \Im\}$ is a filter base on $G$ finer than $\Im$, and conspicuously $g \notin nm\text{-}j\text{-}\omega\text{-}cod\,\wp$. So $\Im$ cannot be *nm- j-ω*- directed towards $g$.

**Definition 8.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is said to be *nm-j-ω*-perfect if for every filter base $\Im$ on $\lambda(G)$, *nm-j-ω*-directed towards some subset $L$ of $\lambda(G)$, the filter base $\lambda^{-1}(\Im)$ is *nm-j-ω*-directed towards $\lambda^{-1}(L)$ in $G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 4.** Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be a mapping. Then the following are equivalent:
(a) $\lambda$ is *nm-j-ω*-perfect.
(b) For every filter base $\Im$ on $\lambda(G)$, which is *nm- j-ω*-convergent to a point $h$ in $H$, $\lambda^{-1}(\Im) nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to \lambda^{-1}(h)$.
(c) For any filter base $\Im$ on $G$, $nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\Im) \subset \lambda(nm\text{-}j\text{-}\omega\text{-}cod\ \Im)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.
**Proof:** (a) ⇒ (b) Proof by Theorem (2).
(b) ⇒ (c) Let $h \in nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\Im)$. By Theorem (1), there is a filter base $\wp$ in $\lambda(G)$ finer than $\lambda(\Im)$, $\wp\ nm\text{-}_j\text{-}\omega \to h$. Let $\upsilon = \{\lambda^{-1}(\mathcal{G}) \cap M : \mathcal{G} \in \wp$ and $M \in \Im\}$. Then $\upsilon$ is a filter base on $G$ finer than $\lambda^{-1}(\wp)$. Since $\wp\ nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to h$, and by Theorem (2) and $\lambda$ is *nm- j-ω*-perfect, $\lambda^{-1}(\wp)\ nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to \lambda^{-1}(h)$. $\upsilon$ Being finer than $\lambda^{-1}(\wp)$, then $\lambda^{-1}(h) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \upsilon) \neq \phi$. It is then clear that $\lambda^{-1}(h) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \Im) \neq \phi$. Then, $h \in \lambda(nm\text{-}j\text{-}\omega\text{-}cod\ \Im)$.
(c) ⇒ (a) Suppose $\Im$ be a filter base on $\lambda(G)$, it is *nm-j-ω*-directed towards some subset $L$ of $\lambda(G)$. Let $\wp$ be a filter base on $G$ finer than $\lambda^{-1}(\Im)$. Hence, $\lambda(\wp)$ is a filter base on $\lambda(G)$ finer than $\Im$ and so $L \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\wp)) \neq \phi$. Then by (c) $L \cap \lambda(nm\text{-}j\text{-}\omega\text{-}cod\ \wp)) \neq \phi$, so that $\lambda^{-1}(L) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \wp) \neq \phi$. Then, $\lambda^{-1}(\Im)$ is *nm-j-ω*-directed towards $\lambda^{-1}(L)$. Thus, $\lambda$ is *nm-j-ω*-perfect.

**Definition 9.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be *nm-j-ω*-closed if the image of every *nm-j-ω*-closed set in $G$ is *nm-j-ω*-closed in $H$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 5.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-closed if $nm\text{-}\omega\text{-}\operatorname{cl}_j^{\omega}\ \lambda(K) \subset \lambda(nm\text{-}\omega\text{-}\operatorname{cl}_j^{\omega}(K))$, for $n, m$= 1and 2 such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.
**Proof:** Straightforward.

**Theorem 6.** The *nm-j-ω*-perfect mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-closed, where $j = \theta, \delta, \alpha, pre, b, \beta$.
**Proof:** Follow from Theorem (5) and Theorem (3) (a) ⇒ (c) taking $\Im = \{K\}$.

