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Solving Whitham-Broer-Kaup-Like Equations Numerically by using Hybrid Differential Transform Method and Finite Differences Method

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Abstract:

This paper aims to propose a hybrid approach of two powerful methods, namely the differential transform and finite difference methods, to obtain the solution of the coupled Whitham-Broer-Kaup-Like equations which arises in shallow-water wave theory. The capability of the method to such problems is verified by taking different parameters and initial conditions. The numerical simulations are depicted in 2D and 3D graphs. It is shown that the used approach returns accurate solutions for this type of problems in comparison with the analytic ones.

Keywords: Differential Transform Method, Finite Differences Method, Hybrid Differential Transform - Finite Differences Method, Whitham-Broer-Kaup-Like Equations.

Introduction:

In various scientific fields, the vast majority of the arising phenomena are known to be described by partial differential equations (PDEs). For instance, wave propagation, heat flow and other physical phenomena. It is therefore important to be aware of all the traditional techniques recently developed to solve PDEs and to implement these techniques. A growing interest in this topic has been shown up in recent activities of researchers [1]. In this work, our investigations restricted to solve non-linear PDEs according to the initial conditions of the adopted variable (IVP).

The double-equation (WBKL) is considered to describe the propagating shallow-water waves which have different dispersion relationships [2]. The associated WBKL equations studied by Whitham[3], Broer [4] and Caup [5] are given as follows:

$$\begin{cases} \psi_t + \psi\psi_x + \gamma\psi_x + \beta\psi_{xx} = 0, \\ \varphi_t + \psi\varphi_x + \varphi\psi_x + \alpha\psi_{xxx} - \beta\varphi_{xx} = 0. \end{cases} \quad (1)$$

with exact solutions

$$\psi(x,t) = \lambda - 2k\coth\left(k(x+x_0) - \lambda t\right),$$

$$\varphi(x,t) = -2k^2 csch^2(k(x+x_0) - \lambda t),$$

where $\psi = \psi(x,t)$denotes the horizontal velocity, $\varphi = \varphi(x,t)$denotes the height that deviates the liquid from its equilibrium position, and α, β are constants that could appear indifferent diffusion powers [6].

Many researchers have discussed various numerical and analytical methods to solve the shallow wave equation since its emergence. For instance, the Whitham-Broer-Kaup equations and their variances were solved by using the homotopy perturbation method [7],the bifurcation method [2],the Adomian decomposition method [8], and the power series method [9].

As a combination of differential transform and finite difference methods, the hybrid differential transform-finite difference method(HDTFDM)has been developed to handle plenty of problems and attracted the attention of a broad group of scholars. For instance Chu and Ghen [10] have utilized it to solve the nonlinear heat construction problem. Maerefat et al. [11] have used it to solve heat transfer model in an annular fin with variable thermal conductivity. Singu and Demir [12] have applied it to solve some nonlinear equations. Che [13] has studied the nonlinear heat combustion problem via the hybrid method. Mosayebidorcheh et al. [14] have analyzed the turbulent MHD Couettenanofluid flow and heat transfer by using the hybrid method.

The finite difference method is one of the most important methods in the field of numerical analysis because of its accurate and detailed results. It is one of the oldest methods used to solve the normal and partial differential equations, This method was proposed in the eighteenth century by Euler And relies on the sequential Tyler [15,16]. However, the differential method alone needs a lot of time, so Hybrid DTFD Methods has been suggested [17].

Our motivation in this paper is to construct the approximate solution of the coupled nonlinear WBKL equations by using a hybrid approach of two well-known methods, namely, the differential transform and finite difference methods. To this end, the HDTFDM is described in Sec. 2. The underlying method is analyzed to gain the approximate analytical solution of the coupled WBKL equations in Sec. 3. Finally, some conclusions are exhibited in Sec.4.

