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Fuzzy-assignment Model by Using Linguistic Variables

Zainb Hassan Radhy[*] *Firas Hussean Maghool* *Nagham Kamal Hady*

Department of Mathematics, College of Computer Science and Information Technology, University of Al-Qadisiya
[*]Corresponding author: Zainb.Hassan@qu.edu.iq[*] ,Firas.maghool@qu.edu.iq, tomhom937@gmail.com
[*]ORCID ID https://orcid.org/0000-0003-0540-1957[*] , http://orcid.org/0000-0003-0829-9425

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Abstract:

This work addressed the assignment problem (AP) based on fuzzy costs, where the objective, in this study, is to minimize the cost. A triangular, or trapezoidal, fuzzy numbers were assigned for each fuzzy cost. In addition, the assignment models were applied on linguistic variables which were initially converted to quantitative fuzzy data by using the Yager's ranking method. The paper results have showed that the quantitative date have a considerable effect when considered in fuzzy mathematic models.

Key words: Fuzzy-assignment problem, Hungarian method, Fuzzy number, Ranking of fuzzy numbers

Introduction:

Assignment problem is a special type of linear programming problem. This model assumes that the number of sources, supplies or job matches the same number of destinations, demands or persons. The two numbers are equal, and correspondingly the number of columns and rows, in the cost matrix, will be identical.

By this paper, a more accurate problem was studied, specifically the assignment-problem with fuzzy costs $\tilde{c_{ij}}$ which are represented by fuzzy quantifier that are switched by fuzzy numbers of trapezoidal or triangular forms. The objective function is regarded in this work as a fuzzy-function, because it requires to minimize the total cost according to some crisp constraints. Initially, the fuzzy ranking is employed to rank the objective values of the objective function (1,2).

Basic Definitions:

A fuzzy number $\mathcal{A}^{\sim} = (\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3)$ is said to be a triangular fuzzy number if its embership function is specified

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & If\ a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & If\ a_2 \le x \le a_3 \\ 0 & \text{others} \end{cases} \quad \dots(1)$$

where $(a_1, a_2, a_3) \in R$

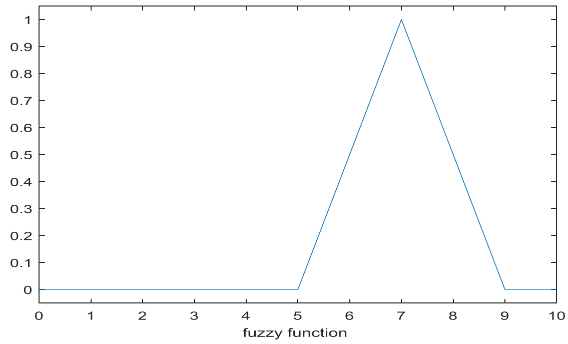

Figure 1. Represent triangular fuzzy number

The triangular fuzzy number is based on three-value ruling, the minimum possible value $\mathfrak{a}_1$, and the most possible value $\mathfrak{a}_2$ and maximum possible value $\mathfrak{a}_3$ as in Fig.1

Trapezoidal fuzzy number:

A fuzzy number $\mathcal{A}^{\sim} = (\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_4)$ is supposed to be Trapezoidal fuzzy number as in Fig. 2 if its membership function is given by:

$$\mu_{\tilde{\mathcal{A}}}(x) = \begin{cases} \frac{x-\mathfrak{a}_1}{\mathfrak{a}_2-\mathfrak{a}_1} & \text{If } \mathfrak{a}_1 \le x \le \mathfrak{a}_2 \\ 1 & \text{If } \mathfrak{a}_2 \le x \le \mathfrak{a}_3 \\ \frac{\mathfrak{a}_3-x}{\mathfrak{a}_3-\mathfrak{a}_2} & \text{If } \mathfrak{a}_3 \le x \le \mathfrak{a}_4 \\ 0 & \text{Otherwise} \end{cases} \quad \dots(2)$$

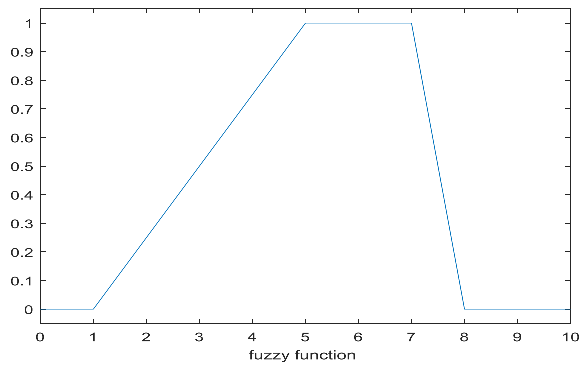

Figure 2. Represent trapezoidal fuzzy number

Linguistic variables (3,4)

The linguistic variables concept and its application in approximating the linguistic variables refers to those variables whose values are sentences or words in an artificial, or natural, language. For example, if a "speed" is a linguistic variable , then its values indicate either low, medium or high speed. Accordingly, these values are denoted as fuzzy numbers.

