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Some Games Via Ĩ- Semi- g- Separation Axioms

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Abstract:

The research demonstrates new species of the games by applying separation axioms via Ĩ-$semi$-g-$open$ sets, where the relationships between the various species that were specified and the strategy of winning and losing to any one of the players, and their relationship with the concepts of separation axioms via Ĩ-$semi$-g-$open$ sets have been studied.

Key words: Ĩsg-open set, Ĩsg-closed set, Ĩsgo-function, Ĩsg-continuous-function and game.

Introduction:

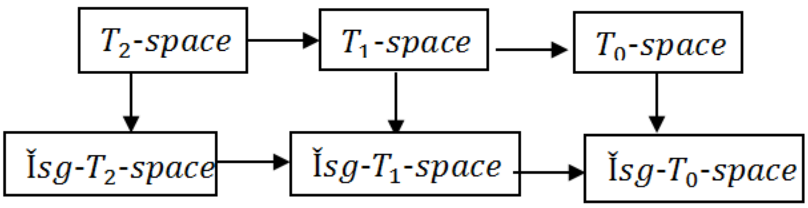
This previously connotation of ideals on a science of topology was proffered by Kuratowski in 1933(1) and Vaidynathaswamy in 1945(2). An ideal Ĩ on Ӿ $\neq \phi$ is a family of subsets of Ӿ that encompass the two prerequisites finite additivity (Ạ and Ḅ ∈ Ĩ $implies$ Ạ ∪ Ḅ ∈ Ĩ) and heredity (Ạ ⊆ Ḅ and Ḅ ∈ Ĩ $implies$ Ạ ∈ Ĩ) (1, 2).Let (Ӿ, τ) be a topological space, a subset $\mathcal{U}$ of Ӿ is a semi-open if $\mathcal{U}$ ⊆ int(cl($\mathcal{U}$)) (4).In a newly time, there are those who have insert a topic Ĩ-$semi$-g-$closed$ set for the ideal spaces (Ӿ, τ, Ĩ) by use the connotation of semi-open sets (3), if Ạ-$\mathcal{U}$ ∈ Ĩ where $\mathcal{U}$ is semi-open in (Ӿ, τ, Ĩ) implies cl(Ạ)-$\mathcal{U}$ ∈ Ĩ, then the set Ạ be Ĩ-$semi$-g-$closed$ set (artlessly, Ĩsg-$closed$). So, the set Ạ denominates Ĩ-$semi$-g-$open$ (frugally,Ĩsg-$open$) if Ӿ-Ạ is Ĩsg-$closed$. And the symbol Ĩsg-C(Ӿ) (respectively,Ĩsg-O(Ӿ)) is the family of all Ĩsg-$closed$ (respectively, Ĩsg-$open$) sets in (Ӿ, τ, Ĩ).Notes that; every open set in (Ӿ, τ) is Ĩsg-$open$ in (Ӿ, τ, Ĩ).The separation axioms via Ĩsg-$open$ sets were proffered by (5) and which include many definitions as follows:

i- A space (Ӿ, τ, Ĩ) is Ĩ-$semi$-g-T_0-$space$ (artlessly,Ĩsg-T_0-$space$), if for any elements $ɕ_1 \neq ɕ_2$, there is an Ĩsg-$open$ set $\mathcal{U}$ containing only one of them. For any ideal Ĩ on Ӿ, the fact of a space (Ӿ, τ) T_0-$space$ is enough to be the space (Ӿ, τ, Ĩ) Ĩsg-T_0-$space$.

ii- A space (Ӿ, τ, Ĩ) is Ĩ-$semi$-g-T_1-$space$ (artlessly,Ĩsg-T_1- $space$), if for any elements $ɕ_1 \neq ɕ_2$, there are two Ĩsg-$open$ sets $\mathcal{U}$ and $\mathcal{V}$ such that ($ɕ_1 \in \mathcal{U}$-$\mathcal{V}$) and ($ɕ_2 \in \mathcal{V}$-$\mathcal{U}$). For any ideal Ĩ on Ӿ, the fact of a space (Ӿ, τ) T_1-$space$ is enough to be the space (Ӿ, τ, Ĩ) Ĩsg-T_1-$space$.

iii- A space (Ӿ, τ, Ĩ) is Ĩ-$semi$-g-T_2-$space$ (artlessly, Ĩsg-T_2-$space$), if for any elements $ɕ_1 \neq ɕ_2$, there are two disjoint Ĩsg-$open$ sets $\mathcal{U}$ and $\mathcal{V}$ such that $ɕ_1 \in \mathcal{U}$ and $ɕ_2 \in \mathcal{V}$. Obviously, if (Ӿ, τ) is T_2-$space$, then (Ӿ, τ, Ĩ) is Ĩsg-T_2-$space$.

The relations among the different types of separation axioms that introduced above are discussed in (5), as the following scheme shows:



Scheme 1. The relations among the different types of separation axioms.

All games that are species in this research from a kind"Two-Zero-Sum Games", where the games will be defined between the two players Player I and Player II and there are choices; $M_1, M_2, M_3, \dots, M_r$ for each player, are called moves or options. If player chooses one move, then this process of it is called one step. When the game has more than one step, it is called a repeated or iterated game. This number of repetition of this game is finite or infinite. The player gets a certain reward after each step. The payoff for any one of the players equal to the loss of the other player (6,7).

These games have two types; alternating games and simultaneous games and the game is alternating if one of players (Player I) chooses one of the moves, then the other (Player II) chooses one of the other moves after that Player I know the move of the first player. In these alternating games we must note the player who will start for playing. The game is simultaneous if both players choose their moves in the same time, but none of them knows the choice of the other (6,7).

A function S is a strategy for (Player I) define as follows; $S = \{S_m : A_{m-1} \times B_{m-1} \rightarrow A_m$, such that $(A_1, B_1, \dots, A_{m-1}, B_{m-1}) = A_m\}$. Similarly, a function T is a strategy for (Player II) define as follows; $T = \{T_m ; A_m \times B_{m-1} \rightarrow B_m$, such that $(A_1, B_1, \dots, A_{m-1}, B_{m-1}, A_m) = B_m\}$. (6, 7).

Analytical study of games in ideal topological spaces

In this section, we insert new games by linking them with separation axioms via open (respectively, $\tilde{I}sg\text{-}open$) sets. At the beginning of this explanation there are two entrants which are the first player (frugally, Player I) and second player (frugally, Player II). And in these games there are finite or infinite innings and every inning has two steps for the two players, first step and second step (6, 7). The players are playing an inning for each positive integer in the game. In these games we provide the winning strategy and the losing strategy for each player, we symbolize to the winning strategy by ↗ and the losing strategy by ↘ and we will symbolize by ⤱ (respectively,⤰) when a player does not have a winning strategy (respectively, losing strategy).

Definition 1: For an ideal space$(X, \tau, \tilde{I})$, determine a game $\acute{G}(T_0, X)$(respectively,$\acute{G}(T_0, X, \tilde{I})$) as fallows

Player I and Player II are playing an inning for each positive integer in this game in the $r\text{-}th$ inning:

In the first step, Player I : Choose $ç_r \neq ѵ_r$ where $ç_r, ѵ_r \in X$

In second step, Player II : Choose $\mathcal{U}_r$ an open (respectively,$\tilde{I}sg\text{-}open$) set containing only one of the two elements $ç_r, ѵ_r$.

