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Copulas in Classical Probability Sense

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Abstract:

Copulas are simply equivalent structures to joint distribution functions. Then, we propose modified structures that depend on classical probability space and concepts with respect to copulas. Copulas have been presented in equivalent probability measure forms to the classical forms in order to examine any possible modern probabilistic relations. A probability of events was demonstrated as elements of copulas instead of random variables with a knowledge that each probability of an event belongs to [0,1]. Also, some probabilistic constructions have been shown within independent, and conditional probability concepts. A Bay's probability relation and its properties were discussed with respect to copulas. Moreover, an extension of multivariate constructions of each probabilistic copula has been presented. Finally, we have shown some examples that explain each relation of copula in terms of probability space instead of distribution functions.

Key words: Conditionality, Copula Concepts, Independent Events, Probability Measure Theory, Statistical Concepts.

Introduction:

Recently, copulas have played an essential role in many applications that their structures need a perfect statistical inference. Their role is central and valuable in statistics and even in probability measure theory because their features are highly rigid, smooth and various.

Nowadays, copulas are implemented in many different kinds of sciences like mechanics, financial analysis, physics, and others. In particular, copulas are used as an efficient statistical instrument to describe the dependence structures of random variables. One of the most important usage of copulas has firstly occurred in the study of finance, risk management, and insurance.

Moreover, copula is a modern phenomenon in the world of statistics and especially in statistical inference of non-linear data. It is a better approach than linear correlation coefficient approach when our analysis is used to a set of data that cannot follow normal distribution.

Historically, copulas have explicitly been mentioned by many researchers and scientific literatures. In particular, we could firstly refer to the study of Frechet and Hoeffding, in the 1942, throughout their study of the extremes of t-norms, (1). They have derived a type of copulas without referring to the word copula.

Afterwards, in the 1959, an impressive article has been published by one brilliant statistician named Sklar. He has stated his essential well-known theorem which is known by Sklar's theorem. He has borrowed a Latin word "copula" to refer to its name that means the link or the join, (2). His theorem illustrates the base of many concepts of copulas and their structures, (3, 4). It extended the nature of describing and analyzing the dependence structure with respect to the joint distribution function and its marginal distribution functions, (5). Indeed, one should explain that the importance of this historical survey of copula is to demonstrate its important in various field of science and applications.

Moreover, there are several aspects of science that used copulas in their structures and one can mainly refer to the literature of Nelsen in 1986 within his book entitled "introduction to copula".

He has collected most related works to copulas, their constructions, and their properties, (1). In the last fifty years, the concern with copulas has a very wide impact in different aspects of life, especially in finance. One of the most popular results associated with the study of statistical inference of portfolio and analysis of assets has been shown in (6-8). There are some other results that generalized the notions of copulas and connected them to some algebras and quantum logic spaces, (9-12).

Indeed, this work is an attempt to demonstrate copulas in association with the classical probability space. This means that our concern focuses on the language of events as elements of assigned copulas rather than the language of random variables and distribution functions. Also, we look for testing the properties of classical probability space with respect to copulas and their different properties, formulas, and results. For instance, test the Bayesian probabilities in terms of copulas, and comparing their formulas and results to the classical.

Indeed, there are various copulas that have been built in terms of probabilistic events. Also, we have shown some examples that are relevant to the classical examples which are shown in many literatures, for details.

Eventually, we should refer to the structure of this study. In the next section, we have presented some basic concepts related to copulas and probability space. Section three is devoted to presenting the main constructions of our study. Finally, we have shown some conclusions and future studies.

Basic Concepts

In this section, we present the most common definitions, and properties related to probability space structure, copulas and their constructions. We begin with the essential definition of measure space:

Definition 1: (12) Let $(\Omega, \mathcal{F})$ be a measurable space. A map $\mu: \mathcal{F} \to [0, \infty]$ is called a σ-measure, if the following conditions hold.

1. $\mu(A) \geq 0$, for all $A \in \mathcal{F}$;
2. $\mu(\phi) = 0$;
3. If $A_1, A_2, \ldots \in \mathcal{F}, A_i \cap A_j = \phi$ for $i \neq j$.

$\mu(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} A_i$.

We say that μ is σ-additive.

A measure μ is called finite, if there is $k \in \mathcal{R}$ such that $\mu(\Omega) = k$.

Remark 1: If $k = 1$, then the measure μ is called a probability measure denoted by P. While the triple $(\Omega, \mathcal{F}, P)$ is well-known by probability space, where $\Omega \neq \emptyset$, $\mathcal{F}$ is a σ-algebra on Ω, and P is a probability measure, (4).

Definition 2: (12) Let $(\Omega, \mathcal{F}, P)$ be a probability space. The σ-algebra $\mathcal{F}$ is the set of all subsets that are called random events, which are the set of outcomes of an experiment for which one can ask a probability.

Note that, for any event $A \in \mathcal{F}$, $P(A)$ belongs to [0,1]. In fact, this notion is essential in our constructed definitions.

Moreover, we review the notions of bivariate copulas in order to use them in our future structures. Indeed, there are two main functions which are known by copula and co-copula.

Definition 3: (1, 2, 4) A **two-dimensional copula** is a function $C: [0,1]^2 \to [0,1]$ with the following conditions.

1. For each $x \in [0,1], C(x, 0) = C(0, x) = 0$;
2. For each $x \in [0,1], C(x, 1) = C(1, x) = x$;
3. Let $x_1, x_2, y_1, y_2 \in [0,1]$. If $x_1 \leq x_2, y_1 \leq y_2$, then

$C(x_1, y_1) + C(x_2, y_2) \geq C(x_1, y_2) + C(x_2, y_1)$.

Definition 4: (1, 2, 4) A two-dimensional co-copula is a function $\mathrm{H}: [0,1]^2 \to [0,1]$that holds the following conditions.

1. For each $x \in [0,1], H(x, 0) = H(0, x) = x$;
2. For each $x \in [0,1], H(x, 1) = C(1, x) = 1$;
3. Let $x_1, x_2, y_1, y_2 \in [0,1]$. If $x_1 \leq x_2, y_1 \leq y_2$, then

$H(x_1, y_1) + H(x_2, y_2) \leq C(x_1, y_2) + C(x_2, y_1)$.

