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On Free Resolution of Weyl Module and Zero Characteristic Resolution In The Case of Partition (8,7,3)

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Abstract:

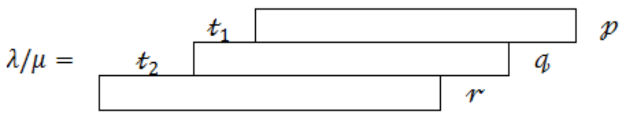
This treatise is an application of the characteristic-free resolution of $K_{(8,7,3)}\mathcal{F}$ to the Lascoux resolution of $K_{(8,7,3)}\mathcal{F}$ (characteristic zero resolution). From this, study, we gain the connection between the resolution of Weyl module $K_{(8,7,3)}\mathcal{F}$ in characteristic free mode and in the Lascoux mode.

Keywords: Weyl module, resolution, free resolution, characteristic-free resolution, characteristic zero resolution.

Introduction:

Let $\mathcal{R}$ be a commutative ring with 1 and $\mathcal{F}$ be a free $\mathcal{R}$ -module. The divided power algebra $\mathcal{DF} = \sum_{i\geq 0}\mathcal{D}_i\mathcal{F}$ can be acquainted as the graded abelian algebra generated by x^i where $x \in \mathcal{F}$ and i is a non-negative integer, and $\mathcal{D}_i\mathcal{F}$ is the divided power algebra of degree$i$.

The resolution of partition $(p + t_1 + t_2, q + t_2, r)$ which is represented by below diagram and in our case $t_1 = t_2 = 0$.



The authors in (1) and (2) clarify the description of the characteristic zero skeleton by Lascoux in the resolution of skew-shapes. While the authors in (3), (4) and (5) exhibit the formulation of the terms of Lascoux resolution.

The authors in (6) exhibit the terms and the exactness of the Weyl resolution in the case of partition (8,7). As well in (7) they discuss the terms of characteristic zero complex in the case of the partition (8,7,3) and the diagram for the complex of characteristic zero in the case of the partition (8,7,3).

In the next section, we survey the terms of characteristic free resolution of Weyl module in the case of the partition (8,7,3) which is the generalization of the partition (3,3,3).

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While in the last section we stratify the resolution gain it in the below section to the Lascoux resolution by itself track of authors in (8) and (9) with Capelli identities as in (10).

Characteristic-Free Resolution of the Partition (8,7,3)

As in (3) for the case (p, q, r) with $r \geq 2$ the terms of the resolution are:

$$Res([p,q;0])\otimes\mathcal{D}_r \oplus \sum_{e\geq 0} \underline{\mathcal{Z}}_{32}^{(e+1)} y\, Res([p,q + e + 1; e + 1])\otimes\mathcal{D}_{r-e-1} \oplus \sum_{e_1\geq 0, e_2\geq e_1} \underline{\mathcal{Z}}_{32}^{(e_2+1)} y \underline{\mathcal{Z}}_{31}^{(e_1+1)} z Res([p + e_1 + 1, q + e_2 + 1; e_2 - e_1])\otimes\mathcal{D}_{r-(e_1+e_2+2)}$$

where $\underline{\mathcal{Z}}_{ab}^{(m)}$ is the following bar complex

$$0 \to \underbrace{\mathcal{Z}_{ab} w \mathcal{Z}_{ab} w \ldots \mathcal{Z}_{ab}}_{m-times} \longrightarrow \sum_{k_i\geq 1, \sum k_i = m} \mathcal{Z}_{ab}^{(k_1)} w \mathcal{Z}_{ab}^{(k_2)} w \ldots \mathcal{Z}_{ab}^{(k_{m-1})} \to \cdots \to \mathcal{Z}_{ab}^{(m)} \to 0$$

By stratify the above formulation for partition (8,7,3)

$$Res([8,7;0])\otimes\mathcal{D}_3 \oplus \sum_{e\geq 0} \underline{\mathcal{Z}}_{32}^{(e+1)} y\, Res([8,7 + e + 1; e + 1])\otimes\mathcal{D}_{3-e-1} \oplus \sum_{e_1\geq 0, e_2\geq e_1} \underline{\mathcal{Z}}_{32}^{(e_2+1)} y \underline{\mathcal{Z}}_{31}^{(e_1+1)} z Res([8 + e_1 + 1, 7 + e_2 + 1; e_2 - e_1])\otimes\mathcal{D}_{3-(e_1+e_2+2)} \quad \ldots 1$$

So

$$\sum_{e\geq 0} \underline{\mathcal{Z}}_{32}^{(e+1)} y\, Res([8,7 + e + 1; e + 1]) \otimes\mathcal{D}_{3-e-1} = \underline{\mathcal{Z}}_{32} y\, Res([8,8;1])\otimes\mathcal{D}_2 \oplus \underline{\mathcal{Z}}_{32}^{(2)} y\, Res([8,9;2])\otimes\mathcal{D}_1 \oplus \underline{\mathcal{Z}}_{32}^{(3)} y\, Res([8,10;3])\otimes\mathcal{D}_0$$

