

DOI: <http://dx.doi.org/10.21123/bsj.2021.18.3.0531>

## Studying the Magnetohydrodynamics for Williamson Fluid with Varying Temperature and Concentration in an Inclined Channel with Variable Viscosity

Wissam Sadiq Khudair \*

Hasan Hadi Dwail

Directorate of Education Babylon, Ministry of Education, Iraq.

\*Corresponding author: [wissamhsse12@gmail.com](mailto:wissamhsse12@gmail.com)\*, [mathhasan74@gmail.com](mailto:mathhasan74@gmail.com)

\*ORCID ID: <https://orcid.org/0000-0002-2110-7275>\*, <https://orcid.org/0000-0003-2201-5636>

Received 13/11/2019, Accepted 6/9/2020, Published Online First 21/2/2021, Published 1/9/2021



This work is licensed under a [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/).

### Abstract:

In this paper, the Magnetohydrodynamic (MHD) for Williamson fluid with varying temperature and concentration in an inclined channel with variable viscosity has been examined. The perturbation technique in terms of the Weissenberg number ( $We \ll 1$ ) to obtain explicit forms for the velocity field has been used. All the solutions of physical parameters of the Darcy parameter ( $Da$ ), Reynolds number ( $Re$ ), Peclet number ( $Pe$ ) and Magnetic parameter ( $M$ ) are discussed under the different values as shown in plots.

**Key words:** Inclined channel, Magnetohydrodynamic (MHD), Variable viscosity, Williamson fluid.

### Introduction:

In physiology, the Non-Newtonian fluids that modify their consistency in line with the forces they managed. The molecule mixture traps them in situ, nevertheless, the opposite section assumes that hanging these substances expels fluids from the colloids and pushes them nearer along and hardens. However, a recent analysis has tested each assumption. The thickness of this liquid depends primarily on the friction between its molecules and hydraulics forces that play a role for the chemical process once the mixture becomes less dense.

This classic model includes an accurate benefit over non-Newtonian fluids, because it is derived from the kinetic theory of gases and not from the interactions of inquiry. Formulating flows of newly established materials such as black lead residue and glycerol may be common.

In (2016) some necessary contributions may be mentioned by Bhatti et al. (0). In (2018) the Swedish engineer Alexander, initiated the study of Magneto hydrodynamic (MHD) (2).

Viscosity is a fluid's inner asset that has flow stability. Consistency could also be a fundamental fluid property that is essential in some respects as used in fossil fuel, industrial chemistry, packaging and writing, meat and drink, engineering, energy, and environment, etc. Consistency is regarded as a performance of either temperature or strain in natural science.

There are several studies in the scientific works on fluid movement in the channel, for example; the movement of Williamson fluid for two types (Poiseuille flow and Couette flow) in an inclined channel dependent on the viscosity studied by Nadeem et al. (3) in (2015).

In (2016) the effect of temperature on (MHD) for Jeffrey liquid with flexible viscosity model in to porous channel was considered by Al-Khafajy (4).

In (2017) Jassim (5) studied the effects of Williamson fluid of wall tapered and magnetic field on peristaltic movement in an inclined channel. In (2014) the flexible viscosity flows in channel with great temperature generation was studied by Ting (6).

In (2017) Immaculate et al. (7) discussed the Williamson Nano-fluid with unsteady flow MHD in a porous channel and oscillating wall temperature to solve the momentum equations by using the homotopy analysis method.

This study aims to employ a series of perturbation method to fix the issue of an elevated medium with variable viscosity for the impact of (MHD) of Williamson fluid with varying temperature and concentration.

### Formulation of the problem

Let us consider the flow for Magneto hydrodynamic (MHD) of Williamson fluid with varying temperature and concentration in an inclined channel with variable viscosity and at height ( $a$ )

(Fig.1). Consider the Cartesian Coordinates system is considered such that  $(u(y, t), 0, 0)$  is velocity vector in which  $u$  is the  $x$ -component of velocity and  $y$  is perpendicular to  $x$ -axis.

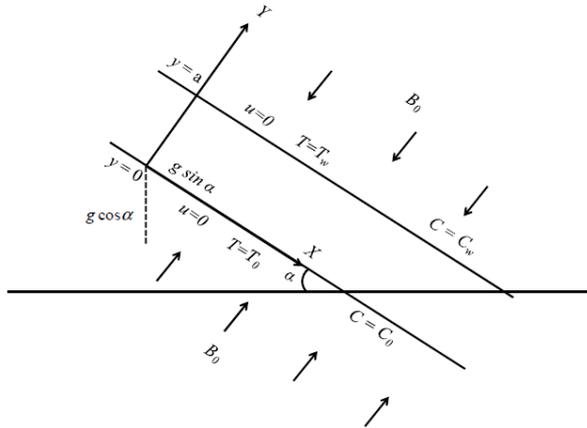


Figure 1. Geometry of the problem

The Williamson fluid governing equations are (8):

$$\mathbf{S} = -\bar{p}\mathbf{I} + \boldsymbol{\tau} \quad (1)$$

$$\bar{\tau} = [\mu_{\infty} + (\mu_0 - \mu_{\infty})(1 + \Gamma\dot{\gamma})^{-1}]A_1 \quad (2)$$

where  $\mu_{\infty}$  is the infinite shear rate viscosity,  $\mu_0 = \mu(T)$ ,  $\Gamma$  is the time constant,  $\mathbf{I}$  is the unit tensor,  $\bar{p}$  is the pressure and  $\bar{\tau}$  is the extra stress tensor. Then  $\dot{\gamma}$  is given by:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\Pi}, \text{ and } \Pi = \text{tr}(A_1)^2, A_1 = \nabla\bar{V} + (\nabla\bar{V})^T \quad (3)$$