**Definition 10.** A subset $K$ of bitopological space $(G, \sigma_1, \sigma_2)$ is said to be *nm*-Supra-*ω*-rigid (written as *nm-j-ω*-rigid ) in $G$ if for every filter base $\Im$ on $G$ with $(nm\text{-}j\text{-}\omega\text{-}cod\ \Im) \cap K = \phi$, there is $S \in \sigma_n$ and $M \in \Im$, such that $K \subset S$ and $\operatorname{cl}_j^{\omega}(S) \cap M = \phi$. or equivalent, if for every filter base $\Im$ on $G$ whenever, $K \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \Im) = \phi$, then for some $M \in \Im$, $K \cap (nm\text{-}\omega\text{-}\operatorname{cl}_j^{\omega}(M)) = \phi$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 7.** If a mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-closed such that for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in $G$, then $\lambda$ is *nm- j-ω*-perfect, where $j = \theta, \delta, \alpha, pre, b, \beta$.
**Proof:** Assume that $\Im$ be a filter base on $\lambda(G)$ such that $\Im\ nm\text{-}_j\text{-}\omega \to h$ in $H$, for some $h \in H$. If $\wp$ is a filter base on $G$ finer than the filter base on $\lambda^{-1}(\Im)$. Thus $\lambda(\wp)$ is a filter base $H$, finer than $\Im$. Since $\Im nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to g$, by Theorem (1), $h \in nm\text{-}j\text{-}\omega\text{-}cod\lambda(\wp)$, i.e., $h \in \cap\{nm\text{-}\omega\text{-}\operatorname{cl}_j^{\omega}\lambda(\mathcal{G}): \mathcal{G} \in \wp\}$ and $h \in \cap\{\lambda(nm\text{-}\omega\text{-}\operatorname{cl}_j^{\omega}(\mathcal{G})): \mathcal{G} \in \wp\}$ by Theorem (5), since $\lambda$ is *nm- j-ω*-closed. Then $\lambda^{-1}(h) \cap nm\text{-}\omega\text{-}\operatorname{cl}_j^{\omega}(\mathcal{G}) \neq \phi$, for all $\mathcal{G} \in \wp$. Hence for all $S \in \sigma_n$ with $\lambda^{-1}(h) \subset S$, $\operatorname{cl}_j^{\omega}(S) \cap \mathcal{G} \neq \phi$, for all $\mathcal{G} \in \wp$. Since $\lambda^{-1}(h)$ is *nm- j-ω*-rigid, it then that $\lambda^{-1}(h) \cap (nm\text{-}j\text{-}\omega\text{-}cod\ \wp) \neq \phi$. Then $\lambda^{-1}(\Im) nm\text{-}_j\text{-}\omega\text{-}dir\text{-}tow \to \lambda^{-1}(h)$, and by Theorem (4 (b) ⇒ (a) ). Thus $\lambda$ is *nm-j-ω*-perfect.

**Definition 11.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be *nm*-Supra-*ω*-continuous (written as *nm-j-ω*-continuous) if for any $\varsigma_n$-open nbd $T$ of $\lambda(g)$, there exists a $\sigma_n$- open nbd $S$ of $g$, $\lambda(\operatorname{cl}_j^{\omega}(S)) \subset \varsigma_{m}\text{-}\operatorname{cl}_j^{\omega}(T)$ , where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Definition 12.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be weakly *nm- j-ω*-continuous if for any $\varsigma_n$-open nbd $T$ of $\lambda(g)$, there exists a $\sigma_n$-open nbd $S$ of $g$ such that $\lambda(S) \subset \varsigma_{m}\text{-}\operatorname{cl}_j^{\omega}(T)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Definition 13.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be strongly *nm-j-ω*-continuous if for any $\varsigma_n$-open nbd $T$ of $\lambda(g)$, there exists a $\sigma_n$- open nbd $S$ of $g$, $\lambda(\operatorname{cl}_j^{\omega}(S)) \subset T$ , where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Definition 14.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be super *nm-j-ω*-continuous if for any $\varsigma_n$-open nbd $T$ of $\lambda(g)$, there exists a $\sigma_n$-open nbd $S$ of $g$, $\lambda(\operatorname{int}_j^{\omega}(\operatorname{cl}_j^{\omega}(S)) \subset \varsigma_{m}\text{-}\operatorname{cl}_j^{\omega}(T)$, for $n, m$ = 1and 2 such that $(n \neq m)$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Definition 15.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to be almost *nm-j-ω*-continuous if for any $\varsigma_n$-open nbd $T$ of $\lambda(g)$, there exists a $\sigma_n$-open nbd $S$ of $g$, $\lambda(S) \subset (\varsigma_{m}\text{-}\operatorname{int}_j^{\omega}(\operatorname{cl}_j^{\omega}(T))$ , for $n, m$ =