$\alpha - cut$ And strong $\alpha - cut$ (5,6)

For a fuzzy set($\mathcal{A}$) is defined on the interval $\mathcal{X}$ = [0,1], then $(\alpha \in [0,1])$,where $\alpha - cut\ \alpha_{\mathcal{A}}$ and the strong $\alpha - cut\ \alpha_{\mathcal{A}}^{+}$, are the crisp set

$a_{\mathcal{A}} = \{\mathcal{X}/\mathcal{A}(\mathcal{X}) \geq \alpha\}$

$\alpha_{\mathcal{A}}^{+} = \{\mathcal{X}/\mathcal{A}(\mathcal{X}) > \alpha\}$

Assignment problem:

The mathematical formulation of this model can be described as in Table 1:

Table 1. The mathematical formulation of the assignment problem

Source → destination ↓	1	2	3	….j….	n
1	c_{11}	c_{12}	c_{13}	c_{1j}	c_{1n}
2	c_{21}	c_{22}	c_{23}	c_{2j}	c_{2n}
3	-	-	-	-	-
-	-	-	-	-	-
I	c_{i1}	c_{i2}	c_{i3}	c_{ij}	c_{in}
-	-	-	-	-	-
N	c_{n1}	c_{n2}	c_{n3}	c_{nj}	c_{nn}

$Minimize\quad z = \sum_{i=1}^{n}\sum_{j=1}^{n} c\text{ij}\, x_{ij} \qquad i,j = 1,2,\dots\dots,n$ ….(3)

$subuject\ \ to{:}\ \sum_{j=1}^{n} X_{ij}\ ,\ \sum_{i=1}^{n} X_{ij} \qquad i,j = 1,2,\dots\dots n$ ……(4)

Where $\mathcal{X}_{ij}$ = 1if the i th person is assignment the j thjob and (0) otherwise is the decision variable denoting the assignment of the person i to j

The work's objective is represented by minimizing the total assignment cost and all the sources to the destinations such that one source for each destination. In the matrix of mathematical model, this will mean there is a certain row for each column.

When representing the costs by fuzzy numbers, $\mathcal{C}_{ij}^{\sim}$, then the problem of fuzzy-assignment becomes:

$Y_{Z}^{\sim} = \sum_{i=1}^{n}\sum_{ij}^{n} Y(\mathcal{C}_{ij}^{\sim})$ This equation under the same conditions at which the assignment model is transformed, by a fuzzy number ranking method, to the coefficients of fuzzy cost in form of crisp ones. Where a ranking function $\mathcal{F}(\mathcal{R}) \rightarrow \mathcal{R}$, $\mathcal{F}(\mathcal{R})$ is set of fuzzy of all fuzzy numbers such that each fuzzy number is represented by a real number

The triangular fuzzy number$\mathcal{A}^{\sim} = (\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3)$ $\mathcal{R}$ Is given by $\mathcal{R}(\mathcal{A}) = \frac{\mathfrak{a}_1 + 2\mathfrak{a}_2 + \mathfrak{a}_3}{4}$ and the trapezoidal fuzzy number $\mathcal{A}^{\sim} = (\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_4)$ $\mathcal{R}$ Is given by $\mathcal{R}(\mathcal{A}) = \frac{\mathfrak{a}_1 + \mathfrak{a}_2 + \mathfrak{a}_3 + \mathfrak{a}_4}{4}$ Yager' s Ranking index [5] is defined by:

$Y(\mathcal{C}^{\sim}) = \int_{0}^{1} 0.5(\mathcal{C}_{a}^{\ell} + \mathcal{C}_{a}^{u})$ Where $(\mathcal{C}_{a}^{\ell}, \mathcal{C}_{a}^{u})$ is the $a - level\ cut$ of the fuzzy number $\mathcal{C}^{\sim}$

The representative value of the fuzzy number $\mathcal{C}^{\sim}$ is obtained from the Yager's Ranking index $Y(\mathcal{C}^{\sim})$.

Later, $Y(\mathcal{C}_{ij}^{\sim})$ are crisp values. Obviously, this problem is the crisp assignment problem of the form (minimize $= \sum_{i=1}^{n}\sum_{j=1}^{n} \mathcal{C}_{ij}\mathcal{X}_{ij}$) where the Hungarian method could be used to solve it.

Steps of assuming this method are:

i. Substitute of the cost matrix $\mathcal{C}_{ij}$with linguistic variables by triangular fuzzy number
ii. Find (YRI)
iii. Replace triangular number by their respective ranking indices
iv. The resulting AP are solved to find the optimal assignment by using the Hungarian technique.

Numerical example:

A fuzzy-assignment problem, which is signified by a matrix of (3x3) dimension, is considered. The three matrix columns refer to classification of vehicles a according to type of fuel. While the three matrix row indicate three corresponding features for each type of vehicle consequently, the matrix element $[\mathcal{C}_{ij}^{\sim}]$ will be represented by linguistic variables. These variables will be defined later by fuzzy number. The problem is substituted, to find the optimal assignment. Thereby, the assignment cost of becomes:

$$\begin{array}{c} \\ w \\ x \\ z \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} high & medium & low \\ simple & complected & specific \\ cheap & reasonable & expensive \end{array}\right] \end{array}$$

Where (1, 2, 3) (deasil, hybrid, electric power) and $(\mathcal{W}, \mathcal{X}, \mathcal{Z})$ are (environmental pollution, maintenance, economical) Respectively.