And

$\sum_{e_1\geq 0, e_2\geq e_1} \underline{\mathcal{Z}}_{32}^{(e_2+1)} y \underline{\mathcal{Z}}_{31}^{(e_1+1)} z$
$Res([8+e_1+1, 7+e_2+1; e_2-e_1])\otimes$
$\mathcal{D}_{3-(e_1+e_2+2)} =$
$\underline{\mathcal{Z}}_{32} y \underline{\mathcal{Z}}_{31} z Res([9,8;0])\otimes\mathcal{D}_1 \oplus \underline{\mathcal{Z}}_{32}^{(2)} y \underline{\mathcal{Z}}_{31} z$
$Res([9,9;1])\otimes\mathcal{D}_0$
Where $\underline{\mathcal{Z}}_{32} y$ is the bar complex

$$0 \longrightarrow \mathcal{Z}_{32} y \xrightarrow{\partial_y} \mathcal{Z}_{32} \longrightarrow 0$$

$\underline{\mathcal{Z}}_{32}^{(2)} y$ is the bar complex

$$0 \longrightarrow \mathcal{Z}_{32} y \mathcal{Z}_{32} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(2)} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(2)} \longrightarrow 0$$

$\underline{\mathcal{Z}}_{32}^{(3)} y$ is the bar complex

$$0 \longrightarrow \mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y \xrightarrow{\partial_y} \begin{matrix} \mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \\ \oplus \\ \mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \end{matrix} \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(3)} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(3)} \longrightarrow 0$$

and $\underline{\mathcal{Z}}_{31} z$ is the bar complex

$$0 \longrightarrow \mathcal{Z}_{31} z \xrightarrow{\partial_z} \mathcal{Z}_{31} \longrightarrow 0$$

Where x, y and z stand for the separator variables, and the boundary map is $\partial_x + \partial_y + \partial_z$.

Let Bar(M,A;S) be the free bar module on the set $S=\{x, y, z\}$, where A is the free associative algebra generated by $\mathcal{Z}_{21}$,$\mathcal{Z}_{32}$ and $\mathcal{Z}_{31}$ and their divided powers with the pursue relations:
$\mathcal{Z}_{32}^{(a)}\mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)}\mathcal{Z}_{32}^{(a)}$ and $\mathcal{Z}_{21}^{(a)}\mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)}\mathcal{Z}_{21}^{(a)}$
and the module M is the direct sum of $\mathcal{D}_p \otimes \mathcal{D}_q \otimes \mathcal{D}_r$ for suitable p, q and r with the action of $\mathcal{Z}_{21}$, $\mathcal{Z}_{32}$, and $\mathcal{Z}_{31}$ and their divided powers.

The terms of the characteristic-free resolution 1 are:

- In dimension zero $(\mathcal{X}_0)$ we have $\mathcal{D}_8\otimes\mathcal{D}_7\otimes\mathcal{D}_3$
- In dimension one $(\mathcal{X}_1)$ we have
  - $\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{8+b}\otimes\mathcal{D}_{7-b}\otimes\mathcal{D}_3$; $with\ 1\leq b\leq 7,\ b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(b)} y\mathcal{D}_8\otimes\mathcal{D}_{7+b}\otimes\mathcal{D}_{3-b}$; $with\ 1\leq b\leq 3,\ b\in\mathbb{Z}^+$
- In dimension two $(\mathcal{X}_2)$ we have the sum of the following terms:
  - $\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{7-|b|}\otimes\mathcal{D}_3$; $with\ 2\leq |b| = b_1+b_2\leq 7;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{8+b}\otimes\mathcal{D}_{8-b}\otimes\mathcal{D}_2$; $with\ 2\leq b\leq 8, b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(2)} y\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{8+b}\otimes\mathcal{D}_{9-b}\otimes\mathcal{D}_1$; $with\ 3\leq b\leq 9, b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(3)} y\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{8+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_0$; $with\ 4\leq b\leq 10, b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(b_1)} y\mathcal{Z}_{32}^{(b_2)} y\mathcal{D}_8\otimes\mathcal{D}_{7+|b|}\otimes\mathcal{D}_{3-|b|}$; $with\ 2\leq |b| = b_1+b_2\leq 3;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(b)} y\mathcal{Z}_{31} z\mathcal{D}_9\otimes\mathcal{D}_{7+b}\otimes\mathcal{D}_{2-b}$; $with\ 1\leq b\leq 2,\ b\in\mathbb{Z}^+$
- In dimension three $(\mathcal{X}_3)$ we have the sum of the following terms:
  - $\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{Z}_{21}^{(b_3)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{7-|b|}\otimes\mathcal{D}_3$; $with\ 3\leq |b| = b_1+b_2+b_3\leq 7\ and\ b_1\geq 1;\ b_1,b_2,b_3\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_2$; $with\ 3\leq |b| = b_1+b_2\leq 8\ and\ b_1\geq 2;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(2)} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; $with\ 4\leq |b| = b_1+b_2\leq 9\ and\ b_1\geq 3;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{8+b}\otimes\mathcal{D}_{9-b}\otimes\mathcal{D}_1$; $with\ 3\leq b\leq 9, b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(3)} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; $with\ 5\leq |b| = b_1+b_2\leq 10\ and\ b_1\geq 4;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(c_1)} y\mathcal{Z}_{32}^{(c_2)} y\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{8+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_0$; $with\ c_1+c_2 = 3\ and\ 4\leq b\leq 10;\ b,c_1,c_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{D}_8\otimes\mathcal{D}_{10}\otimes\mathcal{D}_0$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{31} z\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{9+b}\otimes\mathcal{D}_{8-b}\otimes\mathcal{D}_1$; $with\ 1\leq b\leq 8, b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(2)} y\mathcal{Z}_{31} z\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{9+b}\otimes\mathcal{D}_{9-b}\otimes\mathcal{D}_0$; $with\ 2\leq b\leq 9, b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{Z}_{31} z\mathcal{D}_9\otimes\mathcal{D}_9\otimes\mathcal{D}_0$
- In dimension four $(\mathcal{X}_4)$ we have the sum of the following terms:
  - $\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{Z}_{21}^{(b_3)} x\mathcal{Z}_{21}^{(b_4)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{7-|b|}\otimes\mathcal{D}_3$; $with\ 4\leq |b| = \sum_{i=1}^{4} b_i\leq 7\ and\ b_1\geq 1;\ b_1,b_2,b_3,b_4\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{Z}_{21}^{(b_3)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_2$; $with\ 4\leq |b| = b_1+b_2+b_3\leq 8\ and\ b_1\geq 2;\ b_1,b_2,b_3\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(2)} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{Z}_{21}^{(b_3)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; $with\ 5\leq |b| = b_1+b_2+b_3\leq 9\ and\ b_1\geq 3;\ b_1,b_2,b_3\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; $with\ 4\leq |b| = b_1+b_2\leq 9\ and\ b_1\geq 3;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(3)} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{Z}_{21}^{(b_3)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; $with\ 6\leq |b| = b_1+b_2+b_3\leq 10\ and\ b_1\geq 4;\ b_1,b_2,b_3\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(c_1)} y\mathcal{Z}_{32}^{(c_2)} y\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; $with\ c_1+c_2 = 3\ and\ 5\leq |b| = b_1+b_2\leq 10\ and\ b_1\geq 4;\ b_1,b_2,c_1,c_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{8+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_0$; $with\ 4\leq b\leq 10, b\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{31} z\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_1$; $with\ 2\leq |b| = b_1+b_2\leq 8\ and\ b_1\geq 1;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32}^{(2)} y\mathcal{Z}_{31} z\mathcal{Z}_{21}^{(b_1)} x\mathcal{Z}_{21}^{(b_2)} x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_0$; $with\ 3\leq |b| = b_1+b_2\leq 9\ and\ b_1\geq 2;\ b_1,b_2\in\mathbb{Z}^+$
  - $\mathcal{Z}_{32} y\mathcal{Z}_{32} y\mathcal{Z}_{31} z\mathcal{Z}_{21}^{(b)} x\mathcal{D}_{9+b}\otimes\mathcal{D}_{9-b}\otimes\mathcal{D}_0$; $with\ 2\leq b\leq 9, b\in\mathbb{Z}^+$