The Hybrid DTFD Method

In order to describe the hybrid method, the differential transformation of the analytic function $\psi(x)$ in a given domain can be written as [10]

$$\Psi(\lambda)=\frac{1}{\lambda!}\left[\frac{d^{\lambda}\psi(x)}{dx^{\lambda}}\right]_{x=x0},$$

The function $\psi(x)$ is analytic and differentiated continuously in the domain of interest [18]. where $\Psi(\lambda)$ stands for the transformed (spectrum) function. The original function could be regained by taking the inverse transform as follows

$$\psi(x)=\sum_{\lambda=0}^{\infty}\Psi(\lambda)(x-x_0)^{\lambda},$$

this series converges if $\exists\ 0<\gamma<1$ such that $\|\psi_{\lambda+1}(x)\|\le\gamma\|\psi_{\lambda}(x)\|,\forall\lambda\ge\lambda_0$, for some $\lambda_0\in IN$ [19].

Now, upon combining the above two equations, Taylor's series expansion of $\psi(x)$ can be readily obtained as follows

$$\psi(x)=\sum_{\lambda=0}^{\infty}(x-x_0)^{\lambda}\frac{1}{\lambda!}\frac{d^{\lambda}\psi(x)}{dx^{\lambda}}\bigg|_{x=x0},$$

Therefore, one can easily deduce that the DTM is based on the Taylor's series expansion. According to the DTM, the derivatives are not evaluated symbolically. In particular, the function $\psi(x)$can be expressed in a finite series as follows

$$\psi(x)=\sum_{\lambda=0}^{n}\Psi(\lambda)(x-x_0)^{\lambda},$$

To solve the PDE in the domain [0, T] and $x\in[x_{first},x_{end}]$ using the hybrid method, the finite differences and DTM are applied on space variable and time variable respectively. The time domain is divided into N sections. Assume that the time ranges are H = T / N (18). The partial derivatives are approximated, with respect to the space-variable, x, in the PDE by the finite differences formulas. The area $0<x<a$ is divided into several equal time periods and the length of the interval is equal to h and to the approximation of the central difference with respect to the first three derivatives, so that equations are recalculated.

It is worth mentioning that this method can be applied directly to non-linear differential equations without the need for linear system. Although the DTM series solution has a good approximation to the exact solution of many equations, it is not applicable to solve the PDEs. For this reason, the hybrid method can be used instead [20].

Approximate Solution of the WBKL equations

To apply the hybrid method to the system

$$\begin{cases}\psi_t=-\psi\psi_x-\gamma\psi_x-\beta\psi_{xx}\,,\\ \varphi_t=-\psi\varphi_x-\varphi\psi_x-\alpha\psi_{xxx}+\beta\varphi_{xx}\,,\end{cases}\qquad(2)$$

Taking differential transform of equation (2) with respect to the time only with the time interval$H=1$. The above system is turned to be:

$$\begin{aligned}(k+1)\Psi(x,k+1)&=-\Psi(x,k)\frac{\partial\Psi(x,k)}{\partial x}-\gamma\frac{\partial\varphi(x,k)}{\partial x}-\beta\frac{\partial^2\Psi(x,k)}{\partial x^2},\\ (k+1)\Phi(x,k+1)&=-\Phi(x,k)\frac{\partial\Psi(x,k)}{\partial x}-\Psi(x,k)\frac{\partial\Phi(x,k)}{\partial x}-\gamma\frac{\partial^3\Phi(x,k)}{\partial x^3}+\beta\frac{\partial^2\Phi(x,k)}{\partial x^2},\end{aligned}\qquad(3)$$