Indeed, these notions were discussed in detail by Nelsen, (10). Any function with the properties that are grounded, 2-increasing, and $C(a, 1) = a$ simply leads to decide that the function is copula. While, the co-copula has a reverse properties that are $H(a, 0) = a, H(a, 1) = 1$, and 2-nonincreasing property, (8).

Copulas in Terms of Probability Space

The main aim of this section focuses on the structures of copulas and their conditions that we have proposed in terms of classical probability space. In other words, we propose constructions of copulas in terms of probability measure space instead of distributing functions (whether univariate or bivariate distribution functions). It is an approach that allows us to investigate the properties of those functions via probability measure space and test some characteristics that might differ from classical properties. We begin with the proposed definition of copula with respect to probability space $(\Omega, \mathcal{F}, P)$.

Definition 5: A function $PC: [0,1]^2 \to [0,1]$ is said to be a bivariate probabilistic intersection copula, denoted by b.p.i.c, if the following conditions hold:

1.For each $A \in \mathcal{F},\ PC\big(P(A), P(\emptyset)\big) = PC\big(P(\emptyset), P(A)\big) = 0$;

2.For each $A \in \mathcal{F},\ PC\big(P(A), P(\Omega)\big) = P(A)$;

Similarly, for each $B \in \mathcal{F},\ PC\big(P(\Omega), P(B)\big) = P(B)$;

3.Let $A_1, A_2, B_1, B_2 \in \mathcal{F}$, such that $P(A_1) \leq P(A_2)$, $P(B_1) \leq P(B_2)$. Then

$PC\big(P(A_1), P(B_1)\big) + PC\big(P(A_2), P(A_2)\big) \geq PC\big(P(A_1), P(B_2)\big) + PC\big(P(A_2), P(B_1)\big)$.

Example 1: Let $(\Omega, \mathcal{F}, P)$ be a probability space such that the sample space of an experiment for throwing a dice is $\Omega = \{1,2,3,4,5,6\}$ and let the events of Ω be $A, B \in \mathcal{F}$ that represent the events of even numbers, and the prime numbers, respectively. That means, $A = \{2,4,6\}$, and $B = \{1,2,3,5\}$. Now, suppose that

$$PC\big(P(A), P(B)\big) = P(A)P(B), \text{ for all } A, B \in \mathcal{F}.$$

Then, let us examine whether the conditions of b.p.i.c are satisfied or not. First of all, it is clear that $P(A) = 0.5$, whereas $P(B) = \frac{2}{3}$. Then

1.For $A, B \in \mathcal{F}$, $PC\big(P(A), P(\emptyset)\big) = P(A)P(\emptyset) = 0.5 * 0 = 0$;

Similarly, $PC\big(P(\emptyset), P(B)\big) = 0 * \frac{2}{3} = 0$;

2.For $A, B \in \mathcal{F}$. We have

$$PC\big(P(A), P(\Omega)\big) = P(A)P(\Omega) = 0.5 * 1 = 0.5 \in [0,1];$$

Similarly, $PC\big(P(\Omega), P(B)\big) = P(\Omega)P(A) = 1 * \frac{2}{3} = \frac{2}{3} \in [0,1]$;

3. Let $A, B \in \mathcal{F}$, such that $P(\emptyset) \leq P(A)$, and $P(B) \leq P(\Omega) \Longrightarrow 0 \leq 0.5,\ \frac{2}{3} \leq 1$. Then

$$\begin{aligned} &PC\big(P(\emptyset), P(B)\big) + PC\big(P(A), P(\Omega)\big) \\ &\quad - PC(P(\emptyset), P(\Omega) \\ &\quad - PC\big(P(A), P(B)\big) \\ &= P(\emptyset)P(B) + P(A)P(\Omega) - P(\emptyset)P(\Omega) \\ &\quad - P(A)P(\mathrm{B}) \\ &= 0 * \frac{2}{3} + 0.5 * 1 - 0 * 1 - 0.5 * \frac{2}{3} = \frac{1}{6} \geq 0 \end{aligned}$$

Then PC satisfies the 2-increasing property. Since the given PC in this example fulfills the three conditions of copula, so it is simply a b.p.i.c.

Moreover, let's propose a construction of the definition of dual copula that is:

Definition 6: A function $\mathrm{PC}^*: [0,1]^2 \to [0,1]$ is said to be a bivariate probabilistic union copula, denoted by b.p.u.c, if the following conditions hold:

1.For each $\mathrm{A}, \mathrm{B} \in \mathcal{F},\ \mathrm{PC}^*\big(\mathrm{P(A)}, \mathrm{P}(\emptyset)\big) = \mathrm{P(A)}$, similarly, $\mathrm{PC}^*\big(\mathrm{P}(\emptyset)\mathrm{P(B)}\big) = \mathrm{P(B)}$;

2.For each $\mathrm{A} \in \mathcal{F},\ \mathrm{PC}^*\big(\mathrm{P(A)}, \mathrm{P}(\Omega)\big) = \mathrm{PC}^*\big(\mathrm{P}(\Omega), \mathrm{P(A)}\big) = 1$;

3.Let $\mathrm{A}_1, \mathrm{A}_2, B_1, B_2 \in \mathcal{F}$, such that $P(A_1) \leq P(A_2)$, $P(B_1) \leq P(B_2)$. Then

$PC^*\big(P(A_1), P(B_1)\big) + PC^*\big(P(A_2), P(B_2)\big) \leq PC^*\big(P(A_1), P(B_2)\big) + PC^*\big(P(A_2), P(B_1)\big)$.

Example 2 Back to **Example 1** and suppose that $PC^*\big(P(A), P(B)\big) = \max(P(A), P(B))$. Again, let's try to figure out whether PC^* fulfils the conditions of b.p.u.c or not. So, we can see that

1. Let $A, B \in \mathcal{F},\ PC^*\big(P(A), P(\emptyset)\big) = \max\big(P(A), P(\emptyset)\big) = \max(0.5,0) = 0.5 = P(A)$;

Similarly,

$PC^*\big(P(\emptyset), P(B)\big) = \max\big(P(\emptyset), P(B)\big) = P(B) = \frac{2}{3}$.

2. Let $A, B \in \mathcal{F},\ PC^*\big(P(A), P(\Omega)\big) = PC^*\big(P(\Omega), P(B)\big) = \max\big(P(A), P(\Omega)\big) = \max(0.5,1) = \max\left(1, \frac{2}{3}\right) = 1$.