∘ In dimension five $(\mathcal{X}_5)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{7-|b|} \otimes \mathcal{D}_3$; $with\ 5 \le |b| = \sum_{i=1}^{5} b_i \le 7\ and\ b_1 \ge 1;$ $b_1, b_2, b_3, b_4, b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_2$; $with\ 5 \le |b| = \sum_{i=1}^{4} b_i \le 8\ and\ b_1 \ge 2;$ $b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_1$; $with\ 6 \le |b| = \sum_{i=1}^{4} b_i \le 9\ and\ b_1 \ge 3;$ $b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_1$; $with\ 5 \le |b| = b_1 + b_2 + b_3 \le 9\ and\ b_1 \ge 3;$ $b_1, b_2, b_3 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(3)} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ 7 \le |b| = \sum_{i=1}^{4} b_i \le 10\ and\ b_1 \ge 4;$ $b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(c_1)} y \mathcal{Z}_{32}^{(c_2)} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ c_1 + c_2 = 3\ and$ $6 \le |b| = b_1 + b_2 + b_3 \le 10\ and\ b_1 \ge 4;$ $b_1, b_2, b_3 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ 5 \le |b| = b_1 + b_2 \le 10\ and\ b_1 \ge 4;$ $b_1, b_2 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_1$; $with\ 3 \le |b| = b_1 + b_2 + b_3 \le 8\ and$ $b_1 \ge 1;\ b_1, b_2, b_3 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_0$; $with\ 4 \le |b| = b_1 + b_2 + b_3 \le 9\ and$ $b_1 \ge 2;\ b_1, b_2, b_3 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_0$; $with\ 3 \le |b| = b_1 + b_2 \le 9\ and\ b_1 \ge 2;$ $b_1, b_2 \in \mathbb{Z}^+$

∘ In dimension six $(\mathcal{X}_6)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{Z}_{21}^{(b_6)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{7-|b|} \otimes \mathcal{D}_3$; $with\ 6 \le |b| = \sum_{i=1}^{6} b_i \le 7\ and$ $b_1 \ge 1;\ b_1, b_2, b_3, b_4, b_5, b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y\ \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_2$; $with\ 6 \le |b| = \sum_{i=1}^{5} b_i \le 8\ and$ $b_1 \ge 2;\ b_1, b_2, b_3, b_4, b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)} y\ \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_1$; $with\ 7 \le |b| = \sum_{i=1}^{5} b_i \le 9\ and$ $b_1 \ge 3;\ b_1, b_2, b_3, b_4, b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_1$; $with\ 6 \le |b| = \sum_{i=1}^{4} b_i \le 9\ and$ $b_1 \ge 3;\ b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(3)} y\ \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ 8 \le |b| = \sum_{i=1}^{5} b_i \le 10\ and$ $b_1 \ge 4;\ b_1, b_2, b_3, b_4, b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(c_1)} y \mathcal{Z}_{32}^{(c_2)} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ c_1 + c_2 = 3\ and$ $7 \le |b| = \sum_{i=1}^{4} b_i \le 10\ and\ b_1 \ge 4;$ $b_1, b_2, b_3, b_4, c_1, c_2 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ 6 \le |b| = b_1 + b_2 + b_3 \le 10$ $and\ b_1 \ge 4;\ b_1, b_2, b_3 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_1$; $with\ 4 \le |b| = \sum_{i=1}^{4} b_i \le 8\ and$ $b_1 \ge 1;\ b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_0$; $with\ 5 \le |b| = \sum_{i=1}^{4} b_i \le 9\ and$ $b_1 \ge 2;\ b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_0$; $with\ 4 \le |b| = b_1 + b_2 + b_3 \le 9$ $and\ b_1 \ge 2;\ b_1, b_2, b_3 \in \mathbb{Z}^+$