The central-difference formula is used on the first three derivatives in Eq. (3) to obtain the following difference equations:

$$\begin{aligned}(k+1)\Psi_i(k+1)&=-\Psi_i(k)\frac{\left(\Psi_{i+1}(k)-\Psi_{i-1}(k)\right)}{2h}\\&-\gamma\frac{\left(\Phi_{i+1}(k)-\Phi_{i-1}(k)\right)}{2h}\\&-\beta\frac{\left(\Psi_{i+1}(k)-2\Psi_i(k)+\Psi_{i-1}(k)\right)}{h^2},\end{aligned}$$

$$\begin{aligned}(k+1)\Phi_i(k+1)&=-\Phi_i(k)\frac{\left(\Psi_{i+1}(k)-\Psi_{i-1}(k)\right)}{2h}\\&-\eta_i(k)\frac{\left(\Phi_{i+1}(k)-\Phi_{i-1}(k)\right)}{2h}\\&-\alpha\frac{\left(\Psi_{i+2}(k)-2\Psi_{i+1}(k)+2\Psi_{i-1}(k)-\Psi_{i-2}(k)\right)}{2h^3}\\&+\beta\frac{\left(\Phi_{i+1}(k)-2\Phi_i(k)+\Phi_{i-1}(k)\right)}{h^2},\end{aligned}$$

The differential transform result of the initial condition reads

$$\psi(x,0) = \Psi_i(0) = \lambda - 2k\coth\left(k(x+x_0)\right),$$
$$\varphi(x,0) = \Phi_i(0) = -2k^2 csch^2\left(k(x+x_0)\right), \quad (4)$$

where $\Psi_i(k)$ and $\Phi_i(k)$ are differential transformations of $\psi(x,t)$ and $\varphi(x,t)$ respectively at the point $x = x_i$.

$$\Psi_i(k+1) = \frac{-\Psi_i(k)}{k+1}\frac{\left(\Psi_{i+1}(k)-\Psi_{i-1}(k)\right)}{2h} - \frac{\gamma}{k+1}\frac{\left(\Phi_{i+1}(k)-\Phi_{i-1}(k)\right)}{2h} - \frac{\beta}{k+1}\frac{\left(\Psi_{i+1}(k)-2\Psi_i(k)+\Psi_{i-1}(k)\right)}{h^2}$$

$$\Phi_i(k+1) = -\frac{\Phi_i(k)}{k+1}\frac{\left(\Psi_{i+1}(k)-\Psi_{i-1}(k)\right)}{2h} - \frac{\Psi_i(k)}{k+1}\frac{\left(\Phi_{i+1}(k)-\Phi_{i-1}(k)\right)}{2h} - \frac{\alpha}{k+1}\frac{\left(\Psi_{i+2}(k)-2\Psi_{i+1}(k)+2\Psi_{i-1}(k)-\Psi_{i-2}(k)\right)}{2h^3} + \frac{\beta}{k+1}\frac{\left(\Phi_{i+1}(k)-2\Phi_i(k)+\Phi_{i-1}(k)\right)}{h^2}$$

When $k = 0,1,2$, yeilds

$$\Psi_i(1) = \frac{-\Psi_i(0)}{1}\frac{\left(\Psi_{i+1}(0)-\Psi_{i-1}(0)\right)}{2h} - \frac{\gamma}{1}\frac{\left(\Phi_{i+1}(0)-\Phi_{i-1}(0)\right)}{2h} - \frac{\beta}{1}\frac{\left(\Psi_{i+1}(0)-2\Psi_i(0)+\Psi_{i-1}(0)\right)}{h^2}$$

$$\Phi_i(1) = -\frac{\Phi_i(0)}{1}\frac{\left(\Psi_{i+1}(0)-\Psi_{i-1}(0)\right)}{2h} - \frac{\Psi_i(0)}{1}\frac{\left(\Phi_{i+1}(0)-\Phi_{i-1}(0)\right)}{2h} - \frac{\alpha}{1}\frac{(\Psi_{i+2}(0)-2\Psi_{i+1}(0)+2\Psi_{i-1}(0)-\Psi_{i-2}(0))}{2h^3} + \frac{\beta}{1}\frac{(\Phi_{i+1}(0)-2\Phi_i(0)+\Phi_{i-1}(0))}{h^2}$$