3. Suppose that $P(\emptyset) \leq P(A)$, and $P(B) \leq P(\Omega)$, implies that $0 \leq 0.5,\ \frac{2}{3} \leq 1$

Then

$$\begin{aligned} &\max\big(P(\emptyset), P(B)\big) + \max\big(P(A), P(\Omega)\big) \leq \max\big(P(\emptyset), P(\Omega)\big) + \max\big(P(A), P(B)\big) \\ &\max\left(0, \frac{2}{3}\right) + max(0.5,1) \\ &\qquad \leq max(0,1) + max\left(0.5, \frac{2}{3}\right) \\ &\qquad = \frac{2}{3} + 1 \leq 1 + \frac{2}{3} \end{aligned}$$

Since all the three conditions of **Definition 5** are fulfilled, so PC^* is b.p.u.c.

Corollary 1: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F}$, and PC be a b.p.i.c. Then

$$PC^*\big(P(A), P(B)\big) = 1 - PC\big(P(A^c), P(B^c)\big)$$

is a b.p.u.c

Proof:

We need to show that PC^* in the relation above holds the conditions of being b.p.u.c. Thus

1. $PC^*\big(P(A), P(\emptyset)\big) = 1 - PC\big(P(A^c), P(\emptyset^c)\big) = 1 - PC\big(P(A^c), P(\Omega)\big) = 1 - P(A^c) = P(A)$

Similarly, $PC^*\big(P(\emptyset), P(B)\big) = P(B)$

2. $PC^*\big(P(A), P(\Omega)\big) = 1 - PC\big(P(A^c), P(\Omega^c)\big) = 1 - PC\big(P(A^c), P(\emptyset)\big) = 1 = PC^*\big(P(\Omega), P(\mathrm{B})\big)$

3. To prove that PC^* holds third property of b.p.u.c, we suppose that $A_1, A_2, B_1, B_2 \in \mathcal{F}$, such that $A_1 \subseteq A_2, B_1 \subseteq B_2$, implies that $P(A_1) \leq P(A_2), P(B_1) \leq P(B_2)$. Then, suppose that

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$$PC^*\big(P(A_1), P(\mathrm{B}_1)\big) + PC^*\big(P(A_2), P(\mathrm{B}_2)\big) - PC^*\big(P(A_1), P(\mathrm{B}_2)\big) - PC^*\big(P(A_2), P(\mathrm{B}_1)\big) \geq 0$$

Thus

$$1 - PC\big(P(A_1^c), P(\mathrm{B}_1^c)\big) + 1 - PC\big(P(A_2^c), P(\mathrm{B}_2^c)\big) - \left[1 - PC\big(P(A_1^c), P(\mathrm{B}_2^c)\big)\right] - \left[1 - PC\big(P(A_2^c), P(\mathrm{B}_1^c)\big)\right] \geq 0$$

Hence

$$PC\big(P(A_1^c), P(\mathrm{B}_2^c)\big) + PC\big(P(A_2^c), P(\mathrm{B}_1^c)\big) - PC\big(P(A_1^c), P(\mathrm{B}_1^c)\big) - PC\big(P(A_2^c), P(\mathrm{B}_2^c)\big) \geq 0$$

Or equivalently

$$PC\big(P(A_1^c), P(\mathrm{B}_2^c)\big) + PC\big(P(A_2^c), P(\mathrm{B}_1^c)\big) \geq PC\big(P(A_1^c), P(\mathrm{B}_1^c)\big) + PC\big(P(A_2^c), P(\mathrm{B}_2^c)\big)$$

Therefore, PC^* is a b.p.u.c.

Another structure that combines copula to one of the well-known operations of probability space can be shown in the following definition.

Moreover, in the sense of probability space operations and according to Sklar's theorem (2), it is possible to show that the following relations are true:

Proposition 1: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F}$, and PC be a b.p.i.c. Then the following relation is true

$$PC\big(P(A), P(B)\big) = P(A \cap B)$$

Proof:

The can prove easily be achieved by showing that the conditions of b.p.i.c. hold.

Let $A, B \in \mathcal{F}$, $PC\big(P(A), P(\emptyset)\big) = P(A \cap \emptyset)$, but$A \cap \emptyset = \emptyset$. Thus $P(A \cap \emptyset) = P(\emptyset) = 0 = PC\big(P(\emptyset), P(B)\big)$

1. From probability space, it is well-known that $P(\Omega) = 1$. Thus, $PC\big(P(A), P(\Omega)\big) = P(\mathrm{A} \cap \Omega)$, but $A \cap \Omega = A$, implies that

$PC\big(P(A), P(\Omega)\big) = P(A \cap \Omega) = P(A)$.

Similarly, $PC\big(P(\Omega), P(B)\big) = P(B)$;

3. Further, we are obliged to prove the 2-increasing property. Let $A_1, A_2, B_1, B_2 \in \mathcal{F}$, such that $A_1 \subseteq A_2$, $B_1 \subseteq B_2$ implies that $P(A_1) \leq P(A_2), P(B_1) \leq P(B_2)$ respectively. Then

$$PC\big(P(A_1), P(B_1)\big) + PC\big(P(A_2), P(B_2)\big) - PC\big(P(A_1), P(B_2)\big) - PC\big(P(A_1), P(B_1)\big) \geq 0$$

But

$PC\big(P(A_1), P(B_1)\big) = P(A_1 \cap B_1), PC\big(P(A_2), P(B_2)\big) = P(A_2 \cap B_2)$ $, PC\big(P(A_1), P(B_2)\big) = P(A_1 \cap B_2)$, and $PC\big(P(A_2), P(B_1)\big) = P(A_2 \cap B_1)$. Thus $P(A_1 \cap B_1) + P(A_2 \cap B_2) - P(A_1 \cap B_2) - P(A_2 \cap B_1) \geq 0$, and since P is monotone and increasing, then the relation above has 2-increasing property. Hence, PC has already the 2-increasing property.