1and 2 such that ($n \neq m$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

The relation between weakly and strongly *nm-j-ω*-continuous mappings are given by the following

| | | | | |
|---|---|---|---|---|
| Strongly *nm- j-ω*- continuous mapping | ⇒ | *nm- j-ω*- continuous mapping | ⇒ | Weakly *nm-j-ω*- continuous mapping |
| ⇓ | | ⇓ | | |
| Super *nm- j-ω*- continuous mapping | | almost *nm- j-ω*- continuous mapping | | |

**Figure 1. The relation between weakly and strongly *nm-j-ω*-continuous mappings, where $j = \theta, \delta, \alpha, pre, b, \beta$.**

In the higher figure the converses not be true such that the demonstrated by the following examples:

**Example 1.** Let $A$ be the upper half of the plane and $B$ be the x-axis. Let $G = A \cup B$. If $\tau_{hdis}$ be the half disc topology on $G$ and $\tau_r$ be the relative topology that $G$ inherits by virtue of being a subspace of $\Re^2$. The identity mapping $\lambda : (G, \tau_r) \rightarrow (G, \tau_{hdis})$ . Then, $\lambda$ is weakly *nm- j-ω*-continuous mapping but it is not *nm- j-ω*-continuous mapping.

**Example 2.** Let $\lambda :(G, \sigma_1, \sigma_2) \rightarrow (G, \varsigma_1 , \varsigma_2 )$ be a mapping such that $G = \{u,v, w\}$, and $\sigma_1 = \{G , \varphi\}$, $\sigma_2 = \{G, \varphi,\{u, v\}\}$ and $\varsigma_1 = \{G , \varphi\}$, $\varsigma_2 =\{G, \varphi, \{w\}\}$. Such that $\lambda (u) = \lambda (v) =\lambda (w) = u$. Then $\lambda$ is almost *nm-j-ω*-continuous mapping but it is not *nm- j-ω*-continuous mapping.

**Example 3.** Let $\lambda : (\mathscr{R}, \tau) \rightarrow (\mathscr{R}, \tau)$ be a mapping. Define by $\lambda(g) = g$, and let $(\mathscr{R}, \tau)$ where $\tau$ is the topology with basis whose members are of the form $(a, b)$ and $(a, b)$ -$N$ such that $N = \{1/n;\ n \in Z^+\}$. Then $(\mathscr{R}, \tau)$ is Hausdorff but is not *ω*-regular. Then $\lambda$ is *nm-j-ω*-continuous mapping but is not strongly *nm- j-ω*-continuous mapping.

**Example 4.** Let $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (G, \sigma_1, \sigma_2)$ be identity mapping, such that $G = \{u, v, w\}$ and $\sigma_1 = \{G, \varphi, \{u , v\}\}$, $\sigma_2 = \{\varphi, G, \{u\}, \{v\}, \{u, v\}\}$. Then $\lambda$ is super *nm- j-ω*-continuous mapping but it is not strongly *nm- j-ω*-continuous mapping.