The Williamson fluid extra stress tensor as follows:

$$\left. \begin{aligned} x = \frac{\bar{x}}{a}, y = \frac{\bar{y}}{a}, u = \frac{\bar{u}}{U}, \theta = \frac{T-T_0}{T_w-T_0}, p = \frac{\bar{p}a}{\mu U}, Pe = \frac{\rho a U c_p}{K}, N^2 = \frac{4b^2 a^2}{K} \\ \mu(\theta) = \frac{\mu(T)}{\mu_0}, We = \frac{\Gamma U}{a}, \tau_{xx} = \frac{a}{\mu_0 U} \bar{\tau}_{xx}, \tau_{xy} = \frac{a}{\mu_0 U} \bar{\tau}_{xy}, \dot{\gamma} = \frac{a}{U} \dot{\bar{\gamma}}, \Phi = \frac{C-C_0}{C_w-C_0} \\ Kr = \frac{aK_r^*}{U}, t = \frac{\bar{t}U}{a}, Re = \frac{\rho a U}{\mu}, Da = \frac{k}{a^2}, Gr = \frac{\rho g \beta_T a^2 (T-T_0)}{\mu U}, Sr = \frac{DK_T (T_w-T_0)}{UT_m h (C_w-C_0)} \\ Fr = \frac{U}{ga}, M^2 = \frac{\sigma B_0^2 a^2}{\mu}, Sc = \frac{Ua}{D}, R = \frac{4\alpha^2 a^2}{K}, Q = \frac{Q_H a^2}{K}, Gc = \frac{\rho g \beta_C a^2 (T-T_0)}{\mu U} \end{aligned} \right\} \quad (11)$$

where  $(U)$  is the mean flow velocity,  $(S_c)$  is the Schmidt number,  $(S_r)$  is the Soret number,  $(T_m)$  is the mean temperature,  $(Q)$  is the heat generation parameter and  $(G_c)$  is Solutal Grashof number. By substituting equation (10) and (11) into equations (5), (6), (7), (8), take following form:

$$\rho \frac{U}{a} \frac{\partial u}{\partial t} = -\frac{\mu_0 U}{a} \frac{dp}{hdx} + \frac{\mu_0 U}{a} \frac{\partial \tau_{xy}}{h\partial y} + \rho g \beta_T (T_w - T_0) \sin(\alpha) \theta + \rho g \beta_C (C_w - C_0) \sin(\alpha) \Phi - \sigma B_0^2 \sin^2(\alpha) U u - \frac{\mu_0 \mu(\theta) U}{k} u + \rho g \sin(\delta) \quad (12)$$

$$\bar{\tau} = \mu(T)[(1 + \Gamma\dot{\gamma})]A_1 \quad (4)$$

The momentum, temperature and concentration equations are given by:

$$\rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{xy}}{\partial y} + \rho g \beta_T (T - T_0) \sin(\alpha) + \rho g \beta_C (C - C_0) \sin(\alpha) - \sigma B_0^2 \sin^2(\alpha) \bar{u} - \frac{\mu(T)}{k} \bar{u} + \rho g \sin(\delta) \quad (5)$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \frac{Q_H}{\rho c_p} (T - T_0) \quad (6)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - K_r^* (C - C_2) + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (7)$$

The stress variable viscosity for Williamson fluid is:

$$\bar{\tau}_{xy} = \mu(T)[(1 + \Gamma\dot{\gamma})] \left( \frac{\partial \bar{u}}{\partial y} \right) \quad (8)$$

where  $(\bar{u})$  is the axial velocity,  $(g)$  is the acceleration due to gravity,  $(\sigma)$  is the electrical conductivity,  $(\rho)$  is the density of the fluid,  $(B_0)$  is the strength of the magnetic field,  $(T, C)$  are temperature and is the concentration,  $(K)$  is thermal conductivity,  $(Q_H)$  is heat generation,  $(D)$  is the coefficient of mass diffusivity  $(C_p)$  is specific heat at constant pressure and  $(K_T)$  is the thermal diffusion ratio.

The corresponding boundary conditions are given below:

$$\left. \begin{aligned} \bar{u} = 0, T = T_0, C = C_0 \text{ at } \bar{y} = 0 \\ \bar{u} = 0, T = T_w, C = C_w \text{ at } \bar{y} = a \end{aligned} \right\} \quad (9)$$

$$\frac{\partial q}{\partial y} = 4b^2 (T_0 - T) \quad (10)$$

where  $(b)$  is the radiation absorption and  $(q)$  is the radioactive heat flux. The non-dimensional conditions are as follows (8):

$$\frac{\partial(\theta(T_w-T_0)+T_0)}{\frac{a}{U}\partial t} = \frac{k}{\rho c_p} \frac{\partial^2(\theta(T_w-T_0)+T_0)}{a^2 \partial y^2} - \frac{1}{\rho c_p} 4b^2 (T_0 - T) + \frac{Q_H}{\rho c_p} (T - T_0) \quad (13)$$

$$\frac{\partial(\Phi(C_w-C_0)+C_0)}{\frac{a}{U}\partial t} = \frac{Uh}{S_c} \frac{\partial^2(\Phi(C_1-C_2)+C_2)}{a^2 \partial y^2} - \frac{K_r U}{a} ((\Phi(C_w - C_0) + C_0) - C_0) + \frac{DK_T}{T_m} \frac{\partial^2(\theta(T_w-T_0)+T_0)}{a^2 \partial y^2} \quad (14)$$

where  $\tau_{xx} = 0, \tau_{xz} = 0,$

$$\tau_{xy} = \mu(\theta) \left[ \left( 1 + \Gamma \frac{\partial v}{\partial y} \right) \right] \frac{\partial u}{\partial y}.$$