In fact, there is another way that we could propose to show that property three (2-increasing) is true. Suppose that $\mathrm{A}_1, \mathrm{A}_2, \mathrm{B}_1, \mathrm{B}_2 \in \mathcal{F}$ such that $\mathrm{A}_1 \leq \mathrm{A}_2, \mathrm{B}_1 \leq \mathrm{B}_2$. Now, if $\mathrm{A}_2 \subseteq \mathrm{B}_1$ implies that $\mathrm{P}(\mathrm{A}_2) \leq \mathrm{P}(\mathrm{B}_1)$. Then

$$\begin{aligned} &\mathrm{PC}\big(\mathrm{P}(\mathrm{A}_1), \mathrm{P}(\mathrm{B}_1)\big) + \mathrm{PC}\big(\mathrm{P}(\mathrm{A}_2), \mathrm{P}(\mathrm{B}_2)\big) - \mathrm{PC}\big(\mathrm{P}(\mathrm{A}_1), \mathrm{P}(\mathrm{B}_2)\big) - \mathrm{PC}\big(\mathrm{P}(\mathrm{A}_2), \mathrm{P}(\mathrm{B}_1)\big) \\ &= \mathrm{P}(\mathrm{A}_1 \cap \mathrm{B}_1) + \mathrm{P}(\mathrm{A}_2 \cap \mathrm{B}_2) - \mathrm{P}(\mathrm{A}_1 \cap \mathrm{B}_2) - \mathrm{P}(\mathrm{A}_2 \cap \mathrm{B}_1) \\ &= \mathrm{P}(\mathrm{A}_1) + \mathrm{P}(\mathrm{A}_2) - \mathrm{P}(\mathrm{A}_1) - \mathrm{P}(\mathrm{A}_2) = 0 \end{aligned}$$

Similarly, if $\mathrm{B}_2 \subseteq \mathrm{A}_1$, implies that $\mathrm{P}(\mathrm{B}_2) \leq \mathrm{P}(\mathrm{A}_1)$. Then

$$\begin{aligned} &\mathrm{PC}\big(\mathrm{P}(\mathrm{A}_1), \mathrm{P}(\mathrm{B}_1)\big) + \mathrm{PC}\big(\mathrm{P}(\mathrm{A}_2), \mathrm{P}(\mathrm{B}_2)\big) - \mathrm{PC}\big(\mathrm{P}(\mathrm{A}_1), \mathrm{P}(\mathrm{B}_2)\big) - \mathrm{PC}\big(\mathrm{P}(\mathrm{A}_2), \mathrm{P}(\mathrm{B}_1)\big) \\ &= \mathrm{P}(\mathrm{B}_1) + \mathrm{P}(\mathrm{B}_2) - \mathrm{P}(\mathrm{B}_2) - \mathrm{P}(\mathrm{B}_1) = 0 \end{aligned}$$

Therefore, PC is b.p.i.c and thus the proof is complete.

Proposition 2: Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F}$, and PC^* be a b.p.u.c. Then the following relation is true

$$\mathrm{PC}^*\big(\mathrm{P}(\mathrm{A}), \mathrm{P}(\mathrm{B})\big) = \mathrm{P}(\mathrm{A} \cup \mathrm{B})$$

The proof is similar to the prove of **Proposition 1.**

Proposition 3: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F}$, and PC^* be a b.p.u.c. Then

$$PC\big(P(A), P(B)\big) = PC^*(P(A), P(A)) + PC^*(P(B), P(B)) - PC^*\big(P(A), P(B)\big) \tag{1}$$

is a b.p.i.c.

The proof involves showing that PC satisfies the conditions of b.p.i.c under the relation above. Therefore, the proof is clear.

Remark 2: In terms of the relation in (1), there is a corresponding formula to that relation. That is

$$PC^*\big(P(A), P(B)\big) = PC(P(A), P(A)) + PC(P(B), P(B)) - PC\big(P(A), P(B)\big) \tag{2}$$

We notice that PC^* in equation (2) is simply a b.p.u.c.

Associating with the concept of independent events, we can build the constructions of b.p.i.c, and b.p.u.c, respectively by the following two main ways

Proposition 4: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F}$, and PC be a b.p.i.c. If A, B are two independent, then

$$\mathrm{PC}\big(\mathrm{P}(\mathrm{A}), \mathrm{P}(\mathrm{B})\big) = \mathrm{P}(\mathrm{A})\mathrm{P}(\mathrm{B})$$

Proof:

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Let A, B be two independent events. Then
$\mathrm{PC}(\mathrm{P(A)}, \mathrm{P(B)}) = P(A \cap B)$
But, $P(A \cap B) = P(A)P(B)$ (A, B are independent)
Therefore, $\mathrm{PC}(\mathrm{P(A)}, \mathrm{P(B)}) = P(A)P(B)$
Note that, if $\mathrm{A}, \mathrm{B} \in \mathcal{F}$ are independent, then the following statements are true

1. $\mathrm{PC}(\mathrm{P}(\mathrm{A}^c), P(B^c)) = P(A^c)P(B^c)$;
2. $\mathrm{PC}(\mathrm{P}(\mathrm{A}^c), P(B)) = P(A^c)P(B)$;
3. $\mathrm{PC}(\mathrm{P(A)}, \mathrm{P}(\mathrm{B}^c)) = P(A)P(B^c)$.

Corollary 2: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F} \ni \mathrm{A}, B$ are independents and PC^* be bivariate b.p.u.c. Then

$$PC^*(P(A), P(B)) = 1 - P(A^c)P(B^c)$$

Proof:
From **Proposition 2**, we have shown that $PC^*(\mathrm{P(A)}, \mathrm{P(B)}) = \mathrm{P(A \cup B)}$
But, $P(A \cup B) = 1 - P((A \cup B)^c) = 1 - P(A^c \cap B^c)$. Hence
$PC^*(\mathrm{P(A)}, \mathrm{P(B)}) = 1 - P(A^c \cap B^c)$
But, from classical probability space, we know that, when A, and B are independent, then

$$P(A^c \cap B^c) = P(A^c)P(B^c)$$

Therefore, $PC^*(\mathrm{P(A)}, \mathrm{P(B)}) = 1 - P(A^c)P(B^c)$.
This complete the proof.

Proposition 5: Let $(\Omega, \mathcal{F}, P)$ be a probability space. Then for any two events $\mathrm{A}, \mathrm{B} \in \mathcal{F}$ with b.p.i.c, and b.p.u.c the following properties are true

1. $PC(P(A), P(A)) = PC(P(A), P(\Omega)) = PC(P(\Omega), P(A))$;
2. $PC^*(P(A), P(A)) = PC(P(A), P(\emptyset)) = PC(P(\emptyset), P(A))$;
3. If $A, B \in \mathcal{F}$ are independents, then
a. $PC(P(A), P(B)) = PC(P(A), P(\Omega))PC(P(\Omega), P(B))$;
b. $PC^*(P(A), P(B)) = 1 - PC^*(P(A^c), P(\emptyset))PC^*(P(\emptyset), P(B^c))$.