**Theorem 8.** If an *nm- j-ω*-continuous mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is *nm- j-ω*- perfect , then:
(a) $\lambda$ is *nm-j-ω*- closed.
(b) For every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in $G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Proof:** (a) By Theorem (6) $\lambda$ an *nm- j-ω*- perfect mapping is *nm- j-ω*- closed.
(b) To prove $\lambda^{-1}(h)$ is *nm- j-ω*-rigid, let $h \in H$, and assume that $\Im$ be a filter base on $G$ such that $(nm\text{-}j\text{-}\omega\text{-}cod\ \Im) \cap \lambda^{-1}(h) = \phi$. Then $h \notin \lambda\ (nm\text{-}j\text{-}\omega\text{-}cod\ \Im)$, since $\lambda$ is *nm- j-ω*-perfect, by Theorem (3 (a) ⇒ (c) ). Then, $h \notin (nm\text{-}j\text{-}\omega\text{-}cod\ \lambda(\Im))$, so there exists an $M \in \Im$ such that $h \notin nm\text{-}\omega\text{-}cl_j^{\omega}\lambda(M)$, yond exists an $\varsigma_n$-open nbd $T$ of $h$ also, $\varsigma_{m}\text{-}\ cl_j^{\omega}(T) \cap \lambda(M) = \phi$, since $\lambda$ is *nm- j-ω*-continuous, for every $g \in \lambda^{-1}(h)$, then $\sigma_n$-open nbd $S_g$ of $g$ such that $\lambda(\ cl_j^{\omega}(S_g)) \subset \varsigma_{m}\text{-}\ cl_j^{\omega}(T) \subset H\text{-}\lambda(M)$. Then $\lambda(\ cl_j^{\omega}(S_g)) \cap \lambda(M) = \phi$, so that $cl_j^{\omega}(S_g) \cap M = \phi$, then $g \notin nm\text{-}\omega\text{-}cl_j^{\omega}(M)$, for every $g \in \lambda^{-1}(h)$, then $\lambda^{-1}(h) \cap (nm\text{-}\omega\text{-}cl_j^{\omega}(M)) = \phi$, so $\lambda^{-1}(h)$ is *nm- j-ω*-rigid in $G$, where $j = \theta, \delta, \alpha, pre, b$.

**Corollary 1.** An *nm-j-ω*-continuous mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-perfect if $\lambda$ is *nm- j-ω*- closed and for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in $G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

The results show that thereupon the higher theorem remainders aright if *nm- j-ω*-closeness of $\lambda$ is replaced by a stringently enfeeble condition which will be called as a weak *nm- j-ω*-closeness and strong *nm- j-ω*-closeness of $\lambda$. Thus, these will be predefined as follows:

**Definition 16.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is called weakly *nm- j-ω*-closed if for every $h \in \lambda(G)$, and each $\sigma_n$-open set $S$ containing $\lambda^{-1}(h)$ in $G$, there exists a $\varsigma_m$-open nbd $T$ of $h$, $\lambda^{-1}(\varsigma_{m}\text{-}\ cl_j^{\omega}(T)\ ) \subset cl_j^{\omega}(S)$, for $n, m$ = 1and 2 such that ($n \neq m$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Definition 17.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \varsigma_1, \varsigma_2)$ is said to be strongly *nm- j-ω*-closed if for each $h \in \lambda(G)$, and each $\sigma_n$-open set $S$ containing $\lambda^{-1}(h)$ in $G$, there exists a $\varsigma_m$-open nbd $T$ of $h$, $\lambda^{-1}(\varsigma_{m}\text{-}\ cl_j^{\omega}(T)\ ) \subset (S)$, for $n, m$ = 1and 2 such that ($n \neq m$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

The relation between weakly and strongly *nm-j-ω*-closed mappings are given by the following figure:

Strongly *nm- j-ω*-closed ⇒ *nm- j-ω*-closed ⇒ weakly *nm- j-ω*-closed

**Figure 2. The relation between weakly and strongly *nm-j-ω*-continuous mappings, where $j = \theta, \delta, \alpha, pre, b, \beta$.**