Therefore the non-dimensional equations are:

$$Re \frac{\partial u}{\partial t} = -\frac{dp}{dx} + (\mu(\theta) \left[ \frac{\partial u}{\partial y} + We \left( \frac{\partial u}{\partial y} \right)^2 \right]) + Gr\theta \sin(\alpha) + Gc\Phi \sin(\alpha) - \left( M_1^2 + \frac{\mu(\theta)}{Da} \right) u + \frac{Re}{Fr} \sin(\delta) \quad (15)$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (R + Q)\theta \quad (16)$$

$$\frac{\partial \Phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \Phi}{\partial y^2} - K\Phi + Sr \frac{\partial^2 \theta}{\partial y^2} \quad (17)$$

where  $M_1^2 = M \sin^2(\alpha)$ .

With the boundary conditions:

$$\left. \begin{aligned} u(0) = 0, \theta(0) = 0, \Phi(0) = 0 \text{ at } y = 0 \\ u(1) = 0, \theta(1) = 1, \Phi(1) = 1 \text{ at } y = 1 \end{aligned} \right\} \quad (18)$$

To solve equations (16) and (18), let

$$\theta(y, t) = \theta_f(y)e^{i\omega t} \quad (19)$$

( $\omega$ ) Is the frequency of the oscillation. Substituting equation (19) into equation (16), can be written as:

$$\frac{\partial^2 \theta_f}{\partial y^2} + (R + Q - i\omega Pe)\theta_f = 0 \quad (20)$$

The solution of equation (20) is:

$$\theta_f(y) = \text{Csc}(A) \text{Sin}(Ay) \quad (21)$$

where  $A = \sqrt{R + Q - i\omega Pe}$

Now, the solution of equations (17) and (20), will be discussed. Let

$$\Phi(y, t) = \Phi_f(y)e^{i\omega t} \quad (22)$$

Substituting the equations (22) and (21) into the equation (17),

$$\frac{\partial^2 \Phi_f}{\partial y^2} - Sc(K_r + i\omega)\Phi_f + Sc Sr \frac{\partial^2 \theta_f}{\partial y^2} = 0 \quad (23)$$

The solution of equation (23), is:

$$\begin{aligned} \Phi_f(y) = & e^{\sqrt{B}y} \left( \frac{e^{\sqrt{B}(A+B+A(SrSc))}}{(A+B)(-1+e^{2\sqrt{B}})} \right) + \\ & e^{-\sqrt{B}y} \left( -\frac{e^{\sqrt{B}(A+B+A(SrSc))}}{(A+B)(-1+e^{2\sqrt{B}})} \right) - \frac{A(SrSc)\text{Csc}[\sqrt{A}]\text{Sin}[\sqrt{A}y]}{A+B} \end{aligned} \quad (24)$$

where  $B = \sqrt{Sc(K_r + i\omega)}$ .

### Solution of the problem

To solve equation (15), with the boundary conditions (18), let

$$-\frac{dp}{dx} = \lambda e^{i\omega t} \quad (25)$$

$$u(y, t) = u_f(y)e^{i\omega t} \quad (26)$$

where ( $\lambda$ ) is a real constant.

The Reynold's model and variation of viscosity with temperature is defined as (9):

$$\mu(\theta) = e^{-\eta\theta} \quad (27)$$

by Maclaurin Series:

$$\mu(\theta) = 1 - \eta\theta, \eta \ll 1 \quad (28)$$

By substituting equation (28) into equation (15). It follows that:

$$Re \frac{\partial u}{\partial t} = -\frac{dp}{dx} + \frac{\partial}{\partial y} \left[ (1 - \eta\theta) \left( \frac{\partial u}{\partial y} + We \left( \frac{\partial u}{\partial y} \right)^2 \right) \right] + Gr\theta_f + Gc\Phi_f + \frac{Re}{Fr} \text{Sin}(\delta) - \left( M_1^2 + \frac{(1 - \eta\theta)}{Da} \right) u \quad (29)$$

The "perturbation technique" was used to solve the equation (28), as follows (10):

$$u_f = u_{00} + Weu_{01} + We^2u_{02} + O(We^3) \quad (30)$$

By substituting equation (30) into equation (29) with (18), which equalizes the powers of ( $We$ ):

### i. Zeros-order system ( $We^0$ )

$$Re \frac{\partial u_{00}}{\partial t} = -\frac{dp}{dx} + Gr\theta_f + Gc\Phi_f + \frac{Re}{Fr} \text{Sin}(\delta) - \left( M_1^2 + \frac{(1 - \eta\theta)}{Da} \right) u_{00} + (1 - \eta\theta) \frac{\partial^2 u_{00}}{\partial y^2} \quad (31)$$

$$u_{00}(0) = u_{00}(1) = 0 \quad (32)$$

### ii. First-order system ( $We^1$ )

$$Re \frac{\partial u_{01}}{\partial t} = - \left( M_1^2 + \frac{(1 - \eta\theta)}{Da} \right) u_{01} + (1 - \eta\theta) \frac{\partial^2 u_{01}}{\partial y^2} + 2(1 - \eta\theta) \left[ \frac{\partial u_{00}}{\partial y} \frac{\partial^2 u_{00}}{\partial y^2} \right] \quad (33)$$