Proof:
1. From **Proposition 1**, we have shown that $PC(P(A), P(A)) = P(A \cap A)$
But, from the properties of events on probability space, we know that
$P(A \cap A) = P(A \cap \Omega) = P(\Omega \cap A)$
But $P(A \cap \Omega) = PC(P(A), P(\Omega))$, and similarly $P(\Omega \cap A) = PC(P(\Omega), P(A))$
Therefore, $PC(P(A), P(A)) = PC(P(A), P(\Omega)) = PC(P(\Omega), P(A))$;
2. The proof is similar to the proof of first point.
3. Let A, B be independent events.
a. $PC(P(A), P(B)) = P(A \cap B)$, implies that $P(A \cap B) = P(A)P(B)$.
But, $P(A) = P(A \cap \Omega)$, similarly, $P(B) = P(\Omega \cap B)$.
Thus $P(A \cap B) = P(A \cap \Omega)\, P(\Omega \cap B)$. Hence
$PC(P(A), P(B)) = P(A \cap \Omega)\, P(\Omega \cap B)$
But once again, $P(A \cap \Omega) = PC(P(A), P(\Omega))$, and
$P(\Omega \cap B) = PC(P(\Omega), P(B))$
Therefore $PC(P(A), P(B)) = PC(P(A), P(\Omega))PC(P(\Omega), P(B))$
b. $PC^*(P(A), P(B)) = P(A \cup B) = 1 - P[(A \cup B)^c] = 1 - P(A^c \cap B^c)$
But, A, and B are independent, then $P(A^c \cap B^c) = P(A^c)P(B^c)$, implies that
$P(A^c) = P(A^c \cup \emptyset), P(B^c) = P(\emptyset \cup B^c)$, and
$P(A^c \cup \emptyset) = PC^*(P(A^c), P(\emptyset))$, $P(\emptyset \cup B^c) = PC^*(P(\emptyset), P(B^c))$
Hence
$PC^*(P(A), P(B)) = 1 - PC^*(P(A^c), P(\emptyset))\, PC^*(P(\emptyset), P(B^c))$

Proposition 6: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F}$ be a probability space, with $\mathrm{PC}, \mathrm{PC}^*$ be the b.p.i.c., and b.p.u.c, respectively. Then the following function

$$\mathrm{PC} * \mathrm{PC}^*(\mathrm{P(A)}, \mathrm{P(B)}) = \mathrm{PC}(\mathrm{P(A)}, \mathrm{P(B)})\mathrm{PC}^*(P(A), P(B))$$

is a b.p.i.c.

Proof:
We have to show that $\mathrm{PC} * \mathrm{PC}^*(\mathrm{P(A)}, \mathrm{P(B)})$ satisfies the conditions of being a b.p.i.c. Thus

1. $PC * PC^*(P(A), P(\emptyset)) = PC(P(A), P(\emptyset))PC^*(P(A), P(\emptyset)) = 0.P(A) = 0$;
Similarly, $PC * PC^*(P(\emptyset), P(B)) = 0$
2. $PC * PC^*(P(A), P(\Omega)) = PC(P(A), P(\Omega))PC^*(P(A), P(\Omega)) = P(A).1 = P(A)$;
Similarly, $PC * PC^*(P(\Omega), P(B)) = P(B)$
3. To prove 2-inceasing, let $A_1, A_2, B_1, B_2 \in \mathcal{F}$, such that $P(A_1) \leq P(A_2), P(B_1) \leq P(B_2)$. Then, we need to prove that

$$\begin{aligned} PC * PC^*(P(A_1), P(B_1)) + PC &* PC^*(P(A_2), P(B_2)) - PC \\ &* PC^*(P(A_1), P(B_2)) - PC \\ &* PC^*(P(A_2), P(B_1)) \geq 0 \end{aligned}$$

But,
$PC * PC^*(P(A_1), P(B_1)) = PC(P(A_1), P(B_1))PC^*(P(A_1), P(B_1))$

$PC * PC^*(P(A_2), P(B_2)) = PC(P(A_2), P(B_2))PC^*(P(A_2), P(B_2))$

$$PC * PC^*\big(P(A_1), P(B_2)\big) = PC\big(P(A_1), P(B_2)\big)PC^*(P(A_1), P(B_2))$$

$$PC * PC^*\big(P(A_2), P(B_1)\big) = PC\big(P(A_2), P(B_1)\big)PC^*(P(A_2), P(B_1))$$

Thus,

$$PC\big(P(A_1), P(B_1)\big)PC^*\big(P(A_1), P(B_1)\big) + PC\big(P(A_2), P(B_2)\big)PC^*\big(P(A_2), P(B_2)\big) - PC\big(P(A_1), P(B_2)\big)PC^*\big(P(A_1), P(B_2)\big) - PC\big(P(A_2), P(B_1)\big)PC^*(P(A_2), P(B_1))$$

Now, let $A_2 \subseteq B_1$, implies that $P(A_2) \le P(B_1)$. Hence

$$PC\big(P(A_1), P(B_1)\big)PC^*\big(P(A_1), P(B_1)\big) = P(A_1)P(B_1)$$

Similarly, we obtain that

$$PC\big(P(A_2), P(B_2)\big)PC^*\big(P(A_2), P(B_2)\big) = P(A_2)P(B_2),$$

$$PC\big(P(A_1), P(B_2)\big)PC^*\big(P(A_1), P(B_2)\big) = P(A_1)P(B_2)$$

$$PC\big(P(A_2), P(B_1)\big)PC^*\big(P(A_2), P(B_1)\big) = P(A_2)P(B_1)$$

Then, we have

$$P(A_1)P(B_1) + P(A_2)P(B_2) - P(A_1)P(B_2) - P(A_2)P(B_1) = -P(A_1)[P(B_2) - P(B_1)] + P(A_2)[P(B_2) - P(B_1)] = [P(A_2) - P(A_1)][P(B_2) - P(B_1)]$$

Since, $P(A_1) \le P(A_2)$, and $P(B_1) \le P(B_2)$ are given

Hence $[P(A_2) - P(A_1)][P(B_2) - P(B_1)] \ge 0$

If, we suppose that $B_2 \le A_1$, then this also yields the same result above.