Then Player II wins in the game $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) if $B = \{B_1, B_2, B_3, \dots, B_r, \dots\}$ be a collection of an open (respectively, $Isg\text{-}open$) sets in X such that $\forall\, ç_r \neq ѵ_r$ in X, $\exists B_r \in B$ containing only one of two element $ç_r, ѵ_r$. Otherwise, Player I wins.

Example1: Consider the game $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$), where $X = \{ç, ѵ, ȥ\}$, $\tau = \{X, \phi, \{ç\}, \{ȥ\}, \{ç, ȥ\}\}$ and $\check{I} = \{\phi\}$ as follows:

Player I and Player II are playing for a three innings. Then the first inning is as follows:

Player I : Choose $ç \neq ѵ$ where $ç, ѵ \in X$

Player II : Choose $\mathcal{U}_1 = \{ç\}$ is an open (respectively, $Isg\text{-}open$) set containing only one of the two elements $ç, ѵ$.

Then, the next inning (the second inning) is as follows:

Player I : Choose $ѵ \neq ȥ$ where $ѵ, ȥ \in X$

Player II : Choose $\mathcal{U}_2 = \{ȥ\}$ is an open (respectively, $Isg\text{-}open$) set containing only one of the two elements $ѵ, ȥ$.

Then, the third inning is as follows:

Player I : Choose $ç \neq ȥ$ where $ç, ȥ \in X$

Player II : Choose $\mathcal{U}_1 = \{ç\}$ is an open (respectively, $Isg\text{-}open$) set containing only one of the two elements $ç, ȥ$.

Then $B = \{\mathcal{U}_1, \mathcal{U}_2\}$ is the winning strategy for Player II in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$).

Hence, Player II ↗ $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$).

Remark 1: For a space $(X, \tau, \tilde{I})$:

i- If Player II ↗ $\acute{G}(T_0, X)$, then Player II ↗ $\acute{G}(T_0, X, \tilde{I})$. Therefore, Player II ↘ $\acute{G}(T_0, X)$ then Player II ↘ $\acute{G}(T_0, X, \tilde{I})$.

ii- If Player I ↗ $\acute{G}(T_0, X, \tilde{I})$, then Player I ↗ $\acute{G}(T_0, X)$.

iii- If Player II ↘ $\acute{G}(T_0, X)$ then Player II ↘ $\acute{G}(T_0, X, \tilde{I})$.

Proposition 1:

i- A space (X, τ) is $T_0\text{-}space$ if and only if, Player II ↗ $\acute{G}(T_0, X)$.

ii- A space $(X, \tau, \tilde{I})$ is $Isg\text{-}T_0\text{-}space$ if and only if, Player II ↗ $\acute{G}(T_0, X, \tilde{I})$.

Proof:

i- Necessity; Suppose that (X, τ) is $T_0\text{-}space$. For any choose $ç_1 \neq ç_2$ in X of Player I , then Player II will be find an open set $\mathcal{U}$ containing only one of them. Hence, Player II ↗ $\acute{G}(T_0, X)$.

Sufficient; If Player II ↗ $\acute{G}(T_0, X)$, this mean that for any choose $ç_1 \neq ç_2$ in X of Player I , implies that Player II can be choose an open set $\mathcal{U}$ containing only one of them. So, (X, τ) is $T_0\text{-}space$.

ii- Similar of proof (i).

Corollary 1: For a space $(Ҳ, τ, \tilde{I})$:

i- Player II $\nearrow Ǵ(T_0, Ҳ)$ if and only if $\forall ɕ_1 \neq ɕ_2$ in Ҳ, $\exists Ạ$ is closed set where $ɕ_1 \in Ạ$ and $ɕ_2 \notin Ạ$.

ii- Player II $\nearrow Ǵ(T_0, Ҳ, \tilde{I})$ if and only if $\forall ɕ_1 \neq ɕ_2$ in Ҳ, $\exists Ḅ$ is Isg-$closed$ set where $ɕ_1 \in Ḅ$ and $ɕ_2 \notin Ḅ$.

Proof: Clearly, by using the concept of the complement and Proposition 1.

Corollary 2:

i- A space $(Ҳ, τ)$ is T_0-$space$ if and only if, Player I $\not\nearrow Ǵ(T_0, Ҳ)$.

ii- A space $(Ҳ, τ, \tilde{I})$ is Isg-T_0-$space$ if and only if, Player I $\not\nearrow Ǵ(T_0, Ҳ, \tilde{I})$.

Proof:

i- Necessity; Suppose that $(Ҳ, τ)$ is T_0-$space$. If Player I $\nearrow Ǵ(T_0, Ҳ)$, then there are two elements $ɕ_1 \neq ɕ_2$ in Ҳ such that there is no open set $\mathcal{U}$ containing only one of them. This is a contradiction.

Sufficient; Suppose that Player I $\not\nearrow Ǵ(T_0, Ҳ)$. If a space $(Ҳ, τ)$ is not T_0-$space$, then there are two elements $ɕ_1 \neq ɕ_2$ in Ҳ such that there is no open set $\mathcal{U}$ containing only one of them. So, Player I can choices these two elements $ɕ_1$ and $ɕ_2$ and winning the game. This is a contradiction.

ii- Similar of proof (i).

Proposition 2:

i- A space $(Ҳ, τ)$ is not T_0-$space$ if and only if Player I $\nearrow Ǵ(T_0, Ҳ)$.

ii- A space $(Ҳ, τ, \tilde{I})$ is not Isg-T_0-$space$ if and only if Player I $\nearrow Ǵ(T_0, Ҳ, \tilde{I})$.

Proof:

i- Necessity; Suppose that $(Ҳ, τ)$ is not T_0-$space$, then there are two elements $ɕ_1 \neq ɕ_2$ in Ҳ such that there is no open set $\mathcal{U}$ contain only one of them. So, that the two elements $ɕ_1 \neq ɕ_2$ will be the choice of Player I, then Player II cannot be find an open set $\mathcal{U}$ containing only one of them. Hence, Player I $\nearrow Ǵ(T_0, Ҳ)$.

Sufficient; If Player I $\nearrow Ǵ(T_0, Ҳ)$, this mean that there is a choice $ɕ_1 \neq ɕ_2$ in Ҳ of Player I, such that Player II cannot be choose an open set $\mathcal{U}$ containing only one of them. Hence, $(Ҳ, τ)$ is not T_0-$space$.

ii- Similar of proof (i).

Corollary 3:

i- A space $(Ҳ, τ)$ is not T_0-$space$ if and only if Player II $\not\nearrow Ǵ(T_0, Ҳ)$.

ii- A space $(Ҳ, τ, \tilde{I})$ is not Isg-T_0-$space$ if and only if Player II $\not\nearrow Ǵ(T_0, Ҳ, \tilde{I})$.

Proof:

i- Necessity; Suppose that $(Ҳ, τ)$ is not T_0-$space$. If Player II $\nearrow Ǵ(T_0, Ҳ)$, then for any two elements $ɕ_1 \neq ɕ_2$ in Ҳ, there is an open set $\mathcal{U}$ containing only one of them. This is a contradiction.

Sufficient; Suppose that Player II $\not\nearrow Ǵ(T_0, Ҳ)$. If $(Ҳ, τ)$ is T_0-$space$, then for any two elements $ɕ_1 \neq ɕ_2$ in Ҳ, there is an open set $\mathcal{U}$ containing only one of them. So, Player II will be win the game. This is a contradiction.

ii- Similar of proof (i).