∘ In dimension seven $(\mathcal{X}_7)$ we have the sum of the following terms:

- $\mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{32} y\ \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{Z}_{21}^{(b_6)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_2$; $with\ 7 \le |b| = \sum_{i=1}^{6} b_i \le 8$ $and\ b_1 \ge 2;\ b_1, b_2, b_3, b_4, b_5, b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_1$; $with\ 7 \le |b| = \sum_{i=1}^{5} b_i \le 9$ $and\ b_1 \ge 3;\ b_1, b_2, b_3, b_4, b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)} y\ \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{Z}_{21}^{(b_6)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_1$; $with\ 8 \le |b| = \sum_{i=1}^{6} b_i \le 9$ $and\ b_1 \ge 3;\ b_1, b_2, b_3, b_4, b_5, b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_1$; $with\ 7 \le |b| = \sum_{i=1}^{4} b_i \le 10$ $and\ b_1 \ge 4;\ b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(c_1)} y \mathcal{Z}_{32}^{(c_2)} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ c_1 + c_2 = 3\ and$ $8 \le |b| = \sum_{i=1}^{5} b_i \le 10\ and\ b_1 \ge 4;$ $b_1, b_2, b_3, b_4, b_5, c_1, c_2 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(3)} y\ \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{Z}_{21}^{(b_6)} x \mathcal{D}_{8+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_0$; $with\ 9 \le |b| = \sum_{i=1}^{6} b_i \le$ $10\ and\ b_1 \ge 4;\ b_1, b_2, b_3, b_4, b_5, b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_1$; $with\ 5 \le |b| = \sum_{i=1}^{5} b_i \le 8$ $and\ b_1 \ge 1;\ b_1, b_2, b_3, b_4, b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_0$; $with\ 5 \le |b| = \sum_{i=1}^{4} b_i \le 9\ and$ $b_1 \ge 2;\ b_1, b_2, b_3, b_4 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_0$; $with$ $6 \le |b| = \sum_{i=1}^{5} b_i \le 9\ and\ b_1 \ge 2;$ $b_1, b_2, b_3, b_4, b_5 \in \mathbb{Z}^+$

- In dimension eight $(\mathcal{X}_8)$ we have the sum of the following terms:
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{16}\otimes\mathcal{D}_0\otimes\mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\,\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_2;\ with$ $8 \le |b| = \sum_{i=1}^{6} b_i \le 9\ and\ b_1 \ge 3;$ $b_1,b_2,b_3,b_4,b_5,b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\,\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0;with\ 8 \le |b| = \sum_{i=1}^{5} b_i \le 10\ and\ b_1 \ge 4;\ b_1,b_2,b_3,b_4,b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\,\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0\,;with\ c_1+c_2=3$ $and\ 9 \le |b| = \sum_{i=1}^{6} b_i \le 10\ and\ b_1 \ge 4;$ $b_1,b_2,b_3,b_4,b_5,b_6,c_1,c_2 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_1$
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\,\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_1\,;with$ $7 \le |b| = \sum_{i=1}^{6} b_i \le 8\ and\ b_1 \ge 2;$ $b_1,b_2,b_3,b_4,b_5,b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\,\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_0\,;with$ $6 \le |b| = \sum_{i=1}^{5} b_i \le 9\ and\ b_1 \ge 2;$ $b_1,b_2,b_3,b_4,b_5 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\,\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_0\,;with$ $7 \le |b| = \sum_{i=1}^{6} b_i \le 9\ and\ b_1 \ge 2;$ $b_1,b_2,b_3,b_4,b_5,b_6 \in \mathbb{Z}^+$

- In dimension eight $(\mathcal{X}_9)$ we have the sum of the following terms:
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\,\mathcal{Z}_{21}x\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\,\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{8+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0\,;with$ $9 \le |b| = \sum_{i=1}^{6} b_i \le 10\ and\ b_1 \ge 4;$ $b_1,b_2,b_3,b_4,b_5,b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\,\mathcal{Z}_{21}x\mathcal{D}_{18}\otimes\mathcal{D}_0\otimes\mathcal{D}_0\,;with\ c_1+c_2=3$
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\,\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_1\,;with$ $7 \le |b| = \sum_{i=1}^{6} b_i \le 8\ and\ b_1 \ge 1;$ $b_1,b_2,b_3,b_4,b_5,b_6,b_7 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\,\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_0\,;with$ $7 \le |b| = \sum_{i=1}^{5} b_i \le 9\ and\ b_1 \ge 2;$ $b_1,b_2,b_3,b_4,b_5,b_6 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\,\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_0\,;with$ $8 \le |b| = \sum_{i=1}^{7} b_i \le 9\ \ and\ b_1 \ge 1;$ $b_1,b_2,b_3,b_4,b_5,b_6,b_7 \in \mathbb{Z}^+$