Therefore, $PC * PC^*$ has the 2-increasing property, and this is directly yield that $PC * PC^*$ is a b.p.i.c.

Proposition 7: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}, \mathrm{B} \in \mathcal{F}$, and let $\mathrm{PC}, \mathrm{PC}^*$ be a b.p.i.c., and b.p.u.c, respectively. Then the following function

$$\mathrm{PC} * \mathrm{PC}^*\big(\mathrm{P(A)}, \mathrm{P(B)}\big) = 1 - \mathrm{PC}\big(\mathrm{P(A^c)}, \mathrm{P(B^c)}\big)\mathrm{PC}^*(P(A^c), P(B^c))$$

is a b.p.u.c.

The proof is clear and similar to the proof of **Proposition 6**

Multivariate Representations of Probability Measure Functions

This section is dedicated to show an extension of the most common definitions, propositions, relations and properties of bivariate functions that have been shown in terms of probability space.

Definition 7: A function $PC: [0,1]^n \to [0,1]$ is called a multivariate probability intersection copula, denoted by m.p.i.c, if the following conditions hold:

1. If there exists $A_i \in \mathcal{F} \ni A_i = \emptyset, i = 1, \dots, n,$ then $PC\big(P(A_1), \dots, P(A_i), \dots, P(A_n)\big) = 0;$
2. For any $A_i \in \mathcal{F}$, $A_i \ne \Omega, i = 1, \dots, n$ $\quad PC\big(P(\Omega), \dots, P(A_i), \dots, P(\Omega)\big) = P(A_i);$
3. Let $A_i, B_i \in \mathcal{F}, i = 1, \dots, n$ such that $A_i \subseteq B_i \Longrightarrow P(A_i) \le P(B_i)$. Then $\triangle_{P(A_1)}^{P(B_1)} \triangle_{P(A_2)}^{P(B_2)} \dots \triangle_{P(A_n)}^{P(B_n)} PC(P(U_1), P(U_2), \dots, P(U_n)) \ge 0$

From (8), it has been shown that any copula of volume $\mathcal{B}$ with forward difference operation is non-decreasing. Then the property number three in the above definition is absolutely true.

For instance, suppose that $n = 3$, and let's see how it would look the elements of the 2-increasing property of PC. Hence

$$\begin{aligned}\triangle_{P(A_1)}^{P(B_1)} \triangle_{P(A_2)}^{P(B_2)} \triangle_{P(A_n)}^{P(B_n)} PC\big(P(U_1), P(U_2), P(U_3)\big) &= PC\big(P(B_1), P(B_2), P(B_3)\big) \\ &- PC\big(P(B_1), P(B_2), P(A_3)\big) \\ &- PC\big(P(B_1), P(A_2), P(B_3)\big) \\ &+ PC\big(P(B_1), P(A_2), P(A_3)\big) \\ &- PC\big(P(A_1), P(B_2), P(B_3)\big) \\ &+ PC\big(P(A_1), P(B_2), P(A_3)\big) \\ &+ PC\big(P(A_1), P(A_2), P(B_3)\big) \\ &- PC\big(P(A_1), P(A_2), P(A_3)\big) \ge 0\end{aligned}$$

Another main extension that can be proposed is related to the definition of b.p.u.c, that is:

Definition 8: A function $PC^*: [0,1]^n \to [0,1]$ is called multivariate probability union copula, denoted by m.p.u.c, if the following conditions hold:

1. For any $A_i \in \mathcal{F} \ni A_i \ne \emptyset$, $\quad PC^*\big(P(\emptyset), \dots, P(A_i), \dots, P(\emptyset)\big) = P(A_i);$
2. For any $A_i \in \mathcal{F} \ni A_i = \Omega$, $\quad PC^*\big(P(A_1), \dots, P(\Omega), \dots, P(A_n)\big) = 1;$
3. Let $A_i, B_i \in \mathcal{F}, i = 1, \dots, n$ such that $A_i \subseteq B_i \Longrightarrow P(A_i) \le P(B_i)$. Then $\triangle_{P(A_1)}^{P(B_1)} \triangle_{P(A_2)}^{P(B_2)} \dots \triangle_{P(A_n)}^{P(B_n)} PC^*\big(P(U_1), P(U_2), \dots, P(U_n)\big) \le 0$

Another extension that can be made is related to the m.p.i.c, and m.p.u.c with their relationships through probability of intersection, probability of union, respectively.

Proposition 8: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}_1, \ldots, \mathrm{A}_\mathrm{n} \in \mathcal{F}$, and let PC be a m.p.i.c. Then the following relation is true

$$PC\big(P(A_1), \ldots, P(A_n)\big) = P\left(\bigcap_{i=1}^{n} A_i\right)$$

The proof is similar to the proof of **Proposition 1**

Propositio 9: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathrm{A}_1, \ldots, \mathrm{A}_\mathrm{n} \in \mathcal{F}$, and be PC^* be a m.p.u.c. Then the following relation is true

$$PC^*\big(P(A_1), \ldots, P(A_n)\big) = P\left(\bigcup_{i=1}^{n} A_i\right)$$

The proof is the similar to the proof of **Proposition 2**

Corollary 3: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $A_1, \ldots, A_n \in \mathcal{F}$, PC be a m.p.i.c, and PC^* a m.p.u.c . If $A_1, \ldots, A_n$ are called independents if, and only if

1. $PC\big(P(A_1), \ldots, P(A_n)\big) = \prod_{i=1}^{n} P(A_i)$
2. $PC^*\big(P(A_1), \ldots, P(A_n)\big) = 1 - \prod_{i=1}^{n} P(A_i^c)$

The proof is similar to the proofs of **Proposition 6, Corollary 1**, and **Corollary 2**.

These multivariate representations can also be demonstrated with respect to infinite number of events. This is certainly a situation of continuous case and according to Sklar's theorem leads to a unique copula, see (2).

Conditional Probability and Copulas

One of the basic concepts of probability space theory is the concept of conditional events. The essential formula of conditional probability of an event A with respect to the given event B is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad (3)$$

According to this essential formula of conditional probability in equation (3), it is possible to propose a definition of bivariate conditional probability copula. The definition can be constructed by the following way:

Definition 9: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $A, B \in \mathcal{F}$, and PC be the a b.p.i.c. Then a bivariate conditional probability copula, denoted by b.c.p.c is a function $PC_c: [0,1]^2 \to [0,1]$ that has the following formula

$$PC_c\big(P(A_1), P(A_2) \mid P(B)\big) = \frac{PC\big(P(A_1), P(A_2), P(B)\big)}{PC\big(P(\Omega), P(\Omega), P(\mathrm{B})\big)} \qquad (4)$$

In order to make sure that the relation in equation (4) is true, it is strictly important to prove that the function PC_c satisfies the properties of being b.p.i.c.

