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On The Normality Set of Linear Operators

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Abstract:

In this paper, the Normality set N_A will be investigated. Then, the study highlights some concepts properties and important results. In addition, it will prove that every operator with normality set has non trivial invariant subspace of $\mathcal{H}$.

Key words: Invariant**,** Quasi-similar operator, Normality set, Similar operator, Unitary operator.

Introduction:

The Normality set of linear operators can be defined in the following form: $N_A = \{T \in \mathfrak{B}(\mathcal{H}): AT^* = T^*A\}$, which is in non-empty set and contains restricted normal operators. This set has many of the properties of which we reviewed some with some important relationships. Through them we will try to solve the problem of the invariant subspace which is still not yet solved and thus we discussed in section 5. Many researchers have studied the properties on the normal operators. Fuglede B. (1950) introduced the famous theorem which is proved for any two bounded operators A, B commute and B is normal then also commute with adjoint of operator B (1). Putnam CR. (1951) extended the Fuglede's theorem for two operators (2). The authors (3, 4) showed that the invariant subspace problem is incompletely unresolved problem asking whether every bounded operator on a complex Banach space sends some non-trivial closed subspaces to itself. The first form of the problem as posed by P. Halmos was the first example of an operator without an invariant subspace and constructed by P. Enflo. (For Hilbert spaces, the non-trivial invariant subspace (Briefly n.i.s.) problem remains open). Enflo. Proposed a counterexample to the invariant subspace problem in 1975. The aim of this research is to study the Normality set of $A \in \mathfrak{B}(\mathcal{H})$ also we prove every operator $A: \mathfrak{B}(\mathcal{H}) \to \mathfrak{B}(\mathcal{H})$ has (n.i.s). Furthermore, in this research, first; we recall the properties and other concepts, second; give some important results on the Normality set and relationships it. In Section 5 we try proved the set N_A has (n.i.s) for the operator A.

Preliminaries

In this section, we recall basic properties and other concepts.

Definition 1 (4, 5): For any operator T in $\mathfrak{B}(\mathcal{H})$. The adjoint of $T \in \mathfrak{B}(\mathcal{H})$ is denoted by T^*. The operator $T \in \mathfrak{B}(\mathcal{H})$ is said to be self adjoint, if $T^* = T$, normal if $TT^* = T^*T$, and unitary if $TT^* = T^*T = I$.

Definition 2 (4): If $\mathcal{M}$ is closed subspace of $\mathcal{H}$, and $T(\mathcal{M}) \subseteq \mathcal{M}$, then $\mathcal{M}$ will be invariant subspace for T. Also a subspace $\mathcal{M}$ is a reduced subspace for T, if both $\mathcal{M}$ and $\mathcal{M}^\perp$, where $\mathcal{M}^\perp$ is the orthogonal complement of $\mathcal{M}$, are invariant under T(equivalently, if $\mathcal{M}$ is invariant for both T and T^*). $\mathcal{M}$ is a hyper-invariant subspace for T, if it is invariant for every operator that commutes with T.

Remark 1 (4): It is easily evidenced that $\mathcal{M}$ is invariant subspace for T, if and only if $\mathcal{M}^\perp$ is invariant subspace for T^*.

Definition 3 (6): If $A, B \in \mathfrak{B}(\mathcal{H})$, then A is similar to B, if there exists invertible operator $T \in \mathfrak{B}(\mathcal{H})$, such that $A\mathrm{T} = \mathrm{T}B$. This is denoted by $A \approx B$, when A is similar to B.

Definition 4(6): If $\mathcal{H}_1, \mathcal{H}_2$ are Hilbert spaces, and $A \in \mathfrak{B}(\mathcal{H}_1)$ and $B \in \mathfrak{B}(\mathcal{H}_2)$, then A is quasi-similar to B, if there exists two injective with dense range bounded operators T_1 from $\mathcal{H}_1$ to $\mathcal{H}_2$ and $T_2$from $\mathcal{H}_2$ to $\mathcal{H}_1$, such that $T_1A = BT_1$ and $AT_2 = T_2B$. This is denoted by $A \simeq B$, when A is quasi-similar to B.