**Theorem 9.** An *nm-j-ω*-closed mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is weakly *nm- j-ω*-closed, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Proof:** Assume that $h \in \lambda(G)$ also, let $S$ be a $\sigma_n$-open set containing $\lambda^{-1}(h)$ in $G$, by Theorem (5) and $\lambda$ is *nm- j-ω*-closed mapping, then $nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(G - \mathrm{cl}_j^{\omega}(S)) \subset \lambda[(\sigma_n - \mathrm{cl}_j^{\omega}(G - \mathrm{cl}_j^{\omega}(S)]$. Since $h \notin \lambda[(\sigma_n - \mathrm{cl}_j^{\omega}(G - \mathrm{cl}_j^{\omega}(S)]$, and $h \notin nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(G - \mathrm{cl}_j^{\omega}(S))$. Thus, there exists an $ς_n$-open nbd $T$ of $h$ in $H$, $ς_n\text{-}\mathrm{cl}_j^{\omega}(T) \cap \lambda(G - \mathrm{cl}_j^{\omega}(S)) = \phi$, then $\lambda^{-1}(ς_m\text{-}\mathrm{cl}_j^{\omega}(T)) \cap \lambda(G - \mathrm{cl}_j^{\omega}(S)) = \phi$, i.e $\lambda^{-1}(ς_m\text{-}\mathrm{cl}_j^{\omega}(T)) \subset \mathrm{cl}_j^{\omega}(S)$, then $\lambda$ is weakly *nm- j-ω*-closed.

The inversion of the Theorem (9) is not be right, it will be shown by next example:

**Example 5.** Let $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ be a constant mapping and $\sigma_1$, $\sigma_2$ and $ς_1$, $ς_2$ be any topology, then $\lambda$ is weakly *nm- j-ω*-closed for $n, m$= 1and 2 such that $(n \neq m)$, let $G = H = \Re$. If $ς_1$ or $ς_2$ is discrete topology on $H$, then $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ given by $\lambda(g) = 0$, for every $g \in G$, is neither *12- j-ω*-closed nor *21- j-ω*-closed, regardless of the topologies $\sigma_1$, $\sigma_2$ also, $ς_2$ (or $ς_1$), where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 10.** An strongly *nm-j-ω*-closed mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is *nm- j-ω*-closed, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 11.** If an *nm-j-ω*-continuous mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is *nm-j-ω*-perfect, then:
(a) $\lambda$ is strongly *nm-j-ω*-closed.
(b) for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in $G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 12.** Let $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ be *nm-j-ω*-continuous mapping. Then $\lambda$ is *nm-j-ω*-perfect, if:
(a) $\lambda$ is weakly *nm- j-ω*- closed.
(b) for every $h \in H$, $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in $G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Proof:** Assume that $\lambda$ is *nm-j-ω*-continuous mapping then satisfying the condition for (a) and (b). To show that $\lambda$ is *nm-j-ω*-perfect, Theorem (7) show that $\lambda$ is *nm-j-ω*-closed, let $h \in nm\text{-}j\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for some non- null subset $K$ of $G$. However $h \notin \lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K))$, so $\pounds = \{K\}$ is a filter base on $G$, also $(nm\text{-}j\text{-}\omega\text{-}cod\ \pounds) \cap \lambda^{-1}(h) = \phi$, by *nm-j-ω*-rigidity of $\lambda^{-1}(h)$. There is $\sigma_n$-open set $S$ containing $\lambda^{-1}(h)$ such that $\mathrm{cl}_j^{\omega}(S) \cap K = \phi$, and by a mapping $\lambda$ is weakly *nm- j-ω*-closed, there exists an $ς_n$-open nbd $T$ of $h$, such that $\lambda^{-1}(ς_m\text{-}\mathrm{cl}_j^{\omega}(T)) \subset \mathrm{cl}_j^{\omega}(S)$. Then $\lambda^{-1}(ς_m\text{-}\mathrm{cl}_j^{\omega}(T)) \cap K = \phi$, i.e $(ς_m\text{-}\mathrm{cl}_j^{\omega}(T)) \cap \lambda(K) = \phi$, this is impossible because that $h \in nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$. So $h \in \lambda(nm\text{-}j\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K))$. Then $\lambda$ is *nm-j-ω*-closed.