$$u_{01}(0) = u_{01}(1) = 0 \quad (34)$$

### iii. Second-order system ( $We^2$ )

$$Re \frac{\partial u_{02}}{\partial t} = - \left( M_1^2 + \frac{(1 - \eta\theta)}{Da} \right) u_{02} + (1 - \eta\theta) \frac{\partial^2 u_{02}}{\partial y^2} + 2(1 - \eta\theta) \left[ \frac{\partial u_{00}}{\partial y} \frac{\partial^2 u_{01}}{\partial y^2} + \frac{\partial u_{01}}{\partial y} \frac{\partial^2 u_{00}}{\partial y^2} \right] \quad (35)$$

$$u_{02}(0) = u_{02}(1) = 0 \quad (36)$$

## Results and discussion

The effect of (MHD) for Williamson fluid with varying temperature and concentration in an inclined channel with variable viscosity were discussed. Numerical assessments of analytical consequences and some of the graphically important effects obtained are shown in Figures 2-25. The numerical calculations have been performed using (Mathematical ver.11) using the set of values:  $Fr = 1, \omega = 1, R = 2, Q = 2, Pe = 0.7, \alpha = \frac{\pi}{4}, Sr = 0.1, K_r = 0.5, Sc = 0.6, M = 1, Re = 1, Gr = 1, Gc = 1, Da = 0.8, \lambda = 1, \delta = \frac{\pi}{4}, We = 0.05, t = 0.5$ .

Figure 2 shows that the velocity distribution  $u$  decreases with the increasing of  $\omega$ . Figure 3 displays the effect  $M$  on the velocity distribution function. By increasing  $M$  the velocity distribution decreases. Figure 4 illustrates that velocity distribution increases with increasing the considerations  $Da$ . Figure 5 and 6 illustrate the effect  $Gr$  and  $Gc$ , on the velocity distribution. It has been observed that by the increasing  $Gr$  and  $Gc$  the velocity distribution function  $u$  increases. Figure 7 shows that the velocity distribution  $u$  is rising up by increasing the effect of the consideration  $R$ . Figure 8 illustrates that velocity distribution increases with increasing the considerations  $Q$ .

Figure 9 shows the velocity distribution  $u$  decreases with increasing  $K_r$ . Figure 10 shows the effect of the parameter of  $\lambda$ . By increasing  $\lambda$ , velocity distribution  $u$  rises up. Figure 11 displays the velocity distribution  $u$  increases with increasing  $Re$ . Figure 12 shows the velocity distribution  $u$  increases with the increasing  $\alpha$ . Figure 13 shows that by increasing the  $\delta$  then velocity distribution  $u$  increases. Figure 14 displays that with increasing  $Sc$ , velocity distribution  $u$  decreases. Figure 15 shows that velocity distribution  $u$  decreases with increasing  $Sr$ . Figure 16 displays that increasing  $Fr$ , velocity distribution  $u$  decreases.

Figure 17 illustrates that the temperature increases with increasing  $R$ . Figure 18 shows that the effect  $Q$  on temperature  $\theta$ , that  $\theta$  increases by increasing  $Q$ . Figure 19 shows us that temperature  $\theta$  decreases with increasing  $\omega$ .

The concentration field is shown in Figs. 20 - 25. Figure 20 shows that with increasing  $Sr$ , the concentration field  $\Phi$  decreases. By Fig.21, it is observed that the effect frequency of the oscillation  $\omega$  on concentration field  $\Phi$  by increasing  $\omega$ , which leads to  $\Phi$  decreases.

From Fig.22 and 23 it is noted that by increasing each of parameters  $K_r$  and  $Sc$  then  $\Phi$  decreases. With increasing  $R$ , then concentration field  $\Phi$  decreases in Fig.24. In Fig.25, it is clear that by increasing  $Q$ , the concentration field  $\Phi$  decreases.

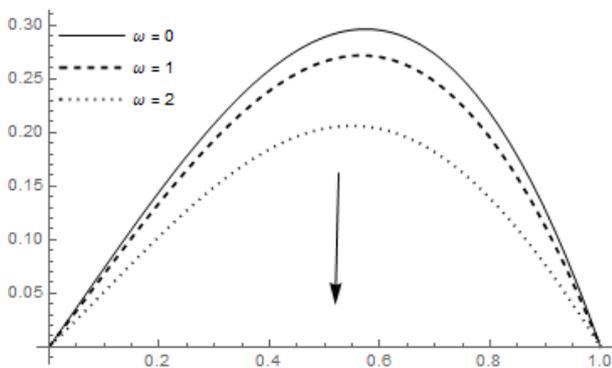


Figure 2. Influence of  $\omega$  on  $u$ .

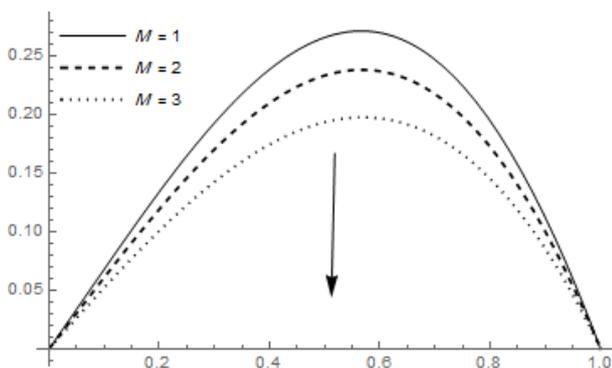


Figure 3. Influence of  $M$  on  $u$ .