By using the following table, the linguistic variables, which show the qualitative data, are changed into quantitative data.

High	(0,3,5)
Simple	(15,18,20)
Medium	(33,36,38)
Cheap	(1,2,3)
Complicated	(23,25,27)
Reasonable	(37,40,41)
Low	(3,8,11)
Expensive	(44,45,50)
specific	(28,30,32)

Then, the variables in linguistic form are represent by fuzzy numbers in triangular form

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} w \\ x \\ z \end{array} & \begin{bmatrix} (0,3,5) & (1,2,3) & (3,8,11) \\ (15,18,20) & (23,25,27) & (28,30,32) \\ (33,36,38) & (37,40,41) & (44,45,50) \end{bmatrix} \end{array}$$

By applying the YRM, Y (0, 3, 5) is calculated, the membership function of the triangular number is (0, 3, 5)

$$\mu_{\tilde{\mathcal{A}}}(x) = \begin{cases} \frac{x-0}{3-0} & 0 \le x \le 3 \\ \frac{x-5}{2-5} & 3 \le x \le 5 \end{cases}$$

The $a - cut$ of the fuzzy number (0, 3, 5) is $(\mathcal{C}_a^{\ell}, \mathcal{C}_a^{\mathcal{U}}) = (2a, 5 - 3\,a)$ for which

$$Y(\tilde{\mathcal{C}_{11}}) = Y(0,3,5) = \int_0^1 0.5(\mathcal{C}_a^{\ell} + \mathcal{C}_a^{\mathcal{U}})\, da$$

$$= \int_0^1 0.5(2a + 5 - 3a)d\ a = 2.35$$

The cost indices of Yager, $\tilde{\mathcal{C}_{ij}}$, are obtained as:
$Y(\tilde{\mathcal{C}_{12}}) = 2, Y(\tilde{\mathcal{C}_{13}}) = 7.5, Y(\tilde{\mathcal{C}_{21}}) = 17.7, Y(\tilde{\mathcal{C}_{22}}) = 25, Y(\tilde{\mathcal{C}_{23}}) = 30, Y(\tilde{\mathcal{C}_{31}}) = 35.75, Y(\tilde{\mathcal{C}_{32}}) = 39.5, Y(\tilde{\mathcal{C}_{33}}) = 46$

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} w \\ x \\ z \end{array} & \begin{bmatrix} 2.35 & 2 & 7.5 \\ 17.75 & 25 & 30 \\ 39.75 & 39.5 & 46 \end{bmatrix} \end{array}$$

Performing row reductions

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} w \\ x \\ z \end{array} & \begin{bmatrix} 0.35 & 0 & 5.5 \\ 0 & 7.25 & 12.25 \\ 0.25 & 0 & 6.25 \end{bmatrix} \end{array}$$

Performing column reductions

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} w \\ x \\ z \end{array} & \begin{bmatrix} 0.35 & 0 & 0 \\ 0 & 7.25 & 6.75 \\ 0.25 & 0 & 0.75 \end{bmatrix} \end{array}$$

The optimal assignment schedule is $\mathcal{W} \to 3, \mathcal{X} \to 1, \mathcal{Z} \to 2$
The cost assignment problem is (Z)=64.75

Conclusion:

In this paper, the assignment costs have been represented by linguistic variables, then triangular fuzzy numbers were applied to be more realistic and general, then the optimal solution was obtained by using Yager ranking indices. This Technique satisfies linearity, compensation and dditivity properties. Moreover, it provides results which are consistent with human intuition, (Yager ranking) technique can be used with another modeling for example transformation model or network business.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Al-Qadisiya.

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نماذج التخصيص الضبابية باستخدام المتغيرات اللغوية

زينب حسن راضي **فراس حسين مجهول** **نغم كامل هادي**

قسم الرياضيات ، كلية علوم الحاسوب وتكنلوجيا المعلومات ، جامعة القادسية

الخلاصة:

يتناول هذا البحث موضوع نماذج التخصيص المعتمدة على الكلف الضبابية حيث ان تقليل الكلف هو الهدف الاساسي المطلوب تحقيقه في اية دراسة تخص نماذج التخصيص. فمن الممكن استخدام الاعداد الضبابية المثلثية او الشبه منحرفة (كما تم في هذا البحث) لكل كلفة ضبابية بالاضافة الى ذلك يتم تطبيق نماذج التخصيص على المتغيرات اللغوية بعد تحويلها الى بيانات كمية ضبابية باعتماد طريقة Yager Ranking Mothed . من خلال نتائج البحث فقد تبين ان المتغيرات اللغوية ذات تأثير مهم عند تطبيقها في النماذج الرياضية الضبابية.

الكلمات المفتاحية: مسائل التخصيص الضيابي، الطريقة الهنكارية الارقام الضبابية، دالة الرتبة للارقام الضبابية.