## Study on some Types of *j*-ω-perfect Mappings in Bitopological Spaces.

In this section, *nm-j-ω*-perfect mappings are given and used the definitions of characterizations theorems for an *nm-j-ω*-continuous mapping and weakly *nm-j-ω*-continuous mapping and strongly *nm-j-ω*-continuous mapping and super *nm-j-ω*-continuous mapping and almost *nm-j-ω*-continuous mapping are indicated to this end, and $n, m$ = 1, 2 where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 13.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is *nm-j-ω*-continuous if $\lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for $n, m$= 1and 2 such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Proof:** ($\Rightarrow$) Assume that $h \in nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)$ and $T$ is $ς_n$- open nbd of $\lambda(g)$. Because of $\lambda$ is *nm-j-ω*-continuous, there exists a $\sigma_n$-open nbd $S$ of $g$ such that $\lambda(\mathrm{cl}_j^{\omega}(S)) \subset ς_m\text{-}\mathrm{cl}_j^{\omega}(T)$. Since, $\mathrm{cl}_j^{\omega}(S) \cap K \neq \phi$, then $ς_m\text{-}\mathrm{cl}_j^{\omega}(T) \cap \lambda(K) \neq \phi$. Thus, $\lambda(g) \in nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$. This shows that $\lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$ for $n, m$ = 1and 2 such that $(n \neq m)$
($\Leftarrow$) Clear.

**Theorem 14.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is weakly *nm-j-ω*-continuous if $\lambda(nm\text{-}\omega\text{-}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for $n, m$ = 1and 2 such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 15.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is strongly *nm-j-ω*- continuous if $\lambda(nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\lambda(K)$, for $n, m$ = 1and 2 such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 16.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is super *nm-j-ω*-continuous if $\lambda(nm\text{-}\omega\text{-int-}\mathrm{cl}_j^{\omega}(K)) \subset nm\text{-}\omega\text{-}\mathrm{cl}_j^{\omega}\lambda(K)$, for $n, m$ = 1and 2 such that $(n \neq m)$, for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 17.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, ς_1, ς_2)$ is almost *nm-ω*-continuous if $\lambda(nm\text{-}\omega\text{-}(K)) \subset nm\text{-}\omega\text{-int-}\mathrm{cl}_j^{\omega}\lambda(K)$, for $n, m$ = 1and 2 such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, pre, b, \beta$.

**Theorem 18.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be *nm-j-ω*-continuous and *nm- j-ω*-perfect, Then $\lambda^{-1}$ preserves *nm-j-ω*-rigidity, where $j = \theta, \delta, \alpha$, *pre, b*, $\beta$.

**Proof:** Assume that $L$ be an *nm- j-ω*-rigid set in $H$ and suppose $\Im$ be a filter base on $G$, then $\lambda^{-1}(L) \cap$ (*nm- j-ω-cod* $\Im$) = ϕ, since $\lambda$ is *nm- j-ω*-perfect and $L \cap \lambda($ *nm- j-ω-cod* $\Im)$ = ϕ. By Theorem (3 (a) ⇒ (c) ) then $L \cap$ ( *nm- j-ω-cod*$\lambda(\Im)$) = ϕ, now $L$ being an *nm- j-ω*-rigid set in $H$ , there exists an $M \in \Im$ such that $L \cap$ ( *nm-ω*-$\mathrm{cl}_j^{\omega}\lambda(M)$) = ϕ, since $\lambda$ is *nm- j-ω*-continuous, by Theorem (14) it follows that $L \cap \lambda$ ( *nm-ω*-$\mathrm{cl}_j^{\omega}(M)$) = ϕ. Then $\lambda^{-1}(L) \cap$ ( *nm-ω*-$\mathrm{cl}_j^{\omega}(M)$) = ϕ. This proves that $\lambda^{-1}(L)$ is *nm- j-ω*-rigid.