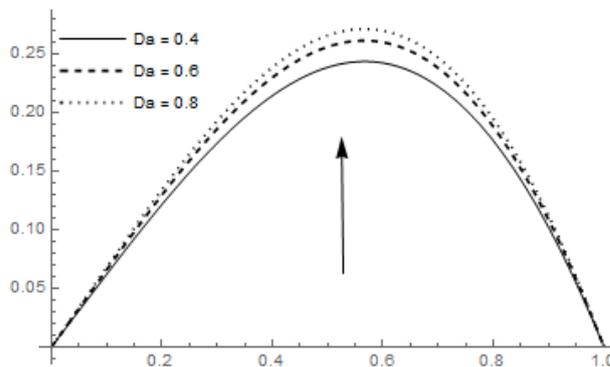


Figure 4. Influence of  $Da$  on  $u$ .

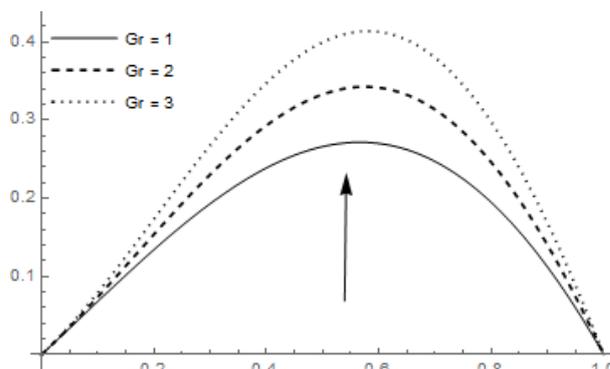


Figure 5. Influence of  $Gr$  on  $u$ .

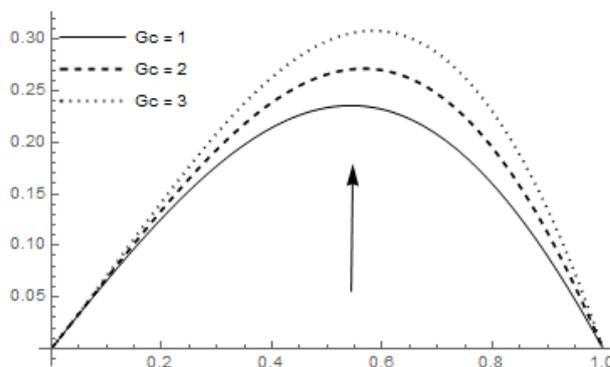


Figure 6. Influence of  $Gc$  on  $u$ .

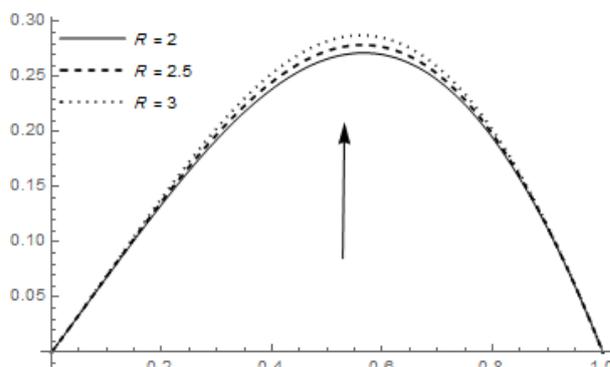


Figure 7. Influence of  $R$  on  $u$ .

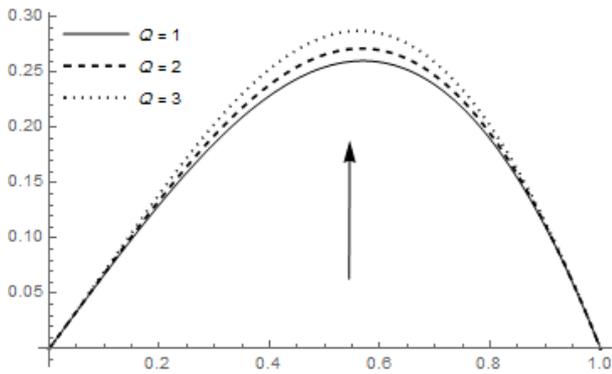


Figure 8. Influence of  $Q$  on  $u$ .

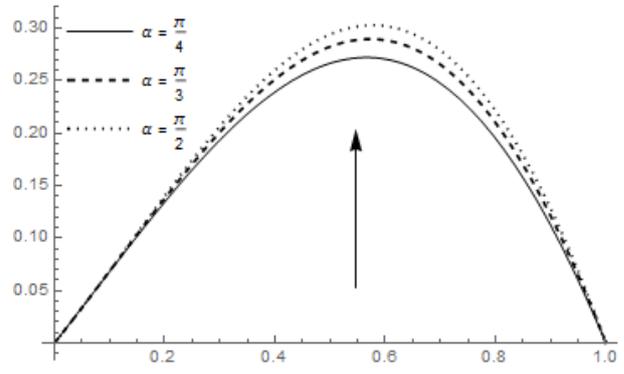


Figure 12. Influence of  $\alpha$  on  $u$ .

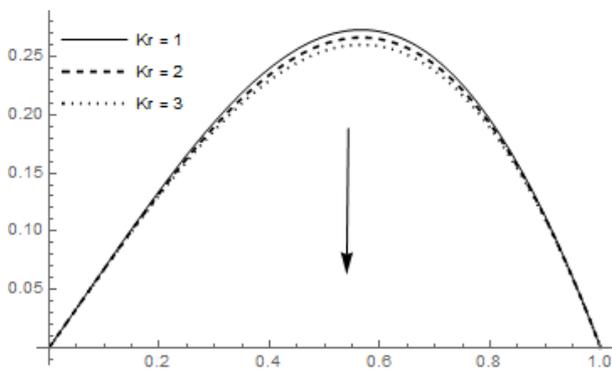


Figure 9. Influence of  $K$  on  $u$ .