Definition 2: For an ideal space $(Ҳ, τ, \tilde{I})$, determine a game $Ǵ(T_1, Ҳ)$(respectively,$Ǵ(T_1, Ҳ, \tilde{I})$) as fallows; Player I and Player II are play an inning with each natural numbers in this game in the r-th inning:

In the first step, Player I: Choose $ɕ_r \neq ѵ_r$ where $ɕ_r, ѵ_r \in Ҳ$

In second step, Player II: Choose $\mathcal{U}_r, \mathcal{V}_r$ are two open (respectively, Isg-$open$) sets such that $ɕ_1 \in \mathcal{U}_r$- $\mathcal{V}_r$ and $ɕ_2 \in \mathcal{V}_r$-$\mathcal{U}_r$.

Then Player II wins in the game $Ǵ(T_1, Ҳ)$ (respectively, $Ǵ(T_1, Ҳ, \tilde{I})$) if $Ḅ = \{\{Ạ_1, Ḅ_1\}, \{Ạ_2, Ḅ_2\}, \{Ạ_3, Ḅ_3\}, \dots, \{Ạ_r, Ḅ_r\}, \dots\}$ be a collection of an open (respectively, Isg-$open$) sets in Ҳ such that $\forall ɕ_r \neq ѵ_r$ in Ҳ, $\exists\{Ạ_r, Ḅ_r\} \in Ḅ$ containing only one of two elements $ɕ_r, ѵ_r$. Otherwise, Player I wins in the game $Ǵ(T_1, Ҳ)$ (respectively, $Ǵ(T_1, Ҳ, \tilde{I})$).

Example 2: Consider the game $Ǵ(T_1, Ҳ)$ (respectively, $Ǵ(T_1, Ҳ, \tilde{I})$), where $Ҳ = \{ɕ, ѵ, ʑ\}$, $τ = Ƥ(Ҳ)$ and $Ǐ = \{\phi\}$ as follows:

Player I and Player II are playing for three innings in this game. Then the first inning is as follows:

Player I: Choose $ɕ \neq ѵ$ where $ɕ, ѵ \in Ҳ$

Player II: Choose $\mathcal{U}_1 = \{ɕ\}$, $\mathcal{V}_1 = \{ѵ\}$ which are open (respectively, Isg-$open$) sets, such that $ɕ \in \mathcal{U}_1$- $\mathcal{V}_1$ and $ѵ \in \mathcal{V}_1$-$\mathcal{U}_1$.

Then the next inning (the second inning) is as follows:

Player I: Choose $ѵ \neq ʑ$ where $ѵ, ʑ \in Ҳ$

Player II: Choose $\mathcal{U}_2 = \{ѵ\}$, $\mathcal{V}_2 = \{ʑ\}$ which are open (respectively, Isg-$open$) sets, such that $ѵ \in \mathcal{U}_2$- $\mathcal{V}_2$ and $ʑ \in \mathcal{V}_2$-$\mathcal{U}_2$.

Then the next inning (the third inning) is as follows:

Player I: Choose $ɕ \neq ʑ$ where $ɕ, ʑ \in Ҳ$

Player II: Choose $\mathcal{U}_3 = \{ɕ\}$, $\mathcal{V}_3 = \{ʑ\}$ which are open (respectively, Isg-$open$) sets, such that $ɕ \in \mathcal{U}_3$- $\mathcal{V}_3$ and $ʑ \in \mathcal{V}_3$-$\mathcal{U}_3$.

Then $Ḅ = \{\{\mathcal{U}_1, \mathcal{V}_1\}, \{\mathcal{U}_2, \mathcal{V}_2\}, \{\mathcal{U}_3, \mathcal{V}_3\}\}$ is the winning strategy for Player II in $Ǵ(T_1, Ҳ)$ (respectively, $Ǵ(T_1, Ҳ, \tilde{I})$).

Hence, Player II $\nearrow Ǵ(T_1, Ҳ)$ (respectively, Player II $\nearrow Ǵ(T_1, Ҳ, \tilde{I})$).

Remark 2: For a space $(Ҳ, τ, \tilde{I})$:

i- If Player II $\nearrow Ǵ(T_1, Ҳ)$, then Player II $\nearrow Ǵ(T_1, Ҳ, \tilde{I})$. Therefore, Player II $\searrow Ǵ(T_1, Ҳ)$ then Player II $\searrow Ǵ(T_1, Ҳ, \tilde{I})$.

ii- If Player I $\nearrow$ Ǵ$(T_1, X, \tilde{I})$, then Player I $\nearrow$ Ǵ(T_1, X).
iii- If Player II $\searrow$ Ǵ(T_1, X), then Player II $\searrow$ Ǵ$(T_1, X, \tilde{I})$.

Proposition 3: For a space $(X, \tau, \tilde{I})$:
i- A space (X, τ) is T_1-*space* if and only if Player II $\nearrow$ Ǵ(T_1, X).
ii- A space $(X, \tau, \tilde{I})$ is *Isg*-T_1-*space* if and only if Player II $\nearrow$ Ǵ$(T_1, X, \tilde{I})$.

Proof:
i- Necessity; Suppose that (X, τ) is T_1-*space*. For any choose $\varsigma_1 \neq \varsigma_2$ in X of Player I, then Player II will be find two open sets $\mathcal{U}$ and $\mathcal{V}$, such that $\varsigma_1 \in \mathcal{U}$-$\mathcal{V}$ and $\varsigma_2 \in \mathcal{V}$-$\mathcal{U}$. Hence, Player II $\nearrow$ Ǵ(T_1, X).
Sufficient; If Player II $\nearrow$ Ǵ(T_1, X), this mean that for any choice $\varsigma_1 \neq \varsigma_2$ in X of Player I, implies that Player II can be choose two open sets $\mathcal{U}$ and $\mathcal{V}$, such that $\varsigma_1 \in \mathcal{U}$-$\mathcal{V}$ and $\varsigma_2 \in \mathcal{V}$-$\mathcal{U}$. Hence, (X, τ) is T_1-*space*.
ii- Similar of proof (i).

Corollary 4: For a space $(X, \tau, \tilde{I})$:
i- Player II $\nearrow$ Ǵ(T_1, X) if $\forall\, \varsigma_1 \neq \varsigma_2$ in X $\exists \mathcal{U}, \mathcal{V}$ two closed sets where $\varsigma_1 \in \mathcal{U}$-$\mathcal{V}$ and $\varsigma_2 \in \mathcal{V}$-$\mathcal{U}$.
ii- Player II $\nearrow$ Ǵ$(T_1, X, \tilde{I})$ if $\forall\, \varsigma_1 \neq \varsigma_2$ in X $\exists \mathcal{U}, \mathcal{V}$ is *Isg-closed* set where $\varsigma_1 \in \mathcal{U}$-$\mathcal{V}$ and $\varsigma_2 \in \mathcal{V}$-$\mathcal{U}$.

Proof: Clearly, by using the concept of the complement and Proposition 3.

Corollary 5:
i- A space (X, τ) is T_1-*space* if and only if Player I $\not\nearrow$ Ǵ(T_1, X).
ii- A space $(X, \tau, \tilde{I})$ is *Isg*-T_1-*space* if and only if Player I $\not\nearrow$ Ǵ$(T_1, X, \tilde{I})$.

Proof: similar of proof Corollary 2.