$$\Psi_i(2) = \frac{-\Psi_i(1)}{2}\frac{\left(\Psi_{i+1}(1)-\Psi_{i-1}(1)\right)}{2h} - \frac{\gamma}{2}\frac{\left(\Phi_{i+1}(1)-\Phi_{i-1}(1)\right)}{2h} - \frac{\beta}{2}\frac{\left(\Psi_{i+1}(1)-2\Psi_i(1)+\Psi_{i-1}(1)\right)}{h^2}$$

$$\Phi_i(2) = -\frac{\Phi_i(1)}{2}\frac{\left(\Psi_{i+1}(1)-\Psi_{i-1}(1)\right)}{2h} - \frac{\Psi_i(1)}{2}\frac{\left(\Phi_{i+1}(1)-\Phi_{i-1}(1)\right)}{2h} - \frac{\alpha}{2}\frac{(\Psi_{i+2}(1)-2\Psi_{i+1}(1)+2\Psi_{i-1}(1)-\Psi_{i-2}(1))}{2h^3} + \frac{\beta}{2}\frac{(\Phi_{i+1}(1)-2\Phi_i(1)+\Phi_{i-1}(1))}{h^2}$$

$$\Psi_i(3) = \frac{-\Psi_i(2)}{3}\frac{\left(\Psi_{i+1}(2)-\Psi_{i-1}(2)\right)}{2h} - \frac{\gamma}{3}\frac{\left(\Phi_{i+1}(2)-\Phi_{i-1}(2)\right)}{2h} - \frac{\beta}{3}\frac{\left(\Psi_{i+1}(2)-2\Psi_i(2)+\Psi_{i-1}(2)\right)}{h^2}$$

$$\Phi_i(3) = -\frac{\Phi_i(2)}{3}\frac{\left(\Psi_{i+1}(2)-\Psi_{i-1}(2)\right)}{2h} - \frac{\Psi_i(2)}{3}\frac{\left(\Phi_{i+1}(2)-\Phi_{i-1}(2)\right)}{2h} - \frac{\alpha}{3}\frac{(\Psi_{i+2}(2)-2\Psi_{i+1}(2)+2\Psi_{i-1}(2)-\Psi_{i-2}(2))}{2h^3} + \frac{\beta}{3}\frac{(\Phi_{i+1}(2)-2\Phi_i(2)+\Phi_{i-1}(2))}{h^2}$$

So, the approximate solution could be achieved as

$$\psi = \Psi_i(0) + \Psi_i(1) + \Psi_i(2) + \Psi_i(3)$$
$$\varphi = \Phi_i(0) + \Phi_i(1) + \Phi_i(2) + \Phi_i(3)$$

With the aid of MAPLE software, the numerical results are given in Tables 1-6 and depicted graphically in two and three dimensions in Figs. 1-6. From which, one can notice that the approximate solutions of the coupled WBKL equations agrees with the analytical ones for different values of k, α, β, x and t. It is worth mentioning that calculating more components will increase the accuracy of the resultant solution but the computational work will be increased as well.

Table 1. The Exact solution for $\psi_{exact}(x,t)$ when $k = 0.1$ and $x_0 = 10$

x / t	0	0.2	0.4	0.6	0.8	1
0	-0.2576070572	-0.2561777488	-0.2547852884	-0.2534285370	-0.2521063994	-0.2508178224
0.2	-0.2577520598	-0.2549229090	-0.2522370860	-0.2496859588	-0.2472615348	-0.2449564036
0.4	-0.2578974440	-0.2523681090	-0.2473798634	-0.2428707508	-0.2387873764	-0.2350834870
0.6	-0.2580432108	-0.2499352426	-0.2429780834	-0.2369833082	-0.2317990990	-0.2273018348
0.8	-0.2581893616	-0.2476174186	-0.2389821738	-0.2318795634	-0.2260040382	-0.2211205480
1	-0.2454082240	-0.2353490436	-0.2274418012	-0.2211762634	-0.2161800660	-0.2121759128

Table 2. The Exact solution for $\varphi_{exact}(x,t)$ when $k = 0.1$ and $x_0 = 10$