**Definition 18.** A subset $K$ of a bitopological space $(G, \sigma_1, \sigma_2)$ is said to be *nm-j-ω*-set in $G$ if for every $\sigma_n$-open cover $\mathcal{K}$ of $K$, there is a finite sub collection $\mathcal{L}$ of $\mathcal{K}$ such that $K \subset \cup \{ \mathrm{cl}_j^{\omega}(S) : L \in \mathcal{L}\}$, where $j = \theta, \delta, \alpha$, *pre, b*, $\beta$.

**Theorem 19.** Let $(G, \sigma_1, \sigma_2)$ be a bitopological space, and a subset $K$ of space for every filter base $\Im$ on $K$ such that (*nm-j-ω-cod* $\Im$) $\cap$ $K \neq$ ϕ, is an *nm-j-ω*-set, where $j = \theta, \delta, \alpha$, *pre, b*, $\beta$.

**Proof:** Let $\mathcal{K}$ be an $\sigma_n$-open cover of $K$, $\sigma_m$- *j-ω*-closed of union of any finite subcollection of $\mathcal{K}$ is not cover $K$. So $\Im = \{ K / \mathrm{cl}_j^{\omega}{}_g(\cup_{\mathcal{L}} (S_{\mathcal{L}})) : \mathcal{L}$ is finite subcollection of $\mathcal{K}\}$ is a filter base on $K$ and (*nm-j-ω-cod* $\Im$) $\cap K$ = ϕ, this contradiction yield that $K$ is an *nm- j-ω*-set.

**Theorem 20.** If $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is *nm-j-ω*-perfect, and $L \subset H$ is *nm- j-ω*-set in $H$, then $\lambda^{-1}(L)$ is an *nm- j-ω*-set in $G$, for $n, m$ = 1and 2 such that $(n \neq m)$, and where $j = \theta, \delta, \alpha$, *pre, b*, $\beta$.

**Proof:** Assume that $\Im$ be a filter base on $\lambda^{-1}(L)$, then $\lambda(\Im)$ is a filter base on $L$. Because $L$ is an *nm-j-ω*-set in $H$, such that $L \cap$ *nm- j-ω-cod* $\lambda(\Im) \neq$ ϕ, by Theorem (12). By Theorem (3 (a) ⇒ (c) ), $L \cap \lambda$(*nm- j-ω-cod* $\Im$) ≠ ϕ, so $\lambda^{-1}(L) \cap$ *nm- j-ω-cod* ($\Im$) ≠ ϕ. Therefore by Theorem (12), $\lambda^{-1}(L)$ is an *nm- j-ω*-set in $G$.

The inversion of the Theorem (20) is not right, as shown by the example following:

**Example 6.** Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be an identity mapping and $\sigma_1$, $\sigma_2$ be the cofinite and discrete topologies respectively on $G$, and $\varsigma_1$ , $\varsigma_2$ respectively denote the indiscrete and usual topologies on $H$ such that $G = H = \Re$,then every subset of either of $(G, \sigma_1, \sigma_2)$ and $(H, \varsigma_1, \varsigma_2)$ is a *12-j-ω*-set. Now , any nonvoid finite set $K \subset G$ is *12- j-ω*-closed in $G$, but $\lambda(K)$ (i.e $K$ ) is not *12-j-ω*-closed in $H$, (in fact, the only *12- j-ω*-closed subset of $H$ are $H$ and ϕ ), where $j = \theta, \delta, \alpha$, *pre, b*, $\beta$.

The Theorem (20) and the above Example (6) allude the definition of a strictly weaker transcription of *nm-j-ω*- perfect mapping as given below.

**Definition 19.** A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ is said to almost *nm-j-ω*-perfect if for every *nm-j-ω*-set $K$ in $H$, $\lambda^{-1}(K)$ is *nm-j-ω*-set in $G$, where $j = \theta, \delta, \alpha$, *pre, b*, $\beta$.