Proposition 4:
i- A space (X, τ) is not T_1-*space* if and only if Player I $\nearrow$ Ǵ(T_1, X).
ii- A space $(X, \tau, \tilde{I})$ is not *Isg*-T_1-*space* if and only if Player I $\nearrow$ Ǵ$(T_1, X, \tilde{I})$.

Proof:
i- Necessity; Suppose that (X, τ) is not T_1-*space*, then there are two elements $\varsigma_1 \neq \varsigma_2$ in X such that there are not two open sets $\mathcal{U}$ and $\mathcal{V}$, where $\varsigma_1 \in \mathcal{U}$-$\mathcal{V}$ and $\varsigma_2 \in \mathcal{V}$-$\mathcal{U}$. So, that the two elements $\varsigma_1 \neq \varsigma_2$ will be the choice of Player I, then Player II cannot be find two open sets $\mathcal{U}$ and $\mathcal{V}$, where $\varsigma_1 \in \mathcal{U}$-$\mathcal{V}$ and $\varsigma_2 \in \mathcal{V}$-$\mathcal{U}$. Hence, Player I $\nearrow$ Ǵ(T_1, X).
Sufficient; If Player I $\nearrow$ Ǵ(T_1, X), this mean that there is a choice $\varsigma_1 \neq \varsigma_2$ in X of Player I, such that Player II cannot be choose two open sets $\mathcal{U}$ and $\mathcal{V}$, where $\varsigma_1 \in \mathcal{U}$-$\mathcal{V}$ and $\varsigma_2 \in \mathcal{V}$-$\mathcal{U}$. Hence, (X, τ) is not T_1-*space*.
ii- Similar of proof (i).

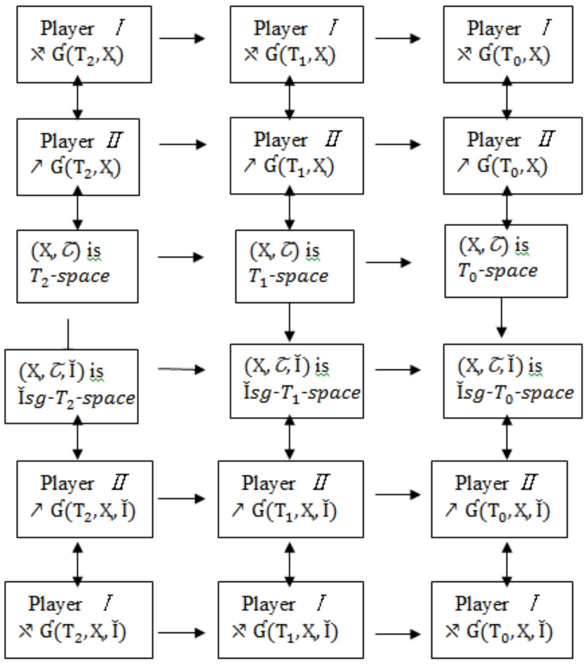
Corollary 6:
i- A space (X, τ) is not T_1-*space* if and only if Player II $\not\nearrow$ Ǵ(T_1, X).
ii- A space $(X, \tau, \tilde{I})$ is not *Isg*-T_1-*space* if and only if Player II $\not\nearrow$ Ǵ$(T_1, X, \tilde{I})$.

Proof: similar of proof Corollary 2.

So, by the same way of Definition 2, the game Ǵ(T_2, X) (respectively, Ǵ$(T_2, X, \tilde{I})$) is such us the game (T_1, X) (respectively, Ǵ$(T_1, X, \tilde{I})$) except $\mathcal{U}, \mathcal{V}$ are two disjoint open (respectively, *Isg-open*) sets and Ḅ be a collection of a disjoint open (respectively, *Isg-open*) sets in X, and it has the same remarks, propositions an corollaries in the game (T_1, X) (respectively, Ǵ$(T_1, X, \tilde{I})$)

Remark 3: For a space $(X, \tau, \tilde{I})$:
i- If Player II $\nearrow$ Ǵ(T_{i+1}, X) (respectively, Ǵ$(T_{i+1}, X, \tilde{I})$); $i = \{0,1\}$, then Player II $\nearrow$ Ǵ(T_i, X) (respectively, Ǵ$(T_i, X, \tilde{I})$).
ii- If Player II $\nearrow$ Ǵ(T_i, X); $i = \{0, 1, 2\}$, then Player II $\nearrow$ Ǵ$(T_i, X, \tilde{I})$.

The following scheme clarifies what exactly Remark 3, refer to.



Scheme 2. Diagram of the implication in Remark 3.

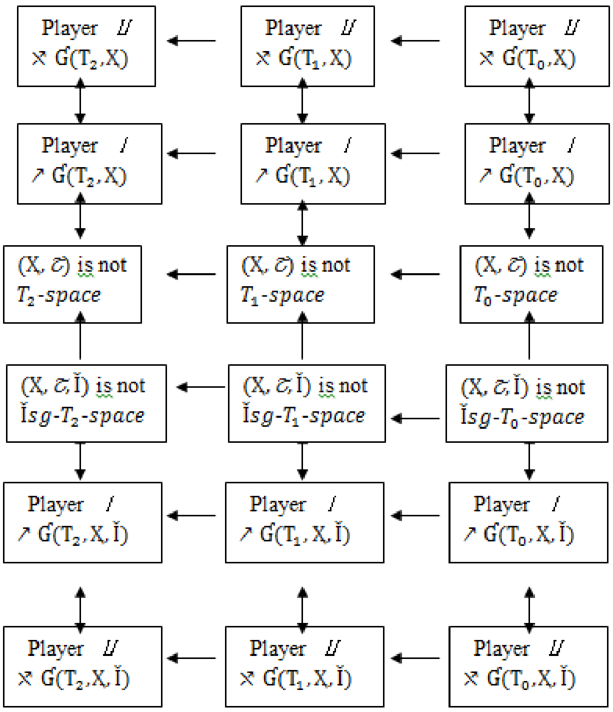
The proof of the meaning conclusion in the above diagram is given by the meaning of figure (3) and Propositions 1, 2, 3, 4 and there corollaries.

Remark 4: For a space $(X, \tau, \tilde{I})$:

i- If Player I $\nearrow \acute{G}(T_i, X)$ (respectively, $\acute{G}(T_i, X, \tilde{I})$); $i = \{0,1\}$ then Player I $\nearrow \acute{G}(T_{i+1}, X)$ (respectively, $\acute{G}(T_{i+1}, X, \tilde{I})$).

ii- If Player I $\nearrow \acute{G}(T_i, X)$; $i = \{0,1,2\}$ then Player I $\nearrow \acute{G}(T_i, X, \tilde{I})$.

The following Scheme clarifies what exactly Remark 4, refer to.



Scheme 3. Diagram of the implication in Remark 4.

Some open functions with games

In this portion we will review some sorts of functions which presented by Abdel Karim and Nasir (3) and study their impact on the games that was previously defined.