$\frac{x}{t}$	0	0.2	0.4	0.6	0.8	1
0	-0.0144812332	-0.0141069082	-0.0137441980	-0.0133926544	-0.0130518503	-0.0127213792
0.2	-0.0145193225	-0.0137799593	-0.0130854592	-0.0124324688	-0.0118179410	-0.0112391019
0.4	-0.0145575330	-0.0131191718	-0.0118477978	-0.0107199546	-0.0097161424	-0.0088200403
0.6	-0.0145958654	-0.0124959890	-0.0107465649	-0.0092779607	-0.0080369066	-0.0069820712
0.8	-0.0146343201	-0.0119077801	-0.0097636506	-0.0080559637	-0.0066814328	-0.0055652511
1	-0.0113521393	-0.0088838314	-0.0070145955	-0.0055778511	-0.0044603108	-0.0035826886

Table 3. The numerical results for $\psi(x,t)$ when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ and $x_0 = 10$, for the approximate solution of the WBKL equation

$\frac{x}{t}$	0	0.2	0.4	0.6	0.8	1
0	-0.2550331063	-0.2536822611	-0.2523684483	-0.2510854467	-0.2498341281	-0.2486145006
0.2	-0. 2550331063	-0.2523684483	-0. 2498341281	-0.2474238524	-0.2451317168	-0.2429495538
0.4	-0. 2550331063	-0.2498341281	-0. 2451317168	-0.2408692910	-0.2370005974	-0.2334838507
0.6	-0. 2550331063	-0.2474238524	-0.2408692910	-0.2352004000	-0.2302819110	-0.2260065976
0.8	-0. 2550331063	-0.2451317168	-0.2370005974	-0.2302819110	-0.2247051410	-0.2200562795
1	-0.2429495538	-0.2334838507	-0.2260065976	-0.2200562795	-0.2153000560	-0.2114790304

Table 4. The numerical results for $\varphi(x,t)$ when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ and $x_0 = 10$, for the approximate solution of the WBKL equation

$\frac{x}{t}$	0	0.2	0.4	0.6	0.8	1
0	-0.0136849919	-0.0133708600	-0. 0130733161	-0.0127249939	-0.0123839941	-0.0120982000
0.2	-0.0136849919	-0.0130733161	-0.0123839941	-0.0118108800	-0.0112289000	-0.0106973600
0.4	-0.0136849919	-0.0123839941	-0.0112289000	-0.0102219941	-0.0092932860	-0.0084769951
0.6	-0.0136849919	-0.0118108800	-0.0102219941	-0.0088735364	-0.0077219963	-0.006704008
0.8	-0.0136849919	-0.0112289000	-0.0092932860	-0.0077219963	-0.0064362303	-0.0053445075
1	-0.0107073600	-0.0084776905	-0.0067063800	-0.0053493405	-0.0043159975	-0.0034899980

Table 5. The numerical results for $e1 = abs(\psi_{exact}(x,t) - \psi(x,t))$ when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ and $x_0 = 10$, for the approximate solution of the WBKL equation

$\frac{x}{t}$	0	0.2	0.4	0.6	0.8	1
0	0.0025750517	0.0024956167	0.0024171341	0.0023431599	0.0022717284	0.0022034549
0.2	0.0027200543	0.0025547547	0.0024024150	0.0022612499	0.0021296239	0.0020071683
0.4	0.0028654385	0.0025334380	0.0022479525	0.0020005069	0.0017863595	0.0016002768
0.6	0.0030112053	0.0025105337	0.0021078395	0.0017833926	0.0015165153	0.0012964358
0.8	0.0031573561	0.0024855077	0.0019811569	0.0015969797	0.0012982744	0.0010636458
1	0.0024589887	0.0018658334	0.0014364022	0.0011193612	0.0008803030	0.0006972816