Definition 5 (6): Linear operators $A, B \in \mathfrak{B}(\mathcal{H})$ are unitarily equivalent (denoted by $A \cong B$), if there exists unitary operator $U \in \mathfrak{B}(\mathcal{H})$, where $UA = BU$; that is, $A = U^*BU$ or equivalently $B = UAU^*$.

Theorem 1 (1): Let N be a normal operator on $\mathcal{H}$, for any bounded linear operator A if $AN = NA$, then $AN^* = N^*A$.

Main Results:

Definition 6: For each operator A in $\mathfrak{B}(\mathcal{H})$, we defined new set is the Normality set of A is denoted by $N_A = \{T \in \mathfrak{B}(\mathcal{H}) : AT^* = T^*A\}$. It is clear that it is non-empty set, since $O, I \in N_A$ and $N_{\alpha I} = N_O = \mathfrak{B}(\mathcal{H})$, for every complex number α, where O, I is the zero, unity operator on $\mathcal{H}$, respectively.

The following proposition shows that the normality set is closed on $\mathfrak{B}(\mathcal{H})$.

Proposition 2: The operator $A \in \mathfrak{B}(\mathcal{H})$, then N_A is a closed linear subspace of $\mathfrak{B}(\mathcal{H})$.

Proof: let $X, Y \in N_A$ and $\alpha, \beta \in \mathbb{C}$. So that $AX^* = X^*A$ and $AY^* = Y^*A$.

Hence, $A(\alpha X + \beta Y)^* = \bar{\alpha}AX^* + \bar{\beta}AY^* = \bar{\alpha}X^*A + \bar{\beta}Y^*A = (\alpha X + \beta Y)^*A$. Therefore, $\alpha X + \beta Y \in N_A$.

Thus, N_A is a linear subspace on $\mathfrak{B}(\mathcal{H})$.

Assume $\{T_n\}$ be a sequence of operators in N_A convergent to T. So, $AT_n{}^* = T_n{}^*A$, for every positive integer n. Since A is continuous and $\{T_n\} \to T$, then $\{T_n{}^*\} \to T^*$, $\{AT_n{}^*\} \to AT^*$, and $\{T_n{}^*A\} \to T^*A$. Therefore, $AT^* = T^*A$, that is $T \in N_A$

Then, N_A is a closed linear subspace of $\mathfrak{B}(\mathcal{H})$. ■

The following theorem shows that N_A is proper subspace of $\mathfrak{B}(\mathcal{H})$; when $A \neq \alpha I$ for every $\alpha \in \mathbb{C}$.

Theorem 3: Let $A \in \mathfrak{B}(\mathcal{H})$. Then $N_A = \mathfrak{B}(\mathcal{H})$, if and only if, there exists $\alpha \in \mathbb{C}$, such that $A = \alpha I$.

Proof: Let $\{e_n\}$ be an orthogonal basis for $\mathcal{H}$ and let U, U^* be the Unilateral shift operator and it is adjoint. Hence, $Ue_i = e_{i+1}$ and $U^*e_{i+1} = e_i$ for every $i = 1, 2, \dots, U^*e_1 = 0$.

If $N_A = \mathfrak{B}(\mathcal{H})$ then $U, U^* \in N_A$. Therefore, $U^*A(e_1) = AU^*e_1 = 0$, that is $Ae_1 = \alpha e_1$ for some $\alpha \in \mathbb{C}$. For every $n \geq 2$, $Ae_n = AU^{n-1}e_1 = U^{n-1}Ae_1 = U^{n-1}\alpha e_1 = \alpha e_n$. So, $Ax = \alpha x$ for every $x \in \mathcal{H}$. Thus, $A = \alpha I$. The prove of the converse is trivial. ■

The evidence of the following corollary is a consequence from the proof of the above theorem.

Corollary 4: If $A \neq \alpha I, \forall \alpha \in \mathbb{C}$, then either $U \notin N_A$ or $U^* \notin N_A$, where U, U^* be the Unilateral, bilateral shift operators, respectively.