- In dimension eight $(\mathcal{X}_{10})$ we have the sum of the following terms:
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\,\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{18}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\,\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\,\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_0\,;with$ $8 \le |b| = \sum_{i=1}^{5} b_i \le 9\ and\ b_1 \ge 2;$ $b_1,b_2,b_3,b_4,b_5,b_6,b_7 \in \mathbb{Z}^+$
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\,\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{18}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

Finally In dimension nine $(\mathcal{X}_{11})$ we have

- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\,\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{18}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

Lascoux Resolution of the Partition (8,7,3)

The Lascoux resolution of the Weyl module associated to the partition (8,7,3) is

$$\mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \longrightarrow \begin{array}{c}\mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\\ \oplus \\ \mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_1\mathcal{F}\end{array}$$

$$\longrightarrow \begin{array}{c}\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}\\ \oplus \\ \mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\end{array} \longrightarrow \mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}$$

Where the position of the terms of the complex determined by the length of the permutations to which they correspond.

Then the Lascaux complex has the correspondence between its terms as pursue:

$\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}\ \leftrightarrow$ identity
$\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}\ \leftrightarrow (12)$
$\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\ \leftrightarrow (23)$
$\mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\ \leftrightarrow (123)$
$\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_1\mathcal{F}\ \leftrightarrow (132)$
$\mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_1\mathcal{F}\ \leftrightarrow (13)$

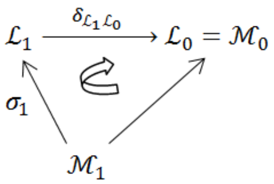
As in (8) the terms can be exhibit as pursue
$\mathcal{X}_0 = \mathcal{L}_0 = \mathcal{M}_0$
$\mathcal{X}_1 = \mathcal{L}_1 \oplus \mathcal{M}_1$
$\mathcal{X}_2 = \mathcal{L}_2 \oplus \mathcal{M}_2$
$\mathcal{X}_3 = \mathcal{L}_3 \oplus \mathcal{M}_3$
$\mathcal{X}_j = \mathcal{M}_j \qquad for\ \ j = 4,5,\ldots,11$
Where $\mathcal{L}_e$ are the sum of the Lascoux terms and $\mathcal{M}_e$ are the sum of the others.

Now, we acquaint the map $\sigma_1: \mathcal{M}_1 \longrightarrow \mathcal{L}_1$ as pursue

- $\mathcal{Z}_{21}^{(2)}x(v) \longmapsto \frac{1}{2}\ \mathcal{Z}_{21}x\partial_{21}(v);$
  $where\ \ v \in \mathcal{D}_{10} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\ \mathcal{Z}_{21}x\partial_{21}^{(2)}(v)\ ;$
  $where\ \ v \in \mathcal{D}_{11} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{4}\ \mathcal{Z}_{21}x\partial_{21}^{(3)}(v);$
  $where\ \ v \in \mathcal{D}_{12} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{5}\ \mathcal{Z}_{21}x\partial_{21}^{(4)}(v);$
  $where\ \ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{6}\ \mathcal{Z}_{21}x\partial_{21}^{(5)}(v);$
  $where\ \ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{7}\ \mathcal{Z}_{21}x\partial_{21}^{(6)}(v)\ ;$
  $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{32}^{(2)}y(v) \longmapsto \frac{1}{2}\ \mathcal{Z}_{32}y\partial_{32}(v)\ ;$
  $where\ \ v \in \mathcal{D}_8 \otimes \mathcal{D}_9 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(3)}y(v) \longmapsto \frac{1}{3}\ \mathcal{Z}_{32}x\partial_{32}^{(2)}(v);$
  $where\ \ v \in \mathcal{D}_8 \otimes \mathcal{D}_{10} \otimes \mathcal{D}_0$

We ought to indicate that the map σ_1 implement the identity

$\delta_{\mathcal{L}_1\mathcal{L}_0}\sigma_1 = \delta_{\mathcal{M}_1\mathcal{M}_0}$ … 2



Where $\delta_{\mathcal{L}_1\mathcal{L}_0}$ the component of the boundary of the fat complex which conveys $\mathcal{L}_1$ to $\mathcal{L}_0$.
We employ the registration $\delta_{\mathcal{L}_{t+1}\mathcal{L}_t}, \delta_{\mathcal{L}_{t+1}\mathcal{M}_1}$ etc.
Thus we can acquaint $\partial_1: \mathcal{L}_1 \longrightarrow \mathcal{L}_0$ as

$$\partial_1 = \delta_{\mathcal{L}_1\mathcal{L}_0}$$

It is plainsman to exhibit that ∂_1 implement 2, for example we adopt one of them:

$$(\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1)\left(\mathcal{Z}_{21}^{(5)}x(v)\right) =$$
$$\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1\left(\frac{1}{5}\ \mathcal{Z}_{21}x\partial_{21}^{(4)}(v)\right)$$
$$= \frac{1}{5}\left(\partial_{21}\partial_{21}^{(4)}(v)\right) = \partial_{21}^{(5)}(v) = \delta_{\mathcal{M}_1\mathcal{M}_0}\left(\mathcal{Z}_{21}^{(5)}x(v)\right)$$

As long as we can acquaint $\partial_2: \mathcal{L}_2 \longrightarrow \mathcal{L}_1$ by $\partial_2 = \delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}$

Proposition 3.1: (4), (5) and (8)

The composition $\partial_1 \circ \partial_2$ equal to zero.