Definition 3 (3): The function $\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{I}, \check{J})$ is;

i- *I-semi-g-open* function, notate as $Isgo\text{-}function$, if $\mathcal{F}(\mathcal{U})$ is $\check{J}sg\text{-}open$ set in $\dot{Y}$ whenever $\mathcal{U}$ is an $\tilde{I}sg\text{-}open$ set in X.

ii- I^**-semi-g-open* function, notate as $I^*sgo\text{-}function$, if $\mathcal{F}(\mathcal{U})$ is $\check{J}sg\text{-}open$ set in $\dot{Y}$ whenever $\mathcal{U}$ is an open set in X.

iii- I^{**}*-semi-g-open* function, notate as $I^{**}sgo\text{-}function$, if $\mathcal{F}(\mathcal{U})$ is open in $\dot{Y}$ whenever $\mathcal{U}$ is an $\tilde{I}sg\text{-}open$ set in X.

Example 3: The function $\mathcal{F} : (X, \tau, \tilde{I}) \to (X, \tau, \check{J})$ where $\mathcal{F}(\wp_1) = \wp_2$, $\mathcal{F}(\wp_2) = \wp_1$ and $\mathcal{F}(\wp_3) = \wp_3$ such that $X = \{\wp_1, \wp_2, \wp_3\}$, $\tau = \{X, \phi, \{\wp_1\}\}$, $\tilde{I} = \{\phi\}$ and $\check{J} = \{\phi, \{\wp_2\}, \{\wp_3\}, \{\wp_2, \wp_3\}\}$. Then $\mathcal{F}$ is $Isgo\text{-}function$ and $I^*sgo\text{-}function$ which is not $I^{**}sgo\text{-}function$ and not open function.

Example4: The identity function $\mathcal{F} : (X, \tau, \tilde{I}) \to (X, \tau, \check{J})$; $\mathcal{F}(\wp) = \wp$ for all $\wp \in X$, where $X = \{\wp_1, \wp_2, \wp_3\}$, $\tau = \{X, \phi, \{\wp_1\}\}$, $\tilde{I} = \{\phi, \{\wp_2\}, \{\wp_3\}, \{\wp_2, \wp_3\}\}$ and $\check{J} = \{\phi\}$. It is possible to see that $\mathcal{F}$ is open function and $I^*sgo\text{-}function$ which is not $Isgo\text{-}function$ and not $I^{**}sgo\text{-}function$.

Proposition 5: Let $\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{I}, \check{J})$ be an open (respectively, *I-semi-g-open*), surjective function "onto- function" and Player II $\nearrow \acute{G}(T_i, X)$ (respectively, $\acute{G}(T_i, X, \tilde{I})$); $i=\{0,1,2\}$. Then Player II $\nearrow \acute{G}(T_i, \dot{Y})$ (respectively, $\acute{G}(T_i, \dot{Y}, \check{J})$).

Proof:

If $i = 0$: Player I in $\acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \check{J})$) choose $\wp_1 \neq \vartheta_1$ where $\wp_1, \vartheta_1 \in \dot{Y}$, Player II in $\acute{G}(T_0, \dot{Y})$ will hold account $\mathcal{F}^{-1}(\wp_1) \neq \mathcal{F}^{-1}(\vartheta_1)$ where $\mathcal{F}^{-1}(\wp_1), \mathcal{F}^{-1}(\vartheta_1) \in X$, but Player II $\nearrow \acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$), $\exists \mathcal{U}_1$ open (respectively, *I-semi-g-open*) set; $\mathcal{U}_1$ contains one of the two elements $\mathcal{F}^{-1}(\wp_1), \mathcal{F}^{-1}(\vartheta_1)$ and since $\mathcal{F}$ is open (respectively, *I-semi-g-open*) function, then $\mathcal{F}(\mathcal{U}_1)$ open (respectively, *I-semi-g-open*) set, then Player II in $\acute{G}(T_0, \dot{Y})$ choose $\mathcal{F}(\mathcal{U}_1)$ contains one of the two elements $\wp_1, \vartheta_1$.In the next inning Player I in $\acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \check{J})$) choose $\wp_2 \neq \vartheta_2$ where $\wp_2, \vartheta_2 \in \dot{Y}$ Player II in $\acute{G}(T_0, \dot{Y})$ will hold account $\mathcal{F}^{-1}(\wp_2) \neq \mathcal{F}^{-1}(\vartheta_2)$ where $\mathcal{F}^{-1}(\wp_2), \mathcal{F}^{-1}(\vartheta_2) \in X$, but Player II $\nearrow \acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$), $\exists \mathcal{U}_2$ open (respectively, *I-semi-g-open*) set; $\mathcal{U}_2$ contains one of the two elements $\mathcal{F}^{-1}(\wp_2), \mathcal{F}^{-1}(\vartheta_2)$ then $\mathcal{F}(\mathcal{U}_2)$ open (respectively, *I-semi-g-open*) set, then for Player II in $\acute{G}(T_0, \dot{Y})$ choose $\mathcal{F}(\mathcal{U}_2)$ which contains one of the two elements $\wp_2, \vartheta_2$.

And in the r-th inning Player I in $\acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \check{J})$) choose $\wp_r \neq \vartheta_r$ where $\wp_r, \vartheta_r \in \dot{Y}$, Player II in $\acute{G}(T_0, \dot{Y})$ will hold account $\mathcal{F}^{-1}(\wp_r) \neq \mathcal{F}^{-1}(\vartheta_r)$ where $\mathcal{F}^{-1}(\wp_2), \mathcal{F}^{-1}(\vartheta_2) \in X$, but Player II $\nearrow \acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$), $\exists \mathcal{U}_r$ open (respectively, *I-semi-g-open*) set; $\mathcal{U}_r$ contains one of the two elements $\mathcal{F}^{-1}(\wp_r), \mathcal{F}^{-1}(\vartheta_r)$, $r = 1,2,\dots$, then $\mathcal{F}(\mathcal{U}_r)$ open (respectively, *I-semi-g-open*) set, then Player II in $\acute{G}(T_0, \dot{Y})$ choose $\mathcal{F}(\mathcal{U}_r)$ which contains one of the two elements $\wp_r, \vartheta_r$, $r = 1,2,\dots$.

Then $\dot{B} = \{\mathcal{F}(\mathcal{U}_1), \mathcal{F}(\mathcal{U}_2), \mathcal{F}(\mathcal{U}_3), \dots, \mathcal{F}(\mathcal{U}_r), \dots\}$ is the winning strategy for Player II in $\acute{G}(T_0, \dot{Y})$.

Hence

Player II $\nearrow \acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \check{J})$).