Table 6: The numerical results for $e2 = abs(\varphi_{exact}(x,t) - \varphi(x,t))$ when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ and $x_0 = 10$, for the approximate solution of the WBKL equation

$\frac{x}{t}$	0	0.2	0.4	0.6	0.8	1
0	0.0007950080	0.0007391400	0.0006866838	0.0006750060	0.0006660058	0.0006218000
0.2	0.0008350080	0.0007066838	0.0006960058	0.0006191200	0.0005811000	0.0005426400
0.4	0.0008725410	0.0007350058	0.0006211000	0.0004980058	0.0004167140	0.0003430451
0.6	0.0009108734	0.0006891200	0.0005250058	0.0004044242	0.0003149102	0.0002780632
0.8	0.0009493281	0.0006811000	0.0004667140	0.0003280036	0.0002437696	0.0002154924
1	0.0006526400	0.0004061404	0.0003082154	0.0002285105	0.0001450024	0.0009000193

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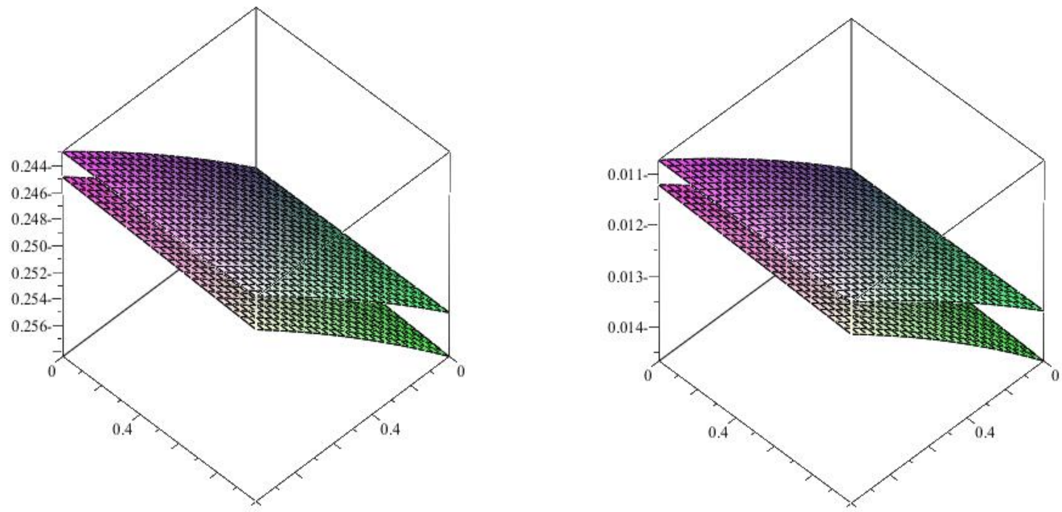
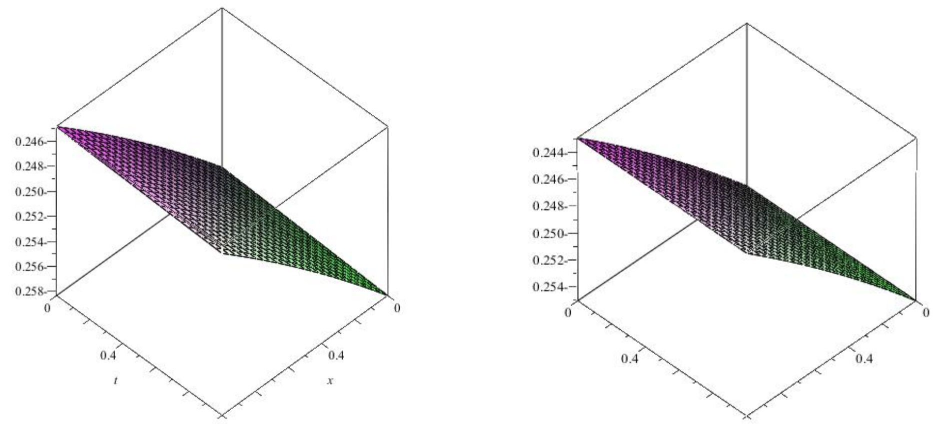