Lemma 5: Let $A \in \mathfrak{B}(\mathcal{H})$. Then, $N_A N_A = N_A$.

Proof: Let $X, \Upsilon \in N_A$. So, $AX^* = X^*A$ and $A\Upsilon^* = \Upsilon^*A$ implies that $AX^*\Upsilon^* = X^*A\Upsilon^*$.

Hence, $A(\Upsilon X)^* = X^*\Upsilon^*A = (\Upsilon X)^*A$. Therefore, $\Upsilon X \in N_A$; that is, $N_A N_A \subset N_A$.

Conversely, assume that $T \in N_A$, so $AT^* = T^*A$ and since $I \in N_A$ or $IT = T$.

Hence, $A(IT)^* = AT^* = T^*A = (IT)^*A$. Therefore, $T \in N_A N_A$. Then $N_A N_A = N_A$.■

Remark 2: It is clear that from Lemma (5), if $\in N_A$, then $T^n \in N_A$ for each n.

Lemma 6: A is normal if and only if $A \in N_A$.

Theorem 7: If A is normal, then $T \in N_A$ if and only if $T^* \in N_A$.

Proof: Let A be normal and $T \in N_A$. So, $AT^* = T^*A$. By using theorem (1).

$AT = TA$. Thus $T^* \in N_A$.

The converse is similar. ■

The following theorem shows the adjoint and invertible of the set N_A.

Theorem 8: Let $A \in \mathfrak{B}(\mathcal{H})$. Then N_A is satisfying the following:

1- $N_A = N_{I+\mu A}$, $\mu \in \mathbb{C}$.
2- $N_A{}^* = N_{A^*}$ that is $N_A = (N_{A^*})^*$. In particular, if A is normal, then $N_A = N_{A^*} = (N_{A^*})^*$. Where $N_A{}^* = \{T^* : T \in N_A\}$.
3- If A is invertible, then $N_A = N_{A^{-1}}$.
4- If Υ is invertible operator, then $\Upsilon \in N_A$ if and only if $\Upsilon^{-1} \in N_A$.

Proof: (1) It is easy by definition of N_A and proposition (2).

(2) Let $S \in N_{A^*}$. So, $A^*S^* = S^*A^*$ take the adjoint, there is $AS = SA$. Therefore, $S^* \in N_A$ that is $S \in N_A{}^*$. Hence, $N_{A^*} \subseteq N_A{}^*$. The converse is similar, and let $T \in N_A$. So, $AT^* = T^*A$ take the adjoint, $TA^* = A^*T$ or $A^*T = TA^*$. Thus $T^* \in N_{A^*}$, that is $T \in (N_{A^*})^*$, it is clear that if A is normal and by Theorem (1), the required result is obtained.

(3) Suppose that A is invertible and $T \in N_A$; so, $AT^* = T^*A$ and $T^*A^{-1} = A^{-1}T^*$, that is $T \in N_{A^{-1}}$ or $N_A \subseteq N_{A^{-1}}$, the converse is similar.

(4) Suppose that Υ is invertible operator then $\Upsilon \in N_A \Leftrightarrow A\Upsilon^* = \Upsilon^*A$.

So, $\Upsilon^{*-1}A = A\Upsilon^{*-1} \Leftrightarrow \Upsilon^{-1} \in N_A$.■

The following propositions shows more properties on the normality set.

Proposition 9: Let $A, \underline{B}, C \in \mathfrak{B}(\mathcal{H})$:

1- If $\underline{B} \in N_A$, then $A \in N_{\underline{B}}$.
2- If $A \in N_{\underline{B}}$ and $\underline{B} \in N_C$, then $\underline{B} \in N_{AC} \cap N_{CA}$.

Proof: (1) Assume that $\underline{B} \in N_A$, so that $A\underline{B}^* = \underline{B}^*A$ by take the adjoint, we have $\underline{B}A^* = A^*\underline{B}$; that is, $A \in N_{\underline{B}}$.