Proof:

$$\partial_1 \circ \partial_2(g) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ (\delta_{\mathcal{L}_2\mathcal{L}_1}(g) + \sigma_1\delta_{\mathcal{L}_2\mathcal{M}_1}(g))$$
$$= \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(g) +$$
$$\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1\delta_{\mathcal{L}_2\mathcal{M}_1}(g)$$

But $\delta_{\mathcal{L}_1\mathcal{L}_0}\sigma_1 = \delta_{\mathcal{M}_1\mathcal{M}_0}$. Then we attain

$$\partial_1 \circ \partial_2(g) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(g) + \delta_{\mathcal{M}_1\mathcal{M}_0}\delta_{\mathcal{L}_2\mathcal{M}_1}(g)$$

Which equal to zero, because of the properties of the boundary map δ, so we attain $\partial_1 \circ \partial_2 = 0$.

Now, we have to acquaint $\sigma_2: \mathcal{M}_2 \longrightarrow \mathcal{L}_2$ such that

$\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1} = \left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right) \circ \sigma_2$ … 3

We acquaint this map as pursue:

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0\ ; where\ v \in \mathcal{D}_{10} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{11} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{11} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0; where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}(v);$
  $where\ \ v \in \mathcal{D}_{11} \otimes \mathcal{D}_5 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{6}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}(v)\ ;$
  $where\ \ v \in \mathcal{D}_{12} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{10}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}(v)\ ;$
  $where\ \ v \in \mathcal{D}_{13} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{15}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}(v);$
  $where\ \ v \in \mathcal{D}_{14} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{21}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}(v);$
  $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{28}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}(v);$
  $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y(v) \longmapsto 0; where\ \ v \in \mathcal{D}_8 \otimes \mathcal{D}_9 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\ (\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) -$
  $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v))\ ; where\ \ v \in \mathcal{D}_{11} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{12}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}(v) -$
  $\frac{1}{4}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}(v)\ ; where\ \ v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{30}\ \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) -$
  $\frac{1}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(4)}(v)\ \ ; where\ \ v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$

- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{60}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(5)}(v)$; $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{105}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{7}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(6)}(v)$; $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{168}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{1}{8}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(7)}(v)$; $where\ v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{252}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(7)}(v)$; $where\ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}(v) \longmapsto 0; where\ v \in \mathcal{D}_8 \otimes \mathcal{D}_{10} \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}(v) \longmapsto 0 where\ v \in \mathcal{D}_8 \otimes \mathcal{D}_{10} \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v)$;
  $where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}^{(2)}(v) - \frac{7}{90}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v) - \frac{2}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(4)}\partial_{32}(v)$;
  $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}^{(2)}(v) - \frac{2}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(5)}\partial_{32}(v)$;
  $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{30}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}^{(2)}(v) - \frac{1}{35}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}\partial_{32}^{(2)}(v) - \frac{2}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(6)}\partial_{32}(v)$;
  $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) - \frac{5}{252}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{32}^{(2)}(v) - \frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(7)}\partial_{32}(v)$;
  $where\ v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{63}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}^{(2)}(v) + \frac{1}{84}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{32}\partial_{31}(v)$;
  $where\ v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(10)}x(v) \longmapsto \frac{1}{84}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{31}^{(2)}(v)$;
  $where\ v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}z(v) \longmapsto \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{32}(v)$;
  $where\ v \in \mathcal{D}_9 \otimes \mathcal{D}_9 \otimes \mathcal{D}_0$

It is plainsman to exhibit that σ_2 which is acquainting above implement 3, for example we adopt one of them:

$Where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$

$(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1})(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x(v))$

$= \sigma_1(\mathcal{Z}_{21}^{(7)}x\partial_{32}^{(2)}(v) + \mathcal{Z}_{21}^{(6)}x\partial_{32}\partial_{31}(v) + \mathcal{Z}_{21}^{(5)}x\partial_{31}^{(2)}(v) - \mathcal{Z}_{32}^{(2)}\mathcal{y}\partial_{21}^{(7)}(v))$

$= \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}^{(2)}(v) + \frac{1}{6}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}\partial_{31}(v) + \frac{1}{5}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\partial_{32}\partial_{21}^{(7)}(v)$

And

$(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1})(\frac{1}{105}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{7}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(6)}(v))$

$= \sigma_1\left(\frac{1}{105}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}\partial_{31}(v) + \frac{2}{105}\mathcal{Z}_{21}^{(2)}x\partial_{31}\partial_{21}^{(4)}\partial_{31}(v)\right) + \frac{2}{105}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) - \frac{1}{7}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(4)}\partial_{31}(v) - \sigma_1\left(\mathcal{Z}_{32}^{(2)}\mathcal{y}\partial_{21}^{(7)}(v)\right) + \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}^{(2)}(v) + \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}\partial_{31}(v) + \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) + \frac{1}{7}\mathcal{Z}_{32}\mathcal{y}\partial_{31}\partial_{21}^{(6)}(v)$

$= \frac{1}{42}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}\partial_{31}(v) + \frac{6}{105}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) - \frac{1}{7}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\partial_{32}\partial_{21}^{(7)}(v) + \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}^{(2)}(v) + \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}\partial_{31}(v) + \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) + \frac{1}{7}\mathcal{Z}_{32}\mathcal{y}\partial_{31}\partial_{21}^{(6)}(v)$

$= \frac{1}{6}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}\partial_{31}(v) + \frac{1}{5}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\partial_{32}\partial_{21}^{(7)}(v) + \frac{1}{7}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}^{(2)}(v)$

Proposition 3.2:
We have exactness at $\mathcal{L}_i$, $i = 1,2,3$.
Proof: see (4), (5) and (8).