If $i = 1$: In the r-th inning of the game $\acute{G}(T_1, \dot{Y})$ (respectively, $\acute{G}(T_1, \dot{Y}, \check{J})$) Player I in $\acute{G}(T_1, \dot{Y})$ (respectively, $\acute{G}(T_1, \dot{Y}, \check{J})$) choose $c_r \neq v_r$ where $c_r, v_r \in \dot{Y}$, $r = 1,2, \dots$, Player II in $\acute{G}(T_1, \dot{Y})$ will hold account $\mathcal{F}^{-1}(c_r) \neq \mathcal{F}^{-1}(v_r)$ where $\mathcal{F}^{-1}(c_r), \mathcal{F}^{-1}(v_r) \in X$, but Player II $\nearrow \acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$), $\exists \mathcal{U}_r$ open (respectively, I-$semi$-g-$open$) set; $\mathcal{U}_r, \mathcal{V}_r$ are two open (respectively, Isg-$open$) sets such that $\mathcal{F}^{-1}(c_r) \in \mathcal{U}_r$- $\mathcal{V}_r$ and $\mathcal{F}^{-1}(v_r) \in \mathcal{V}_r$-$\mathcal{U}_r$ and since $\mathcal{F}$ is open (respectively, I-$semi$-g-$open$) function then $\mathcal{F}(\mathcal{U}_r), \mathcal{F}(\mathcal{V}_r)$ are two open (respectively, I-$semi$-g-$open$) sets, then for Player II in $\acute{G}(T_1, \dot{Y})$ choose $\mathcal{F}(\mathcal{U}_r), \mathcal{F}(\mathcal{V}_r)$ are two open (respectively, Isg-$open$) sets such that $c_r \in \mathcal{F}(\mathcal{U}_r)$- $\mathcal{F}(\mathcal{V}_r)$ and $v_r \in \mathcal{F}(\mathcal{V}_r)$-$\mathcal{F}(\mathcal{U}_r)$, $r = 1,2, \dots$.
Then
$Ḅ = \{\{\mathcal{F}(\mathcal{U}_1), \mathcal{F}(\mathcal{V}_1)\}, \{\mathcal{F}(\mathcal{U}_2), \mathcal{F}(\mathcal{V}_2)\}, \{\mathcal{F}(\mathcal{U}_3), \mathcal{F}(\mathcal{V}_3)\}, \dots, \{\mathcal{F}(\mathcal{U}_r), \mathcal{F}(\mathcal{V}_r)\}, \dots\}$ is the winning strategy for Player II in $\acute{G}(T_1, \dot{Y})$.
Hence Player II $\nearrow \acute{G}(T_1, \dot{Y})$ (respectively, $\acute{G}(T_1, \dot{Y}, \check{J})$).

If $i = 2$: By the same method as above then Player II $\nearrow \acute{G}(T_2, \dot{Y})$ (respectively, $\acute{G}(T_2, \dot{Y}, \check{J})$) except $\mathcal{U}_j, \mathcal{V}_j$ are two disjoint open (respectively, Isg-$open$) sets; $j = \{1, \dots, r, \dots\}$.
So
$Ḅ = \{\{\mathcal{F}(\mathcal{U}_1), \mathcal{F}(\mathcal{V}_1)\}, \{\mathcal{F}(\mathcal{U}_2), \mathcal{F}(\mathcal{V}_2)\}, \{\mathcal{F}(\mathcal{U}_3), \mathcal{F}(\mathcal{V}_3)\}, \dots, \{\mathcal{F}(\mathcal{U}_r), \mathcal{F}(\mathcal{V}_r)\}, \dots\}$ is the winning strategy for Player II in $\acute{G}(T_2, \dot{Y})$ (respectively, $\acute{G}(T_2, \dot{Y}, \check{J})$).
Hence Player II $\nearrow \acute{G}(T_2, \dot{Y})$ (respectively, $\acute{G}(T_2, \dot{Y}, \check{J})$).

Proposition 6: Let $\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{I}, \check{J})$ be an $I^{*}semi$-g-$open$, surjective function and Player II $\nearrow \acute{G}(T_i, X)$; $i = \{0,1,2\}$. Then Player II $\nearrow \acute{G}(T_i, \dot{Y}, \check{J})$.

Proof: Similar of proof of Proposition 5.

Corollary 7: Let $\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{I}, \check{J})$ be an open and surjective function and Player II $\nearrow \acute{G}(T_i, X)$); $i = \{0,1,2\}$. Then Player II $\nearrow \acute{G}(T_i, \dot{Y}, \check{J})$.

Proposition 7: Let $\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{I}, \check{J})$ be an I^{**}-$semi$-g-$open$ and surjective function, where Player II $\nearrow \acute{G}(T_i, X, \tilde{I})$; $i = 0,1,2$. Then Player II $\nearrow \acute{G}(T_i, \dot{Y})$.

Proof: Similar of proof of Proposition 5.

Some continuous functions with games

Definition 4 (3)**:** The function
$\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{I}, \check{J})$ is;

i- ***I-semi-g-continuous function***, denoted by ***Isg-continuous-function***, if $\mathcal{F}^{-1}(\mathcal{U})$ is Isg-$open$ set in X for every $\mathcal{U}$ open in $\dot{Y}$.

ii- ***Strongly-I-semi-g-continuous function***, denoted by ***strongly-Isg-continuous function***, if $\mathcal{F}^{-1}(\mathcal{U})$ is $open$ set in X for every $\mathcal{U}$ Isg-$open$ in $\dot{Y}$.

iii- ***I-semi-g-irresolute function***, denoted by ***Isg-irresolute-function***, if $\mathcal{F}^{-1}(\mathcal{U})$ is Isg-$open$ set in X for every $\mathcal{U}$ Isg-$open$ in $\dot{Y}$.

Example 5: The function $\mathcal{F} : (X, \tau, \tilde{I}) \to (X, \tau, \check{J})$ such that $\mathcal{F}(c_1) = c_2$, $\mathcal{F}(c_2) = c_1$, $\mathcal{F}(c_3) = c_3$ where $X = \{c_1, c_2, c_3\}$, $\tau = \{X, \phi, \{c_1\}\}$, $\tilde{I} = \{\phi\}$ and $\check{J} = \{\phi, \{c_2\}, \{c_3\}, \{c_2, c_3\}\}$. It is possible to see clearly that $\mathcal{F}$ is continuous and Isg-$continuous\ function$ which is not Isg-$irresolute\ function$.

Example6: The function $\mathcal{F} : (X, \tau, \tilde{I}) \to (X, \tau, \check{J})$ such that $\mathcal{F}(c) = c$ for every $c \in X$ where $X = \{c_1, c_2, c_3\}$, $\tau = \{X, \phi, \{c_1\}\}$, $\tilde{I} = \{\phi, \{c_2\}, \{c_3\}, \{c_2, c_3\}\}$ and $\check{J} = \{\phi\}$. Hence, $\mathcal{F}$ is Isg-$continuous$ and Isg-$irresolute$-$function$ which is not continuous and not $strongly\ Isg$-$continuous\ function$.

Proposition 8: Let $\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{I}, \check{J})$ be a continuous (respectively, I-$semi$-g-$irresolute$) function and injective function and Player II $\nearrow \acute{G}(T_i, \dot{Y})$ (respectively, $\acute{G}(T_i, \dot{Y}, \check{J})$). Then Player II $\nearrow \acute{G}(T_i, X)$ (respectively, $\acute{G}(T_i, X, \tilde{I})$).

Proof:

If $i = 0$: Player I in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) choose $c_1 \neq v_1$ where $c_1, v_1 \in X$, Player II in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) will hold account $\mathcal{F}(c_1) \neq \mathcal{F}(v_1)$ where $\mathcal{F}(c_1), \mathcal{F}(v_1) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \check{J})$), $\exists \mathcal{U}_1$ open (respectively, I-$semi$-g-$open$) in $\dot{Y}$; $\mathcal{U}_1$ contains one of the two elements $\mathcal{F}(c_1), \mathcal{F}(v_1)$ and since $\mathcal{F}$ is continuous (respectively, I-$semi$-g-$irresolute$) function, then $\mathcal{F}^{-1}(\mathcal{U}_1)$ open (respectively, I-$semi$-g-$open$). For Player II in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) choose $\mathcal{F}^{-1}(\mathcal{U}_1)$ contains one of the two elements c_1, v_1.In the next inning Player I in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) choose $c_2 \neq v_2$ where $c_2, v_2 \in X$ Player II in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) will hold account $\mathcal{F}(c_2) \neq \mathcal{F}(v_2)$ where $\mathcal{F}(c_2), \mathcal{F}(v_2) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \check{J})$), $\exists \mathcal{U}_2$ open (respectively, I-$semi$-g-$open$) set in $\dot{Y}$; $\mathcal{F}^{-1}(\mathcal{U}_2)$ contains one of the two elements $\mathcal{F}(c_2), \mathcal{F}(v_2)$, so $\mathcal{F}^{-1}(\mathcal{U}_2)$ open (respectively, I-$semi$-g-$open$). For Player II in $\acute{G}(T_0, \dot{Y})$ choose $\mathcal{F}^{-1}(\mathcal{U}_2)$ which contains one of the two elements c_2, v_2.