(2) Since $A \in N_{\underline{B}}$ and $\underline{B} \in N_C$ given, so $\underline{B}A^* = A^*\underline{B}$ and $C\underline{B}^* = \underline{B}^*C$.

Hence $A\underline{B}^* = \underline{B}^*A$ and $C\underline{B}^* = \underline{B}^*C$. Therefore $A(C\underline{B}^*) = A(\underline{B}^*C)$. Thus $(AC)\underline{B}^* = \underline{B}^*(AC)$; that is, $\underline{B} \in N_{AC}$. Similar way, we can proof $\underline{B} \in N_{CA}$. Thus $\underline{B} \in N_{AC} \cap N_{CA}$. ■

Proposition 10: Let $A, \underline{B}, C \in \mathfrak{P}(\mathcal{H})$. Then:

1- $N_A \cap N_{\underline{B}} \subseteq N_{A+\underline{B}}$.
2- $N_A \cap N_{\underline{B}} \subseteq N_{A\underline{B}} \cap N_{\underline{B}A}$.
3- ${N_A}^n \subseteq N_{A^n}$ for each integer number n. where ${N_A}^n = \{T^n : T \in N_A\}$.
4- $N_A = \bigcap_{\forall \alpha \in N} N_{A^\alpha}$.

Proof: (1) Suppose that $T \in N_A \cap N_{\underline{B}}$; so, $AT^* = T^*A$, and $\underline{B}T^* = T^*\underline{B}$, by additive, there is $(A+\underline{B})T^* = T^*(A+\underline{B})$. Therefore, $T \in N_{A+\underline{B}}$, that is $N_A \cap N_{\underline{B}} \subseteq N_{A+\underline{B}}$.

(2) If $T \in N_A \cap N_{\underline{B}}$, then $A\underline{B}T^* = AT^*\underline{B} = T^*A\underline{B}$, so, $T \in N_{A\underline{B}}$ or $N_A \cap N_{\underline{B}} \subseteq N_{A\underline{B}}$; similarly, $\underline{B}AT^* = \underline{B}T^*A = T^*\underline{B}A$. So, $T \in N_{\underline{B}A}$. Hence, $N_A \cap N_{\underline{B}} \subseteq N_{\underline{B}A}$.

Thus, $N_A \cap N_{\underline{B}} \subseteq N_{A\underline{B}} \cap N_{\underline{B}A}$.

(3) Assume that $T \in N_A$; so, $AT^* = T^*A$ and $A^2T^* = AT^*A = T^*A^2$. Thus, $A^nT^* = T^*A^n$ for every integer number n, that is $T \in N_{A^n}$ for every n. Therefore, $N_A \subseteq N_{A^n}$ and by Lemma (5), there is ${N_A}^n \subseteq N_A$ for every integer number n. So, ${N_A}^n \subseteq N_A \subseteq N_{A^n}$, the result is obtained.

(4) The prove is similar to prove (3). ■

But the following example shows that $N_{A^n} \neq {N_A}^n$, for some $n > 1$.

Example 1: Let $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where α, a, b, c, d are real number and α, b are non-zero. So, A is not invertible and nilpotent operator. That is, $A^n = 0$, for every $n > 1$.

Hence, $A^nT^* = T^*A^n$ and $T \in N_{A^n}$, for every $n > 1$. But $AT^* \neq T^*A$. Therefore $T \notin N_A$. Since ${N_A}^n \subseteq N_A$. Then $T \notin {N_A}^n$ for each n.

Now, we shows the commute normal operators with N_A.

Theorem 11: If the operator $A \in N_A$, then, $AN_A = N_AA$.

Proof: Assume that A is normal enough to prove $AT = TA$ for every operator $T \in N_A$.

So, $AT^* = T^*A$. Since A is normal and by theorem (1), there exists $A^*T^* = T^*A^*$ by take adjoint, implies $AT = TA$, that is $AN_A = N_AA$ for all $T \in N_A$. ■

Relationships with N_A

In this section, the relation between two operators on the set N_A will be studied. Then, this section will investigate whether the operator is similar, quasi-similar, unitary or more than them.