1. Let $A_1, A_2, B \in \mathcal{F}$. Then $PC_c\big(P(A_1), P(\emptyset) \mid P(B)\big) = \frac{PC(P(\mathrm{A}_1), P(\emptyset), P(\mathrm{B}))}{PC(P(\Omega), P(\Omega), P(\mathrm{B}))} = 0 = PC\big(P(\emptyset), P(\mathrm{A}_2), P(\mathrm{B})\big)$;
2. Let $A_1, A_2, B \in \mathcal{F}$. Then

$$PC_c\big(P(A_1), P(\Omega) \mid P(B)\big) = \frac{PC\big(P(\mathrm{A}_1), P(\Omega), P(\mathrm{B})\big)}{PC\big(P(\Omega), P(\Omega), P(\mathrm{B})\big)} = PC_c\big(P(A_1), P(\mathrm{A}_1) \mid P(B)\big)$$

Similarly, $PC_c\big(P(\Omega), P(A_2) \mid P(B)\big) = PC_c\big(P(A_2), P(A_2) \mid P(B)\big)$

3. Let $A_1, A_2, B_1, B_2 \in \mathcal{F}$ such that $A_1 \subseteq A_2, B_1 \subseteq B_2$, implies that $P(A_1) \le P(A_2), P(B_1) \le P(B_2)$. Then the proof is clear because PC is b.p.i.c. Hence, PC_c is 2-increasing.

Lemma 1: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $A, B \in \mathcal{F}$, PC, and PC^*be a b.p.i.c, and b.p.u.c, respectively. A bivariate conditional dual copula to b.c.p.c, denoted by b.c.d.c. is a function PC_c^* , that has the following formula

$$PC_c^*\big(P(A_1), P(A_2) \mid P(B)\big) = 1 - PC_c\big(P(A_1^c), P(A_2^c) \mid P(B)\big)$$

and satisfies the conditions of b.p.u.c.

The proof of **Lemma 1** is equivalent to the proof of **Corollary 1**

Bayesian's Notions and Copulas

An important result that directly flows from the definition of b.c.p.c is the definition of what we proposed and named as bay's copula. The formula of such copula essentially depends on the bay's theorem.The essential assertion of the theorem is that for any two events

$$A, B \in \mathcal{F}, P(A \mid B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)} \qquad (5)$$

With respect to the relation in equation (5), it can be constructing an equivalent relation to the Bay's relation above with respect to the b.c.p.c by the following way:

Definition 10: Let $(\Omega, \mathcal{F}, P)$ be a probability space, $, B \in \mathcal{F}$, PC be a b.p.i.c, and PC_c be a b.c.p.c. A Bay's copula is a function that has the following formula

$$PC_c\big(P(A_1), P(A_2) \mid P(B)\big) = \frac{PC\big(P(A_1), P(A_2)\big)PC_c\big(B \mid P(A_1), P(A_2)\big)}{PC\big(P(A_1), P(A_2)\big)PC_c\big(B \mid P(A_1), P(A_2)\big) + PC\big(P(A_1^c), P(A_2^c)\big)PC_c\big(B \mid P(A_1^c), P(A_2^c)\big)}$$

and satisfies the following conditions

1. For each $A_1, A_2, B \in \mathcal{F}, PC_c\big(P(A_1), P(\emptyset) \mid P(B)\big) = 0 = PC_c\big(P(\emptyset), P(A_2) \mid P(B)\big)$;

2. For each $A_1, A_2, B \in \mathcal{F}, PC_c\big(P(A_1), P(\Omega) \mid P(B)\big) = 1 = PC_c\big(P(\Omega), P(A_2) \mid P(B)\big)$;
3. PC_c with Bay's condition is 2-increasing

It is important to show that property number three of the definition above is true. Let $C \in \mathcal{F}$ be a given event and $A_1 \subseteq A_2, B_1 \subseteq B_2$ implies that $P(A_1) \leq P(A_2), P(B_1) \leq P(B_2)$. Then it is obvious that we need to prove that

$$PC_c\big(P(A_1), P(B_1) \mid P(C)\big) + PC_c\big(P(A_2), P(B_2) \mid P(C)\big) - PC_c\big(P(A_1), P(B_2) \mid P(C)\big) - PC_c\big(P(A_2), P(B_1) \mid P(C)\big) \geq 0$$

Thus

$$PC_c\big(P(A_1), P(B_1) \mid P(C)\big) = \frac{PC\big(P(A_1), P(B_1)\big) PC_c\big(P(C) \mid P(A_1), P(B_1)\big)}{PC\big(P(A_1), P(B_1)\big) PC_c\big(P(C) \mid P(A_1), P(B_1)\big) + PC\big(P(A_1^c), P(B_1^c)\big) PC_c\big(P(C) \mid P(A_1^c), P(B_1^c)\big)}$$

$$PC_c\big(P(A_2), P(B_2) \mid P(C)\big) = \frac{PC\big(P(A_2), P(B_2)\big) PC_c\big(P(C) \mid P(A_2), P(B_2)\big)}{PC\big(P(A_2), P(B_2)\big) PC_c\big(P(C) \mid P(A_2), P(B_2)\big) + PC\big(P(A_2^c), P(B_2^c)\big) PC_c\big(P(C) \mid P(A_2^c), P(B_2^c)\big)}$$

$$PC_c\big(P(A_1), P(B_2) \mid P(C)\big) = \frac{PC\big(P(A_1), P(B_2)\big) PC_c\big(P(C) \mid P(A_1), P(B_2)\big)}{PC\big(P(A_1), P(B_2)\big) PC_c\big(P(C) \mid P(A_1), P(B_2)\big) + PC\big(P(A_1^c), P(B_2^c)\big) PC_c\big(P(C) \mid P(A_1^c), P(B_2^c)\big)}$$

$$PC_c\big(P(A_1), P(B_1) \mid P(C)\big) = \frac{PC\big(P(A_2), P(B_1)\big) PC_c\big(P(C) \mid P(A_2), P(B_1)\big)}{PC\big(P(A_2), P(B_1)\big) PC_c\big(C \mid P(A_2), P(B_1)\big) + PC\big(P(A_2^c), P(B_1^c)\big) PC_c\big(P(C) \mid P(A_2^c), P(B_1^c)\big)}$$