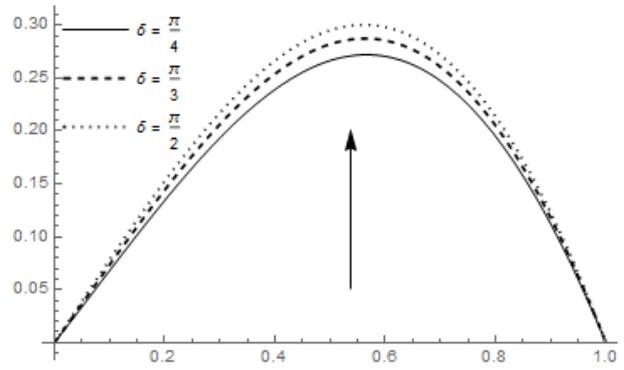


Figure 13. Influence of  $\delta$  on  $u$ .

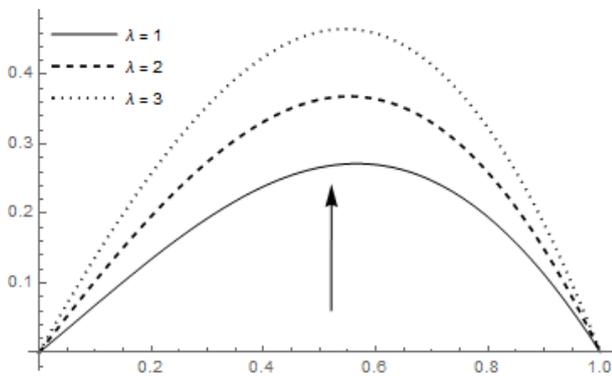


Figure 10. Influence of  $\lambda$  on  $u$ .

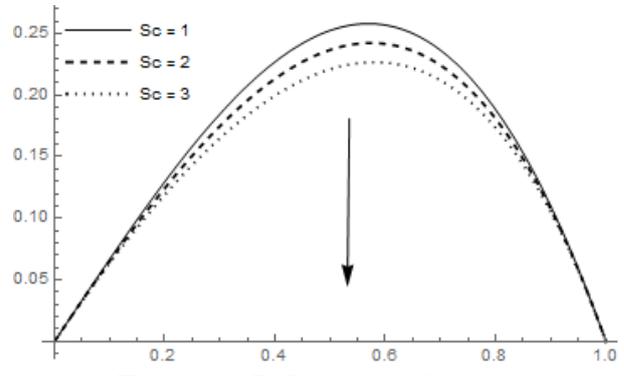


Figure 14. Influence of  $Sc$  on  $u$ .

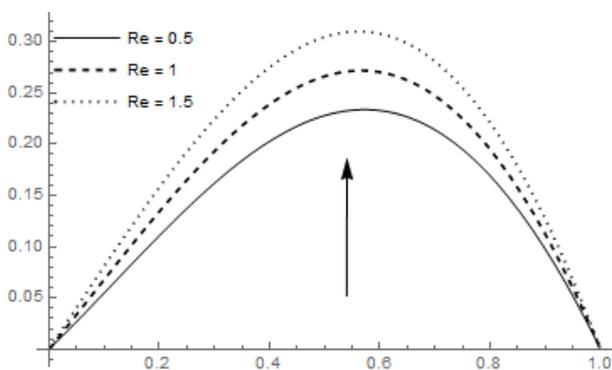


Figure 11. Influence of  $Re$  on  $u$ .

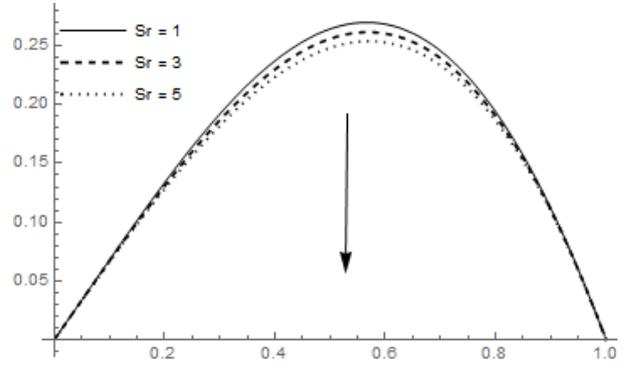


Figure 15. Influence of  $Sr$  on  $u$ .

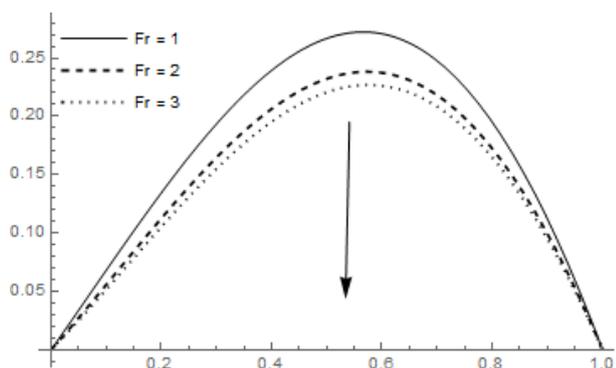


Figure 16. Influence of  $Fr$  on  $u$ .

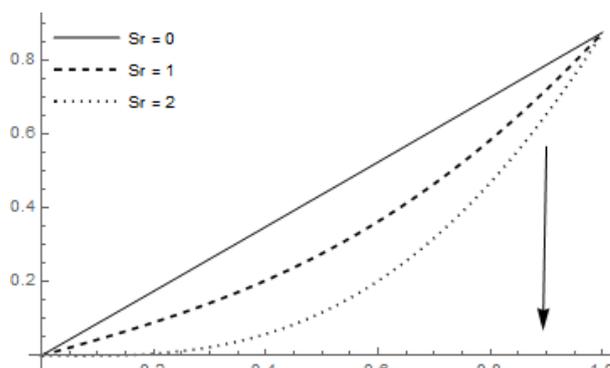


Figure 20. Influence of  $Sr$  on  $C$ .