Now by employ σ_2 we can also acquaint $\partial_3: \mathcal{L}_3 \longrightarrow \mathcal{L}_2$ by $\partial_3 = \delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}$

Proposition 3.3:
The composition $\partial_1 \circ \partial_2$ equal to zero.
Proof: The oneself track employ in proposition 2.

We requirement to acquaint $\sigma_3: \mathcal{M}_3 \longrightarrow \mathcal{L}_3$ which implement

$$\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2} = \left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right) \circ \sigma_3 \quad \ldots 4$$

As pursue:

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$;
  $where\ v \in \mathcal{D}_{11} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$;
  $where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0$;
  $where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$;
  $where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$;
  $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{11} \otimes \mathcal{D}_5 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{12} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{13} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{14} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{15} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{16} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{16} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{16} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0;$ $where\ v \in \mathcal{D}_{16} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$

- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}(v)$; $where\ \ v \in \mathcal{D}_{11} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(4)}x(v) \longmapsto -\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}(v)$; $where\ \ v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(5)}x(v) \longmapsto -\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}(v)$; $where\ \ v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(6)}x(v) \longmapsto -\frac{1}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}(v)$; $where\ \ v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(7)}x(v) \longmapsto -\frac{1}{21}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}(v)$; $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(8)}x(v) \longmapsto -\frac{1}{28}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}(v)$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(9)}x(v) \longmapsto -\frac{1}{36}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}(v)$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}(v) \longmapsto 0$; $where\ \ v \in \mathcal{D}_8 \otimes \mathcal{D}_{10} \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v) - \frac{1}{4}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)$; $where\ \ v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{12}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{7}{60}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)$; $where\ \ v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{20}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; $where\ \ v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{30}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\ z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{3}{70}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; $where\ \ v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{42}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{5}{168}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; $where\ \ v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{56}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) + \frac{1}{72}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; $where\ \ v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$

- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(10)}x(v)$
  $\longmapsto \frac{1}{72}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v)$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x(v)$
  $\longmapsto -\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x(v)$
  $\longmapsto -\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x(v)$
  $\longmapsto -\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x(v)$
  $\longmapsto -\frac{1}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x(v)$
  $\longmapsto -\frac{1}{21}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(9)}x(v) \longmapsto 0$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(10)}x(v) \longmapsto 0$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v)$
  $\longmapsto -\frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v) -$
  $\frac{1}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v)$
  $\longmapsto -\frac{1}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) -$
  $\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v)$
  $\longmapsto -\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) -$
  $\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v)$
  $\longmapsto -\frac{1}{30}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) -$
  $\frac{1}{90}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v)$
  $\longmapsto -\frac{2}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) -$
  $\frac{4}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v)$
  $\longmapsto -\frac{2}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) -$
  $\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v)$
  $\longmapsto -\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) -$
  $\frac{2}{315}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v)$
  $\longmapsto -\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) -$
  $\frac{1}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v)$
  $\longmapsto -\frac{5}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) -$
  $\frac{2}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v)$
  $\longmapsto -\frac{10}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) -$
  $\frac{5}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}x(v)$
  $\longmapsto -\frac{1}{63}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) -$
  $\frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(2)}x(v)$
  $\longmapsto -\frac{2}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) -$
  $\frac{7}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(3)}x(v)$
  $\longmapsto -\frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) -$
  $\frac{7}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(4)}x(v)$
  $\longmapsto -\frac{10}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) -$
  $\frac{35}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(5)}x(v)$
  $\longmapsto -\frac{5}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) -$
  $\frac{7}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}x(v) \longmapsto 0$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(2)}x(v)$
  $\longmapsto -\frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v)$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$