Theorem 12: If $A \approx B$, then, $N_A = (T^*)^{-1}N_BT^* = N_{T^{-1}BT}$.

Proof: Suppose that A and B are similar, by definition (2.4). Then $\exists\, T$ is invertible, where $A = TBT^{-1}$.

Let $X \in N_A$ implies that $AX^* = X^*A$, so $(TBT^{-1})X^* = X^*(TBT^{-1})$.

Hence, $B(T^{-1}X^*T) = (T^{-1}X^*T)B$. Then $B(T^*X\,(T^*)^{-1})^* = (T^*X\,(T^*)^{-1})^*B$. So, $T^*X(T^*)^{-1} \in N_B$. Hence, $X \in (T^*)^{-1}N_BT^*$ and $N_A \subseteq (T^*)^{-1}N_BT^*$.

Conversely, Suppose that $(T^*)^{-1}ST^* \in (T^*)^{-1}N_BT^*$. Hence $S \in N_B$ Implies that $BS^* = S^*B$. So, $(T^{-1}AT)S^* = S^*(T^{-1}AT)$ and $A(TS^*T^{-1}) = (TS^*T^{-1})A$, $(T^*)^{-1}ST^* \in N_A$, $(T^*)^{-1}N_BT^* \subseteq N_A$ and $N_A = (T^*)^{-1}N_BT^*$. ■

Theorem 13: If $A \cong B$, then $N_B = UN_AU^* = N_{UAU^*}$.

Proof: Assume that A and B are unitarily equivalent. So by definition (5) there exists a unitary operator U, where $UA = BU$ or $A = U^*BU$.

Let $T \in N_A$ implies that $AT^* = T^*A$. So, $(U^*BU)T^* = T^*(U^*BU)$.

Hence, $B(UT^*U^*) = (UT^*U^*)B$ and $B(UTU^*)^* = (UTU^*)^*B$. Thus, $UTU^* \in N_B$. That is $UN_AU^* \subset N_B$. Conversely, let $\mathcal{S} \in N_B$ implies that $BS^* = \mathcal{S}^*B$ since $B = UAU^*$.

Hence, $(UAU^*)\mathcal{S}^* = \mathcal{S}^*(UAU^*)$, and $A(U^*\mathcal{S}^*U) = (U^*\mathcal{S}^*U)A$.

Then, $A(U^*\mathcal{S}U)^* = (U^*\mathcal{S}U)^*A$. There exists $U^*\mathcal{S}U \in N_A$, but $\mathcal{S} = U(U^*\mathcal{S}U)U^*$.

Thus, $\mathcal{S} \in UN_AU^*$. Then, $N_B \subset UN_AU^*$. Thus $N_B = UN_AU^*$. ■

Theorem 14: If $A \simeq B$, then ${T_1}^*N_B{T_2}^* \subseteq N_A$ and ${T_2}^*N_A{T_1}^* \subseteq N_B$.

Proof: Suppose that A is quasi-similar to B if there exists two injective with dense range bounded operators T_1 from $\mathcal{H}_1$ to $\mathcal{H}_2$ and T_2 from $\mathcal{H}_2$ to $\mathcal{H}_1$ s.t $T_1A = BT_1$ and $AT_2 = T_2B$.

Let $X \in N_B$. So, $BX^* = X^*B$ and $A(T_2X^*T_1) = (T_2X^*T_1)A$.

Therefore, $A({T_1}^*X{T_2}^*)^* = ({T_1}^*X{T_2}^*)^*A$. Then ${T_1}^*X{T_2}^* \in N_A$, that is ${T_1}^*N_B{T_2}^* \subseteq N_A$.

Now, let $Y \in N_A$. So $AY^* = Y^*A$. Hence $B(T_1Y^*T_2) = (T_1Y^*T_2)B$.

Therefore, $B({T_2}^*Y{T_1}^*)^* = ({T_2}^*Y{T_1}^*)^*B$. Then ${T_2}^*Y{T_1}^* \in N_B$, that is ${T_2}^*N_A{T_1}^* \subseteq N_B$. ■

Invariant Subspace:

It has been clarified that if $A \in \mathfrak{P}(\mathcal{H})$, then N_A is a non-empty set and closed subspace. The following theorem shows that N_{A^*} is invariant under A.