And in the r-th inning of the game $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) Player I in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) choose $ɕ_r \neq ꝩ_r$ where $ɕ_r, ꝩ_r \in X$, $r = 1,2, \ldots$. Player II in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) will hold account $\mathcal{F}(ɕ_r) \neq \mathcal{F}(ꝩ_r)$ where $\mathcal{F}(ɕ_r), \mathcal{F}(ꝩ_r) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \dot{J})$), $\exists \mathcal{U}_r$ open (respectively, *I-semi-g-open*) set in $\dot{Y}$; $\mathcal{U}_r$ contains one of the two elements $\mathcal{F}(ɕ_r), \mathcal{F}(ꝩ_r)$ then $\mathcal{F}^{-1}(\mathcal{U}_r)$ open (respectively, *I-semi-g-open*). For Player II in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$) choose $\mathcal{F}^{-1}(\mathcal{U}_r)$ which contains one of the two elements $ɕ_r, ꝩ_r$.

Then

Ḅ = $\{\mathcal{F}^{-1}(\mathcal{U}_1), \mathcal{F}^{-1}(\mathcal{U}_2), \mathcal{F}^{-1}(\mathcal{U}_3), \ldots, \mathcal{F}^{-1}(\mathcal{U}_r), \ldots\}$ is the winning strategy for Player II in $\acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$). Hence Player II $\nearrow \acute{G}(T_0, X)$ (respectively, $\acute{G}(T_0, X, \tilde{I})$).

If $i = 1$: In the r-th inning of the game $\acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$) Player I in $\acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$) choose $ɕ_r \neq ꝩ_r$ where $ɕ_r, ꝩ_r \in X$, $r = 1,2, \ldots$. Player II in $\acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$) will hold account $\mathcal{F}(ɕ_r) \neq \mathcal{F}(ꝩ_r)$ where $\mathcal{F}(ɕ_r), \mathcal{F}(ꝩ_r) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_0, \dot{Y})$ (respectively, $\acute{G}(T_0, \dot{Y}, \dot{J})$), $\exists \mathcal{U}_r, \mathcal{V}_r$ are two open (respectively, *I-semi-g-open*) sets in $\dot{Y}$; $\mathcal{F}(ɕ_1) \in \mathcal{U}_r$- $\mathcal{V}_r$ and $\mathcal{F}(ꝩ_1) \in \mathcal{V}_r$-$\mathcal{U}_r$ then $\mathcal{F}^{-1}(\mathcal{U}_r), \mathcal{F}^{-1}(\mathcal{V}_r)$ are two open (respectively, *I-semi-g-open*) sets, then Player II in $\acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$) choose $ɕ_r \in \mathcal{F}^{-1}(\mathcal{U}_r)$- $\mathcal{F}^{-1}(\mathcal{V}_r)$ and $ꝩ_r \in \mathcal{F}^{-1}(\mathcal{V}_r)$-$\mathcal{F}^{-1}(\mathcal{U}_r)$.

Then

Ḅ = $\{\{\mathcal{F}^{-1}(\mathcal{U}_1), \mathcal{F}^{-1}(\mathcal{V}_1)\}, \{\mathcal{F}^{-1}(\mathcal{U}_2), \mathcal{F}^{-1}(\mathcal{V}_2)\}, \{\mathcal{F}^{-1}(\mathcal{U}_3), \mathcal{F}^{-1}(\mathcal{V}_3)\}, \ldots, \{\mathcal{F}^{-1}(\mathcal{U}_r), \mathcal{F}^{-1}(\mathcal{V}_r)\}, \ldots\}$ is the winning strategy for Player II in $\acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$). Hence Player II $\nearrow \acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$).

If $i = 2$: By the same method as above then Player II $\nearrow \acute{G}(T_2, X)$ (respectively, $\acute{G}(T_2, X, \tilde{I})$) except $\mathcal{U}_j, \mathcal{V}_j$ are two disjoint open (respectively, *Isg-open*) sets in $\dot{Y}$; $j = \{1, \ldots, r, \ldots\}$.

So

Ḅ = $\{\{\mathcal{F}^{-1}(\mathcal{U}_1), \mathcal{F}^{-1}(\mathcal{V}_1)\}, \{\mathcal{F}^{-1}(\mathcal{U}_2), \mathcal{F}^{-1}(\mathcal{V}_2)\}, \{\mathcal{F}^{-1}(\mathcal{U}_3), \mathcal{F}^{-1}(\mathcal{V}_3)\}, \ldots, \{\mathcal{F}^{-1}(\mathcal{U}_r), \mathcal{F}^{-1}(\mathcal{V}_r)\}, \ldots\}$ is the winning strategy for Player II in $\acute{G}(T_1, X)$ (respectively, $\acute{G}(T_1, X, \tilde{I})$).

Proposition 9: Let $\mathcal{F} : (X, \tau, \tilde{I}) \to (\dot{Y}, \mathfrak{J}, \dot{J})$ be an *I-semi-g-continuous*, injective function and Player II $\nearrow \acute{G}(T_i, \dot{Y})$ then Player II $\nearrow \acute{G}(T_i, X, \tilde{I})$.

Proof:

If $i = 0$: Player I in $\acute{G}(T_0, X, \tilde{I})$ choose $ɕ_1 \neq ꝩ_1$ where $ɕ_1, ꝩ_1 \in X$, Player II in $\acute{G}(T_0, X, \tilde{I})$ will hold account $\mathcal{F}(ɕ_1) \neq \mathcal{F}(ꝩ_1)$ where $\mathcal{F}(ɕ_1), \mathcal{F}(ꝩ_1) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_0, \dot{Y})$, $\exists \mathcal{U}_1$ open in $\dot{Y}$; $\mathcal{U}_1$ contains one of the two elements $\mathcal{F}(ɕ_1), \mathcal{F}(ꝩ_1)$ and since $\mathcal{F}$ is *I-semi-g-continuous* function, then $\mathcal{F}^{-1}(\mathcal{U}_1)$ *I-semi-g-open*. For Player II in $\acute{G}(T_0, X, \tilde{I})$ choose $\mathcal{F}^{-1}(\mathcal{U}_1)$ which contains one of the two elements $ɕ_1, ꝩ_1$.In the next inning Player I in $\acute{G}(T_0, X, \tilde{I})$ choose $ɕ_2 \neq ꝩ_2$ where $ɕ_2, ꝩ_2 \in X$, Player II in $\acute{G}(T_0, X, \tilde{I})$ will hold account $\mathcal{F}(ɕ_2) \neq \mathcal{F}(ꝩ_2)$ where $\mathcal{F}(ɕ_2), \mathcal{F}(ꝩ_2) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_0, \dot{Y})$, $\exists \mathcal{U}_1$ open in $\dot{Y}$; $\mathcal{U}_2$ contains one of the two elements $\mathcal{F}(ɕ_2), \mathcal{F}(ꝩ_2)$ then $\mathcal{F}^{-1}(\mathcal{U}_2)$ *I-semi-g-open*, then Player II in $\acute{G}(T_0, X, \tilde{I})$ choose $\mathcal{F}^{-1}(\mathcal{U}_2)$ which contains one of the two elements $ɕ_2, ꝩ_2$.