By analogy to Theorem (20), amplest condition for a mapping to be almost *nm-j-ω*-perfect, is prove as follows.

**Theorem 21.** Let $\lambda : (G, \sigma_1, \sigma_2) \to (H, \varsigma_1, \varsigma_2)$ be any mapping such that

(a) $\lambda^{-1}(h)$ is *nm-j-ω*-rigid in $G$, such that for every $h \in H$

(b) $\lambda$ is weakly *nm- j-ω*- closed.

Then $\lambda$ is almost *nm- j-ω*-perfect, where $j = \theta, \delta, \alpha$, *pre, b*, $\beta$.

**Proof:** Assume that $L$ be an *nm-j-ω*-set in $H$ and let that $\Im$ be a filter base on $\lambda^{-1}(L)$, then $\lambda(\Im)$ is a filter base on $L$. Also, by Theorem (20), (*nm-j-ω-cod* $\Im$) $\cap L \neq$ ϕ, let $h \in$ [(*nm- j-ω-cod* $\Im$)] $\cap L$. Assume that $\Im$ has no *nm- j-ω*-condensation point in $\lambda^{-1}(L)$, then (*nm- j-ω-cod* $\Im$) $\cap \lambda^{-1}(h)$ = ϕ. Because of $\lambda^{-1}(h)$ is *nm- j-ω*-rigid in $G$, there exists an $M \in \Im$ and a $\sigma_n$-open $S$ containing $\lambda^{-1}(h)$, such that $M \cap \sigma_n$- $\mathrm{cl}_j^{\omega}(S)$ = ϕ. By $\lambda$ is weakly *nm- j-ω*- closed, then there is a $\varsigma_m$ –open nbd $T$ of $h$, $\lambda^{-1}(\varsigma_m - \mathrm{cl}_j^{\omega}(T)) \subset \sigma_n$- $\mathrm{cl}_j^{\omega}(S)$. Therefore which implies that $\lambda^{-1}(\varsigma_m - \mathrm{cl}_j^{\omega}(T)) \cap M$ = ϕ, i.e., $\varsigma_m$ - $\mathrm{cl}_j^{\omega}(T) \cap \lambda(M)$ = ϕ, which is a contradiction . Therefore by Theorem (20), $\lambda^{-1}(L)$ is an *nm- j-ω*-set in $G$. So $\lambda$ is almost *nm-j-ω*-perfect.

## Conclusion.

The main purpose of the present work is the starting point for some application of pairwise supra-*ω*-perfect mappings of abstract topological structures in filter base by using bitopological spaces. Definitions of characterizations theorems are used for an *nm- j-ω*-continuous mapping and weakly *nm- j-ω*-continuous mapping and strongly *nm- j-ω*-continuous mapping and super *nm- j-ω*-continuous mapping and almost *nm- j-ω*-continuous mapping.

## Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
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# بعض انواع التطبيقات في الفضاءات التبولوجية الثنائية

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**الخلاصة:**

قدمنا بعض المفاهيم في الفضاءات التبولوجية الثنائية وهي الاقتراب من المجموعة الجزئية من النمط *nm-j-ω* ، الاتجاه المباشر لمجموعة من النمط *nm- j-ω* ، التطبيقات المغلقة من النمط *nm- j-ω* ، صلابة المجموعة من النمط *nm- j-ω* ، التطبيقات المستمرة من النمط *nm- j-ω* ، والخط الرئيسي لهذا البحث هو التطبيقات التامة من النمط *nm- j-ω* في الفضاءات التبولوجية الثنائية. المميزات المتعلقة بهذه المفاهيم والعديد من المبرهنات قد درسنا حيث $j = \theta, \delta, \alpha, pre, b, \beta$.

**الكلمات المفتاحية :** المرشحات الاساسية ، التقارب من النمط *nm-j-ω* ، التطبيقات المغلقة من النمط *nm-j-ω* ، مجموعة صلبة من النمط *j-ω* ، التطبيقات التامة من النمط *nm-j-ω*.