- $\mathcal{Z}_{32}^{(3)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21}^{(3)} x(v)$
  $\longmapsto -\frac{7}{15} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{31}(v)$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(4)} x(v)$
  $\longmapsto -\frac{7}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{31}(v)$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(5)} x(v)$
  $\longmapsto -\frac{35}{18} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{31}(v)$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(3)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(6)} x(v)$
  $\longmapsto -\frac{7}{3} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{31}(v)$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(2)} x(v)$
  $\longmapsto \frac{1}{3} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}(v)$;
  where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(3)} x(v)$
  $\longmapsto \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)}(v)$;
  where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(4)} x(v)$
  $\longmapsto \frac{1}{10} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)}(v)$;
  where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(5)} x(v)$
  $\longmapsto \frac{1}{15} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)}(v)$;
  where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(6)} x(v)$
  $\longmapsto \frac{1}{21} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)}(v)$;
  where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(7)} x(v)$
  $\longmapsto \frac{1}{28} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)}(v)$;
  where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(8)} x(v)$
  $\longmapsto \frac{1}{36} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)}(v)$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{31} z(v) \longmapsto 0$;
  where $v \in \mathcal{D}_9 \otimes \mathcal{D}_9 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(2)} x(v)$
  $\longmapsto \frac{1}{3} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{31}(v)$;
  where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(3)} x(v)$
  $\longmapsto \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21} \partial_{31}(v) -$
  $\frac{1}{12} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{32}(v)$;
  where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(4)} x(v)$
  $\longmapsto \frac{1}{9} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{31}(v) -$
  $\frac{7}{90} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{32}(v)$;
  where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(5)} x(v)$
  $\longmapsto \frac{1}{12} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{31}(v) -$
  $\frac{1}{15} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)} \partial_{32}(v)$;
  where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(6)} x(v)$
  $\longmapsto \frac{1}{15} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)} \partial_{31}(v) -$
  $\frac{2}{35} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)} \partial_{32}(v)$;
  where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(7)} x(v)$
  $\longmapsto \frac{1}{18} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)} \partial_{31}(v) -$
  $\frac{25}{504} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)} \partial_{32}(v)$;
  where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(8)} x(v)$
  $\longmapsto \frac{1}{21} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)} \partial_{31}(v) +$
  $\frac{1}{36} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{32}(v)$;
  where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(9)} x(v)$
  $\longmapsto \frac{1}{24} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{31}(v)$;
  where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$

Again we can exhibit that σ_3 which is realized above implement 4, and we adopt one of them as an example

Where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$

$$(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2})(\mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21}^{(3)} x(v))$$
$$= \sigma_2(2\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \partial_{32}^{(2)}(v) - \mathcal{Z}_{21} x \partial_{32}^{(2)} \mathcal{Z}_{21}^{(3)} x(v) + \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x \partial_{32}(v) - \mathcal{Z}_{32} y \mathcal{Z}_{32} y \partial_{21}^{(4)}(v)) + \mathcal{Z}_{32} y \mathcal{Z}_{31} z \partial_{21}^{(3)}(v)$$
$$= \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \partial_{21} \partial_{31}(v) + \frac{1}{2} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \partial_{21}^{(3)}(v) + \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \partial_{21}^{(2)} \partial_{32}(v)$$

And

$$(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2})(\frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)}(v))$$
$$= \sigma_2\left(\frac{1}{6} \mathcal{Z}_{21} x \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{32}^{(2)}(v)\right) + \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \partial_{32} \partial_{21}^{(2)}(v) + \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \partial_{21} \partial_{31}(v) - \sigma_2\left(\mathcal{Z}_{32} y \mathcal{Z}_{32} y \partial_{21}^{(4)}(v)\right) + \frac{1}{2} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \partial_{21}^{(3)}(v)$$
$$= \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \partial_{21}^{(2)} \partial_{32}(v) + \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \partial_{21} \partial_{31}(v) + \frac{2}{4} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \partial_{21}^{(3)}(v)$$

So from all, we have done above we have the complex

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0$$

Where ∂_1 is the operation of indicated polarization operators, ∂_2 acquaint as pursue

- $\partial_2(\mathcal{Z}_{21}x(v)) = \partial_{21}(v)$;
  $where\ \ v \in \mathcal{D}_9 \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$
- $\partial_2(\mathcal{Z}_{32}y(v)) = \partial_{32}(v)$;
  $where\ \ v \in \mathcal{D}_8 \otimes \mathcal{D}_8 \otimes \mathcal{D}_2$
- $\partial_2(\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x(v)) = \frac{1}{2}\ \mathcal{Z}_{21}x\partial_{21}\partial_{32}(v) + \mathcal{Z}_{21}x\partial_{31}(v) - \mathcal{Z}_{32}y\partial_{21}^{(2)}(v)$;
  $where\ \ v \in \mathcal{D}_{10} \otimes \mathcal{D}_6 \otimes \mathcal{D}_2$
- $\partial_2(\mathcal{Z}_{32}y\mathcal{Z}_{31}z(v)) = \frac{1}{2}\ \mathcal{Z}_{32}y\partial_{32}\partial_{21}(v) + \mathcal{Z}_{21}x\partial_{32}^{(2)}(v) - \mathcal{Z}_{32}y\partial_{32}^{(2)}(v)$;
  $where\ \ v \in \mathcal{D}_9 \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$

And the map ∂_3 acquaint as

- $\partial_2(\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x(v)) = \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}(v) + \mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}(v);\ where\ \ v \in \mathcal{D}_{10} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$

Proposition 3.4:

The complex

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0 \longrightarrow K_{(8,7,3)}$$

is exact.

Proof: see (4), (5) and (8).

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Conflicts of Interest: None.

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حول التحلل الحر لمقاس وايل وتحلل المميز الصفري في حالة التجزئة (8,7,3)

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الخلاصة:

هذا البحث هو تطبيق المميز الحر للتحلل $K_{(8,7,3)}\mathcal{F}$ الى تحلل لاسكو لـ $K_{(8,7,3)}\mathcal{F}$ (تحلل المميز الصفري) والذي حصلنا عليه من دراسة العلاقة بين تحلل مقاس وايل $K_{(8,7,3)}\mathcal{F}$ بصيغة المميز الحر وصيغة لاسكو.

الكلمات المفتاحية: مقاس وايل، تحلل، التحلل الحر، تحلل المميز الحر، تحلل المميز الصفري.