Theorem 15: If $A \in \mathfrak{P}(\mathcal{H})$. Then $A(N_{A^*}) \subseteq N_{A^*}$.

Proof: Let $AT \in A(N_{A^*})$ where $T \in N_{A^*}$. Hence $A^*T^* = T^*A^*$. Therefore $AT = TA$.

So that $A^*(AT)^* = A^*(T^*A^*) = (TA)^*A^* = (AT)^*A^*$. Thus $AT \in N_{A^*}$.

Corollary 16: Every operator $A: \mathfrak{P}(\mathcal{H}) \to \mathfrak{P}(\mathcal{H})$ has non-trivial invariant subspace.

Proof: If $A = \alpha I$ for some $\alpha \in \mathbb{C}$, then clearly every subspace $M \subseteq \mathfrak{P}(\mathcal{H})$ is invariant. So, it is supposed that $A \neq \alpha I$. It is clear that $N_{A^*} \neq \{0\}$ since $I \in N_{A^*}$ and $N_{A^*} \neq \mathfrak{P}(\mathcal{H})$ theorem (3.3) and N_{A^*} is a closed subspace proposition (2) and by theorem (15). Hence, N_{A^*} is a (n.i.s) for A.

Definition 7: Let $x \in \mathcal{H}$, then we defined the N_A be other form is $N_A(x) = \{T(x): T \in N_A\}$.

Proposition 17: If $x \in \mathcal{H}$, then $N_A(x)$ is a subspace of $\mathcal{H}$.

Proof: Let $T(x),\ S(x) \in N_A(x)$, where $T, S \in N_A$. Since N_A is subspace, then $\alpha T + \beta S \in N_A$ for every $\alpha, \beta \in \mathbb{C}$. So that $\alpha T(x) + \beta S(x) = (\alpha T + \beta S)(x) \in N_A(x)$. Thus $N_A(x)$ is a subspace for every $x \in \mathcal{H}$.

Theorem 18: If $A \in \mathfrak{P}(\mathcal{H})$, then $AN_{A^*}(x) \subseteq N_{A^*}(x)$ for every $x \in \mathcal{H}$.

Proof: Let $AT(x) \in AN_{A^*}(x)$ where $T \in N_{A^*}$. Hence $A^*T^* = T^*A^*$ and $AT = TA$.

So that $A^*(AT)^* = A^*(T^*A^*) = (TA)^*A^* = (AT)^*A^*$. Thus $AT \in N_{A^*}$.

So implies that $AT(x) \in N_{A^*}(x)$. This enough proof.

Corollary 19: If $A \in \mathfrak{P}(\mathcal{H})$, then $\overline{N_{A^*}(x)}$ is a closed invariant subspace for A for every $x \in \mathcal{H}$.

Proof: Since $N_{A^*}(x)$ is subspace and $AN_{A^*}(x) \subseteq N_{A^*}(x)$, then $\overline{N_{A^*}(x)}$ is a closed subspace and $\overline{AN_{A^*}(x)} \subseteq \overline{N_{A^*}(x)}$.

Conclusion:

In this paper, the normality set N_A is studied. This set has many of the properties of which we reviewed some with some important relationships, and tried to solve the famous problem of the invariant subspace which is still not yet solved, and some important results have been submitted in this subject.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Tikrit University.

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حول المجموعة السوية للمؤثرات الخطية

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الخلاصَة:

في هذا البحث، سنحاول تعريف المجموعة السوية من النوع N_A ودراستها. ثم ستسلط الدراسة الضوء على بعض الخصائص والمفاهيم والنتائج المهة. بالأضافة لذلك، سنحاول أثبات انهُ لكل مؤثر محتوى في المجموعة السوية يملك فضاء جزئي لامتغير غير تافه من $\mathcal{H}$.

الكلمات المفتاحية: لا متغير، المجموعة السوية، مؤثر التشابه، مؤثر شبه التشابه، مؤثر أحادي.