And in the r-th inning of the game $\acute{G}(T_0, X, \tilde{I})$ Player I in $\acute{G}(T_0, X, \tilde{I})$ choose $ɕ_r \neq ꝩ_r$ where $ɕ_r, ꝩ_r \in X$, Player II in $\acute{G}(T_0, X, \tilde{I})$ will hold account $\mathcal{F}(ɕ_r) \neq \mathcal{F}(ꝩ_r)$ where $\mathcal{F}(ɕ_r), \mathcal{F}(ꝩ_r) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_0, \dot{Y})$, $\exists \mathcal{U}_r$ open in $\dot{Y}$; $\mathcal{U}_r$ contains one of the two elements $\mathcal{F}(ɕ_r), \mathcal{F}(ꝩ_r)$ then $\mathcal{F}^{-1}(\mathcal{U}_r)$ *I-semi-g-open*, for Player II in $\acute{G}(T_0, X, \tilde{I})$ choose $\mathcal{F}^{-1}(\mathcal{U}_r)$ which contains one of the two elements $ɕ_r, ꝩ_r$.

Then

Ḅ = $\{\mathcal{F}^{-1}(\mathcal{U}_1), \mathcal{F}^{-1}(\mathcal{U}_2), \mathcal{F}^{-1}(\mathcal{U}_3), \ldots, \mathcal{F}^{-1}(\mathcal{U}_r), \ldots\}$ is the winning strategy for Player II in $\acute{G}(T_0, X, \tilde{I})$. Hence Player II $\nearrow \acute{G}(T_0, X, \tilde{I})$.

If $i = 1$: In the r-th inning of the game $\acute{G}(T_1, X, \tilde{I})$ Player I in $\acute{G}(T_1, X, \tilde{I})$ choose $ɕ_r \neq ꝩ_r$ where $ɕ_r, ꝩ_r \in X$, Player II in $\acute{G}(T_1, X, \tilde{I})$ will hold account $\mathcal{F}(ɕ_r) \neq \mathcal{F}(ꝩ_r)$ where $\mathcal{F}(ɕ_r), \mathcal{F}(ꝩ_r) \in \dot{Y}$, but Player II $\nearrow \acute{G}(T_1, \dot{Y})$, $\exists \mathcal{U}_r, \mathcal{V}_r$ are two open sets in $\dot{Y}$; $\mathcal{F}(ɕ_r) \in \mathcal{U}_r$- $\mathcal{V}_r$ and $\mathcal{F}(ꝩ_r) \in \mathcal{V}_r$-$\mathcal{U}_r$ then $\mathcal{F}^{-1}(\mathcal{U}_r), \mathcal{F}^{-1}(\mathcal{V}_r)$ are two *I-semi-g-open* sets, then Player II in $\acute{G}(T_1, X, \tilde{I})$ choose $ɕ_r \in \mathcal{F}^{-1}(\mathcal{U}_r)$- $\mathcal{F}^{-1}(\mathcal{V}_r)$ and $ꝩ_r \in \mathcal{F}^{-1}(\mathcal{V}_r)$-$\mathcal{F}^{-1}(\mathcal{U}_r)$.

Then

Ḅ = $\{\{\mathcal{F}^{-1}(\mathcal{U}_1), \mathcal{F}^{-1}(\mathcal{V}_1)\}, \{\mathcal{F}^{-1}(\mathcal{U}_2), \mathcal{F}^{-1}(\mathcal{V}_2)\}, \{\mathcal{F}^{-1}(\mathcal{U}_3), \mathcal{F}^{-1}(\mathcal{V}_3)\}, \ldots, \{\mathcal{F}^{-1}(\mathcal{U}_r), \mathcal{F}^{-1}(\mathcal{V}_r)\}, \ldots\}$is the winning strategy for Player II in $\acute{G}(T_1, X, \tilde{I})$. Hence Player II $\nearrow \acute{G}(T_1, X, \tilde{I})$.

If $i = 2$: By the same method as above then Player II $\nearrow \acute{G}(T_2, X, \tilde{I})$ except $\mathcal{U}_j, \mathcal{V}_j$ are two disjoint open sets in $\dot{Y}$; $j = \{1, \ldots, r, \ldots\}$. So, B = $\{\{\mathcal{F}^{-1}(\mathcal{U}_1), \mathcal{F}^{-1}(\mathcal{V}_1)\}, \{\mathcal{F}^{-1}(\mathcal{U}_2), \mathcal{F}^{-1}(\mathcal{V}_2)\}, \{\mathcal{F}^{-1}(\mathcal{U}_3), \mathcal{F}^{-1}(\mathcal{V}_3)\}, \ldots, \{\mathcal{F}^{-1}(\mathcal{U}_r),$

$\mathcal{F}^{-1}(\mathcal{V}_r)$ }, ...} is the winning strategy for Player II in Ǵ(T$_2$, X, Ĩ). Hence Player II ↗ Ǵ(T$_2$, X, Ĩ).

Proposition 10: Let $\mathcal{F}$: (X, τ, Ĩ) → (Ẏ, $\mathfrak{J}$, ȷ̃) be a *Strongly-I-semi-g-continuous*, injective function and Player II ↗ Ǵ(T$_i$, Ẏ, ȷ̃); $i = \{0,1,2\}$. Then Player II ↗ Ǵ(T$_i$, X).

Corollary 8: Let $\mathcal{F}$: (X, τ, Ĩ) → (Ẏ, $\mathfrak{J}$, ȷ̃) be a *Strongly-I-semi-g-continuous*, injective function and Player II ↗ Ǵ(T$_i$, Ẏ, ȷ̃); $i = \{0,1,2\}$. Then Player II ↗ Ǵ(T$_i$, X, Ĩ).

Conclusion:

The conclusion of the study is to define many of the matches using the groups under study and using topological properties. The winning strategies were identified which any of the participants can follow in the match. Also, the losing strategies were identified, and they are a warning that they are not followed if possible during the course of the matches. Among these matches is: Ǵ(T$_0$, X) and Ǵ(T$_0$, X, Ĩ) when i = {0,1,2}.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

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بعض المباريات باستخدام بديهيات الفصل من نوع Ĩ- SEMI- g

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الخلاصة:

يحوي البحث على نوع جديد من المباريات من خلال تطبيق بديهيات الفصل عبر مجموعات شبه مفتوحة من نوع(I- semi- g) حيث تمت دراسة العلاقات بين الانواع المختلفة التي تم تحديدها واستراتيجية الفوز والخسارة لاي لاعب وكذلك علاقتها بمفاهيم بديهيات الفصل عبر هذا النوع من المجموعات شبه المفتوحة (I- semi- g) لغرض الوصول الى نوع جديد من المباريات.

الكلمات المفتاحية: مجموعة مفتوحة(Isg)، مجموعة مغلقة (Isg)، دالة (Isgo)، الدالة المستمرة (Isg) و المبارات.