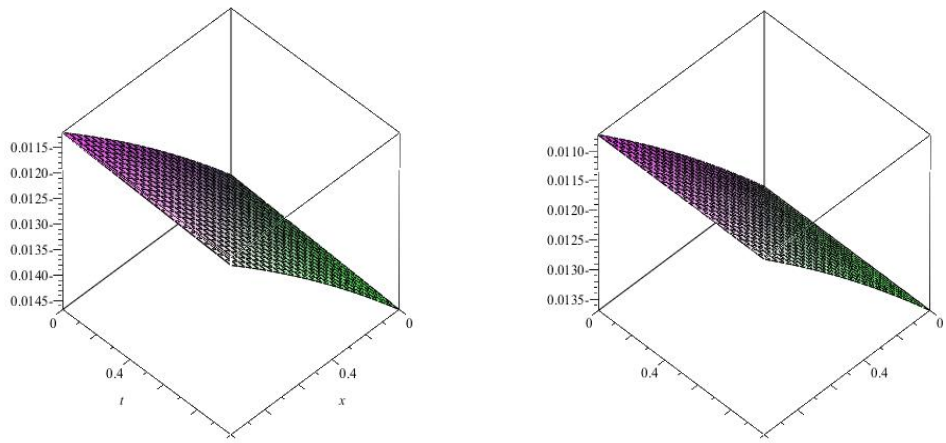
(a) **(b)**

Figure 1.The surface shows the solution $\psi(x, t)$& $\varphi(x,t)$when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ $and\ x_0 = 10$ (a) approximate solution and exact solution for ψ(x,t) , (b)approximate solution and exact solution for φ(x,t)



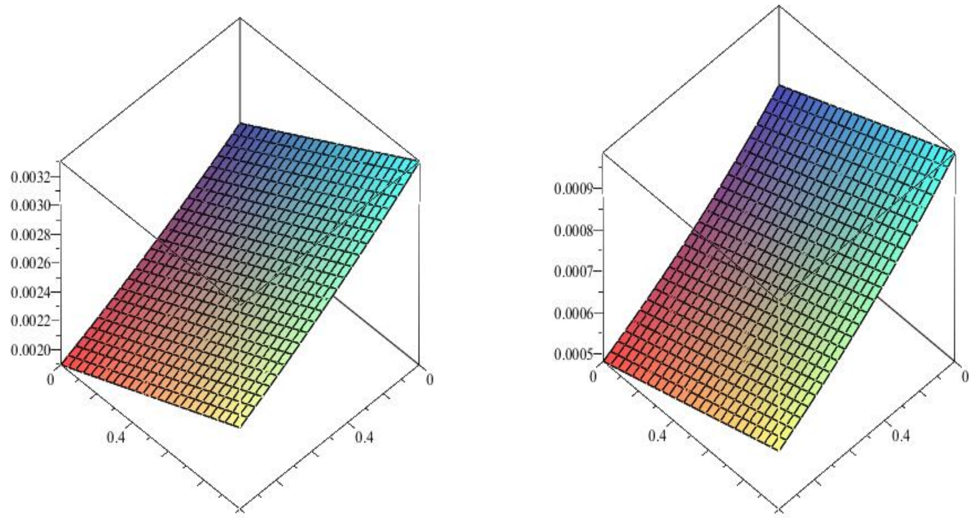
(a) **(b)**

Figure 2. The surface shows the solution $\psi(x,t)$when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ $and\ x_0 = 10$ (a) exact solution , (b)approximate solution



(a) **(b)**

Figure 3. The surface shows the solution $\varphi(x,t)$when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ $and\ x_0 = 10$ (a) exact solution , (b) approximate solution



(a) (b)