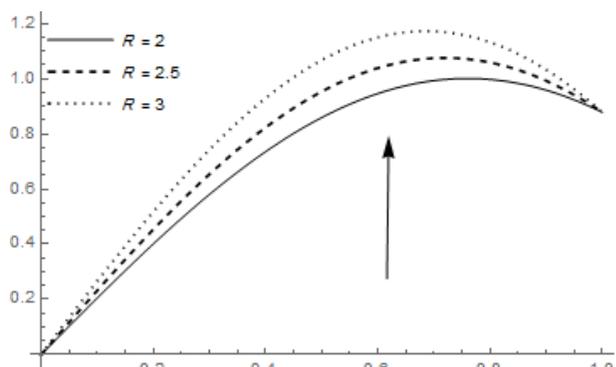


Figure 17. Influence of  $R$  on  $T$ .

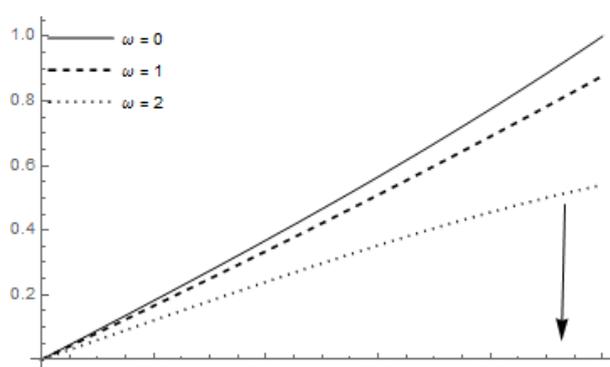


Figure 21. Influence of  $\omega$  on  $C$ .

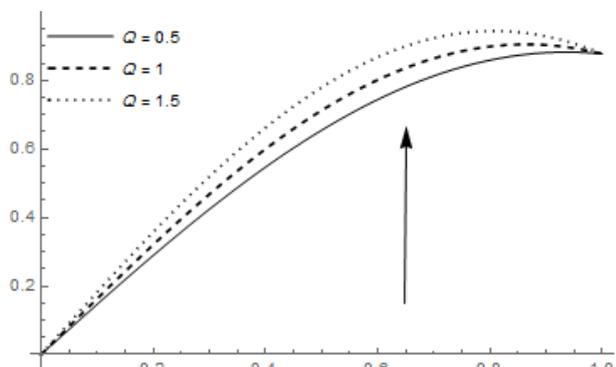


Figure 18. Influence of  $Q$  on  $T$ .

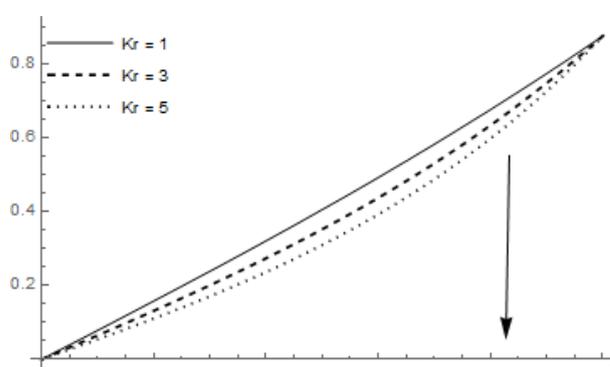


Figure 22. Influence of  $Kr$  on  $C$ .

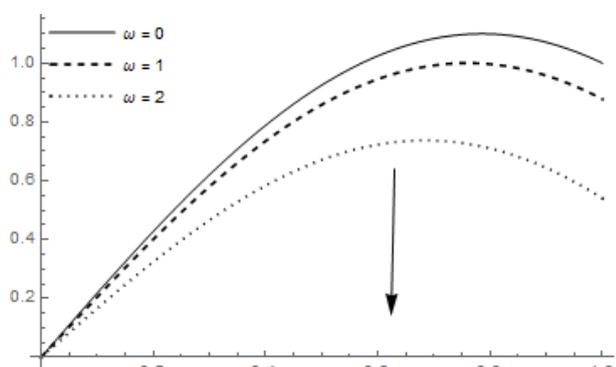


Figure 19. Influence of  $\omega$  on  $T$ .

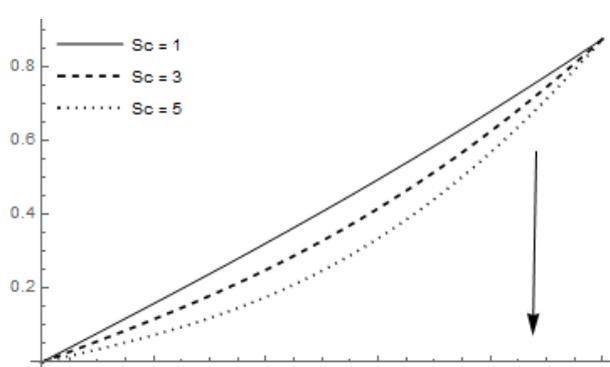


Figure 23. Influence of  $Sc$  on  $C$ .