By using the following technique, the proof goes faster and easier. Let's consider the following case: Let $A_2 \subseteq B_1$. Then

$$PC_c\big(P(A_1), P(B_1) \mid P(C)\big) = \frac{PC\big(P(C), P(A_1)\big)}{PC\big(P(C), P(A_1)\big) + PC\big(P(C), P(B_1^c)\big)}$$

$$PC_c\big(P(A_2), P(B_2) \mid P(C)\big) = \frac{PC\big(P(C), P(A_2)\big)}{PC\big(P(C), P(A_2)\big) + PC\big(P(C), P(B_2^c)\big)}$$

$$PC_c\big(P(A_1), P(B_2) \mid P(C)\big) = \frac{PC\big(P(C), P(A_1)\big)}{PC\big(P(C), P(A_1)\big) + PC\big(P(C), P(B_2^c)\big)}$$

$$PC_c\big(P(A_2), P(B_1) \mid P(C)\big) = \frac{PC\big(P(C), P(A_2)\big)}{PC\big(P(C), P(A_2)\big) + PC\big(P(C), P(B_1^c)\big)}$$

Hence

$$\frac{PC\big(P(C), P(A_1)\big)}{PC\big(P(C), P(A_1)\big) + PC\big(P(C), P(B_1^c)\big)} + \frac{PC\big(P(C), P(A_2)\big)}{PC\big(P(C), P(A_2)\big) + PC\big(P(C), P(B_2^c)\big)} - \frac{PC\big(P(C), P(A_1)\big)}{PC\big(P(C), P(A_1)\big) + PC\big(P(C), P(B_2^c)\big)} - \frac{PC\big(P(C), P(A_2)\big)}{PC\big(P(C), P(A_2)\big) + PC\big(P(C), P(B_1^c)\big)}$$

$$= \frac{\big[PC\big(P(C), P(A_1)\big) + PC\big(P(C), P(B_2^c)\big) - PC(}{\big[PC\big(P(C), P(A_1)\big) + PC\big(P(C), P(B_1^c} + \frac{\big[PC\big(P(C), P(A_2)\big) + PC\big(P(C), P(B_1^c)\big) - PC(}{\big[PC\big(P(C), P(A_2)\big) + PC\big(P(C), P(B_2^c} \quad (6)$$

By rearranging equation (6), we obtain

$$\big[PC\big(P(C), P(A_2)\big) - PC\big(P(C), P(A_1)\big)\big]\big[PC\big(P(C), P(B_1^c)\big) - PC\big(P(C), P(B_2^c)\big)\big]$$

and since $PC\big(P(C), P(B_2^c)\big) \leq PC\big(P(C), P(B_1^c)\big), PC\big(P(C), P(A_1)\big) \leq PC\big(P(C), P(A_2)\big)$, because $B_2^c \subseteq B_1^c, A_1 \subseteq A_2$. Therefore

$$\big[PC\big(P(C), P(A_2)\big) - PC\big(P(C), P(A_1)\big)\big]\big[PC\big(P(C), P(B_1^c)\big) - PC\big(P(C), P(B_2^c)\big)\big] \geq 0$$

Similarly, if we consider that $B_2 \subseteq A_1$, then we absolutely obtain the same result. Therefore, PC_c is b.c.p.c and fulfills the Bay's conditional probability.

Compliance with Ethical Standards

Conflict of interest: Author **Zainalabideen Abdual Samad**, declares that he has no conflict of interest. Author **Ahmed AL-Adilee** declares that he has no conflict of interest. Author **Samir T. A. Al-Shibely** declares that he has no conflict of interest.

Ethical approval: This article does not contain any studies with human participants performed by any of the authors. Also, this article does not contain any studies with animals performed by any of the authors.

Informed consent: Informed consent was obtained from all individual participants included in the study.

Conclusions:

As a summary to this study one could mention that each copula can be written in terms of classical probability space. There are several results that may have modified forms to each type of copula function with respect to intersection, and union, respectively. Associating with probability space definitions, the relations of copulas with respect to independent events property have the same properties to the classical with modified representations. Indeed, these representations of independent events in terms of probability space and copulas are unique. The examples show various successful results that demonstrate copulas with respect to probability space calculations. Moreover, the extension to multivariate events has shown that the constructions are much complicated and still has the same properties to bivariate events. The representation of bay's probability within copula corresponds to the classical Bay's probability. There are several different future studies need to be investigated. For example, generalize the concepts in terms of Bay's theorem so that we could reconstruct copulas of union with respect to conditional probability. Examine the relationships of some algebraic systems like MV-algebra, lattices, and others algebraic systems with copula constructions in probability sense.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Kufa.

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دوال الكوبله بدلالة فضاء الاحتماليه

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الخلاصة:

دوال الكوبلة بشكل مبسط بناءات مكافئة لدوال التوزيع المشتركة. في هذه الدراسة اقترحنا بناءات معدلة تعتمد على الاحتمالية التقليدية ومفاهيم بدلالة دوال الكوبلة. قُدمت دوال الكوبلة بصيغ قياسية احتمالية مكافئة للصيغ التقليدية من اجل اختبار أي علاقات احتمالية جديدة. قمنا بتقديم الاحتمالية للاحداث كعناصر لدالة الكوبلة بدلا من المتغيرات العشوائية مع المعرفة بان كل احتمالية لحدث تنتمي للفترة [0,1]. كذلك، تم اثبات بعض البناءات الاحتمالية من خلال مفاهيم الاحتمالية الاستقلالية والشرطية. ناقشت الدراسة علاقة بيز الاحتمالية وخواصها بالنسبة لدوال الكوبلة. بالإضافة الى ذلك، قُدم التوسع للبناءات المتعددة لكل دالة كوبلة. أخيرا، وضحنا عدد من الأمثلة التي بينت كل علاقة للكوبلة بدلالة الفضاء الاحتمالي بدلا من دوال التوزيع.

الكلمات المفتاحية: الشرطية، مفاهيم الكوبلة، الاحداث المستقلة، نظرية القياس الاحتمالية، المفاهيم الإحصائية.