Figure 4. The surface shows the maximum Absolute value (MAE) for *ψ(x,t)&φ(x,t)*when $k = 0.1$, $\lambda = 0.005$, $\alpha = 0.7$, $\beta = 0.1$, $\gamma = 0.4$ and $x_0 = 10$ (a) MAE for *ψ(x,t)*, (b) MAE for *φ(x,t)*

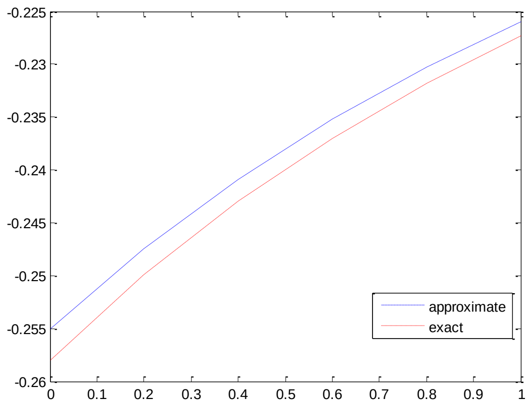


Figure 5. Exact and approximate solution for ψ(x,t) when the time response of the point t=0.6 and x= 0 , 0.2 , 0.4 , 0.6 , 0.8 , 1

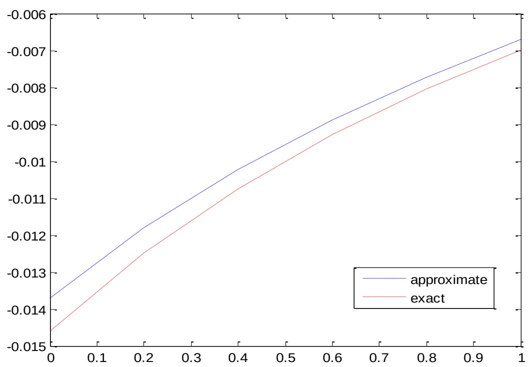


Figure 6. Exact and approximate solution for φ(x,t) when the time response of the point t=0.6 and x= 0, 0.2 , 0.4 , 0.6 , 0.8 , 1

Conclusions:

A physical model of the propagation of shallow water waves is analyzed by the hybrid differential transform-finite difference method. The approximate analytic solutions of the coupled Whitham-Broer-Kaup-Like equations are attained depicted in Figs. 1-6. These solutions are compared with exact ones to measure their accuracy. The results exhibit that the used approach is robust and efficient for solving nonlinear PDEs, whereas the differential transform method is not applicable for such problems.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Mosul.

Authors' contributions statement:

AM. Al-Rozbayani: Conception, Design, Analysis, Interpretation, Methodology, Investigation, Resources, Validation, Revision, Drafting the manuscript and proofreading. *ZM. Al-Botani*: Acquisition of Data, Resources, Software, Visualization, Writing - Original draft, Writing - Review & Editing.

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حل معادلات Whitham-Broer-Kaup-Like عددياً باستخدام هجين طريقة التحويل التفاضلي مع طريقة الفروقات المنتهية

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الخلاصة:

الهدف من هذا البحث هو استخدام طريقة هجينة لطريقتين قويتين، هما التحويل التفاضلي وطريقة الفروقات المنتهية، وللحصول على حل لمعادلات **Whitham-Broer-Kaup-Like** الثنائيةوالتي يناقش سلوكيات انتشار موجات المياه الضحلة. حيث يتم التحقق من كفاءة الطريقة على مثل هذه المسائل من خلال أخذ معلمات وشروط ابتدائية. النتائج العددية موضحة ببعدين وثلاثة أبعاد والتي تم الحصول عليها باستخدام هذه الطريقة ومقارنتها مع الحلول التحليلية تبين أن هناك دقة كبيرة لاقتراب هذه النتائج مع النتائج التحليلية.

الكلمات المفتاحية: طريقة التحويل التفاضلي، طريقة الفروقات المنتهية، معادلات **Whitham-Broer-Kaup-Like**، طريقة هجين التحويل التفاضلي-الفروقات المنتهية.