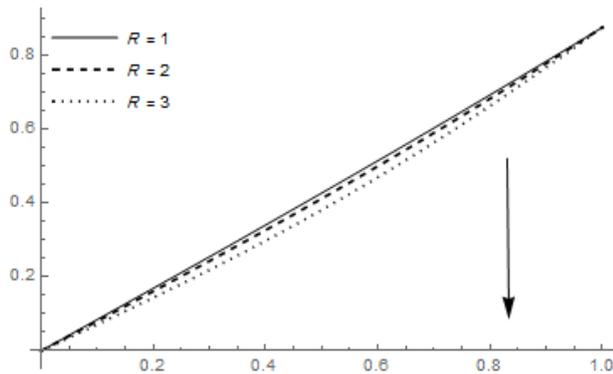


Figure 24. Influence of  $R$  on  $C$ .

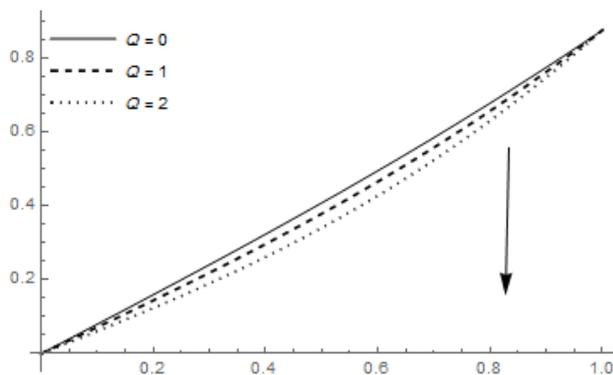


Figure 25. Influence of  $Q$  on  $C$ .

### Conclusion:

The (MHD) oscillatory flow for Williamson fluid with varying temperature and concentration in an inclined channel with variable viscosity are investigated. It is confirmed that the velocity field, concentration profile and temperature analyzed by using the perturbation method is adequate to solve the problem. Different sets of values have been employed to tackle the problem. The conclusions that can be drawn are:

- By increasing the  $Re, \lambda, Da, Q, Gc, \delta, \alpha, R$  and  $Gr$  the velocity distribution increases.
- When increasing the  $M, \omega, Sc, Sr, Fr$  and  $K_r$  the velocity distribution decreases.
- The concentration profile decreases by increasing  $\omega, K_r, Sr, Sc, Q$  and  $R$ .
- It is observed that by increasing  $R$  and  $Q$  the temperature increases, while, by increasing  $\omega$  the temperature decreases  $\theta$ .

### Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for republication attached with the manuscript.

- Ethical Clearance: The project was approved by the local ethical committee in University of Ministry of Education

### References:

1. Bhatti MM, Abbas A. Simultaneous M. effects of slip and MHD on peristaltic blood flow of Jeffrey fluid model through a porous medium. *Alexandria Eng. J.* 2016;55:17–23.
2. Alexander JB. 75th Anniversary of Existence of Electromagnetic–Hydrodynamic Waves. *Sol. Phys.* 2018; 293:83.
3. Nadeem S, Iffat Z, Yousaf M. numerical Solutions of Williamson Fluid with Pressure Dependent Viscosity, *Results in Phys.* 2015;5: 20-25.
4. Al-Khafajy DG. Effects of Heat Transfer on MHD Oscillatory Flow of Jeffrey Fluid with Variable Viscosity Through Porous Medium. *Adv Appl Sci Res.* 2016; 3:179-186.
5. Jassim K. Effects of Wall Tapered and Magnetic Field on Peristaltic Flow of Williamson Fluid, *Math. MSc. Thesis, Baghdad Univ, 2017.*
6. Ting TW, Hung YM, Guo N. Viscous dissipative forced convection in thermal non-equilibrium nano-fluid-saturated porous media embedded in microchannels. *Int. Commun. Heat. Mass. Transf.* 2014;57:309–318.
7. Immaculate L, Muthuraj R, Kant SA, Srinivas S. MHD unsteady flow of a Williamson Nano-fluid in a vertical porous space with oscillating wall temperature. *Front. Heat. Mass. Transf.* 2016;7:1–14.
8. Wissam SK, Al-Khafajy DG. Influence of Heat Transfer on MHD Oscillatory Flow for Williamson Fluid with Variable Viscosity Through a Porous Medium. *IJFMTS* 2018;4:11-17.
9. Hayder KM, Wissam SK, Raheem JM, Qassim AS. Influence of Magnetohydrodynamics Oscillatory Flow for Carreau Fluid Through Regularly Channel With Varying Temperature. *J. of Al-Qadisiyah Com. Sci. Math.* 2019; 11(4):13–22.
10. Romero LA. Perturbation theory for polynomials. *Lecture Notes, Uni. of New Mexico.* 2013.

## دراسة المغناطيسية الديناميكية لمائع وليمسون بتركيز ودرجة حرارة متباينة في قناة مانلة مع لزوجة متغيرة

حسن هادي دويل

وسام صادق خضير

المديرية العامة لتربية بابل، وزارة التربية، العراق.

### الخلاصة:

في هذا البحث تم دراسة الهيدروديناميكا (MHD) لمائع وليمسون خلال قناة مانلة عند التغيير في التركيز و الحرارة عندما اللزوجة متغيرة. استخدمنا طريقة سلسلة الاضطراب باعتماد عدد وزنبرك ( $We \ll 1$ ) للحصول على صيغة معتمده لحقل السرعة. كل النتائج للعوامل الفيزيائية (معامل دارسي (Da)، عدد رينولد (Re)، عدد بيكالت (Pe)، معامل المغناطيسي (M)) حيث نوقشت نتائج المشكلة باستخدام قيم مختلفة كما في الرسوم التوضيحية.

**الكلمات المفتاحية:** قناة مانلة، الهيدروديناميكا (MHD)، الزوجة متغيرة، مائع وليمسون.