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## The Calculation and Analysis of the Total Electron Content Over Different Latitudes and Seasons Using the Numerical Trapezoidal and Simpson Methods

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### Abstract:

It has been shown in ionospheric research that calculation of the total electron content (TEC) is an important factor in global navigation system. In this study, TEC calculation was performed over Baghdad city, Iraq, using a combination of two numerical methods called composite Simpson and composite Trapezoidal methods. TEC was calculated using the line integral of the electron density derived from the International reference ionosphere IRI2012 and NeQuick2 models from 70 to 2000 km above the earth surface. The hour of the day and the day number of the year,  $R12$ , were chosen as inputs for the calculation techniques to take into account latitudinal, diurnal and seasonal variation of TEC. The results of latitudinal variation of TEC show anomalously called equatorial ionization anomaly which presents two crests about the geomagnetic equators. The mean absolute percent errors MAPE for two numerical methods using the electron density profiles shown above were 0.0253, 0.02273 and 0.0213, 0.0124 respectively. The results of seasonal variation of TEC show a larger values for spring and autumn equinoxes other than for summer and winter seasons. The MAPE for autumn equinox has the smallest value than for summer, winter seasons and spring equinox. The MAPE for spring equinox equals to 0.01093 and 0.01015 for Simpson and Trapezoidal methods respectively. For autumn, summer and winter, the MAPE equals to 0.005825 and 0.006629 and 0.04682 and 0.0454, 0.01253 and 0.01231 for Simpson and Trapezoidal methods respectively.

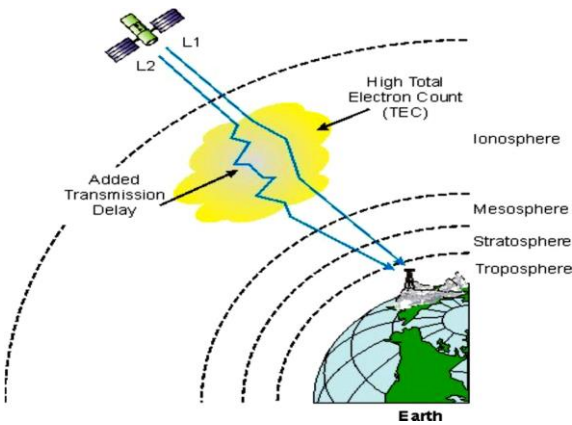
**Key words:** Electron density, GNSS, Global positioning System, Ionosphere, IRI2012 model, NeQuick2 model.

### Introduction:

Users of satellite navigation and satellite communication systems need to assess and monitor ionospheric effects which may degrade their performance (1). The earth's ionosphere is an important error source for global navigation satellite system GNSS signals. The total electron content TEC is the number of free electrons in a column of unit area along a signal path. The ionospheric delay increasing with TEC along the signal trace (2). Transionospheric L-band radio signals used by GNSS may experience range errors up to 100 m which proportional to TEC (3). Over decades, great efforts have been made to model the ionospheric environment through which the radio wave is propagating, as realistically as possible. Empirical modeling means the use of the real data obtained from different stations over the world wide and times, also it is difficult to predict the storm dynamics and abnormal variability (4).

Among ionospheric characteristic parameters, total electron content (TEC) is a parameter of great interest for both applications like Satellite navigation and orbit determination, or satellite altimetry and ionospheric scientific researches (1). TEC empirical models can be constructed by the following two different ways, empirical models of the electron density profile such as IRI and NeQuick and by using different measurements of TEC through regional and global sites (5). Figure 1 shows the ionosphere region according to the height. The receiving signals will cause the receiver to have ranging errors such as ephemeris data, satellite clock, pseudo range and multipath (6).

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**Figure 1. Signal affected at ionosphere region (6).**

In addition to GPS data, we also used an empirical model to derive the TEC and electron density profile. These models estimate the TEC by integrating the electron density profile from the lower boundary to a specified upper boundary (7).

This study aims to calculate and analyse the total electron content (TEC) of ionosphere over different locations and four seasons obtained from the line integral of the electron density through the path of the ionosphere.

**Materials and Methods:**

In this paper, numerical method is used to determine the ionospheric total electron content from Ionosond measurements. The total electron content of the ionosphere is the line integral of electron density profile  $N(h)$  is:

$$TEC = \int_0^{\infty} N(h)dh \quad \dots\dots 1$$

Where  $N(h)$  is the electron density height profile for the study area. One can then write

$$TEC = \int_0^{hmF2} N_B(h)dh + \int_{h>hmF2}^{\infty} N_T(h) dh \quad \dots\dots 2$$

Where  $N_B$  and  $N_T$  are the bottomside and topside profiles. Ionosonde that determine the electron density profiles on line can then calculate TEC in real time if a suitable model for  $N_T$  can be found (8).

Sounding of the ionosphere using ionosondes is an important input for real-time monitoring and forecasting the state of the ionosphere and space weather impacts. The vertical ionospheric sounding is the traditional method for obtaining information about the profile of electron concentration (9).

**The N (h) Profile:**

The electron density profiles used in this study have been predicted from two models. First, the NeQuick2 ionospheric electron model.

NeQuick2 is the latest version of the NeQuick ionospheric electron density model. The NeQuick2 is a quick-run ionospheric electron density model particularly designed for transionospheric propagation applications (10). The NeQuick2 model established in the Abdus Salam International Centre for Theoretical Physics (ICTP) (11).

IRI2012, which is referred to as International Reference Ionosphere model based on all kinds of available data from universal ground observations as well as from satellites. For a given place, day, and time, describing the electron density, electron temperature, ion composition, and ion temperature (12). The IRI2012 website model (13).

**Method of Calculation:**

In this study, two numerical methods have been used to calculate the ionospheric total electron content (TEC). These methods are Composite Simpson's method and Composite Trapezoidal method. The Composite Simpson method is given by (14):

$$TEC = \int_a^b N_e(h)dh = \frac{h}{3} * [N_e(a) + 2 * \sum_{j=1}^{m-1} N_e(h_{2j}) + 4 \sum_{j=1}^m N_e(h_{2j-1}) + N_e(b)] \quad \dots\dots 3$$

Where a,b are the initial and final values of the height electron density profile respectively, and h is the subinterval width and is given by

$$h = \frac{b-a}{2m}$$

and  $n=2m$  subintervals of [a,b]. Where for n-subintervals, the composite trapezoidal method can be written as:

$$TEC = \int_a^b N_e(h)dh = \frac{h}{2} * [N_e(a) + 2 * \sum_{j=1}^{n-1} N_e(h_j) + N_e(b)] \quad \dots\dots 4$$

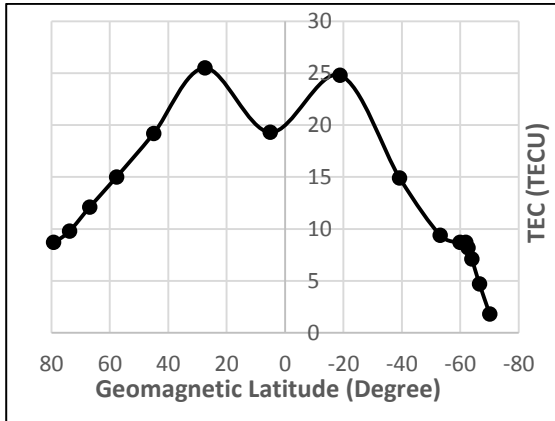
The previous numerical methods have been programmed using Matlab2013a.

**Results and Discussion:**

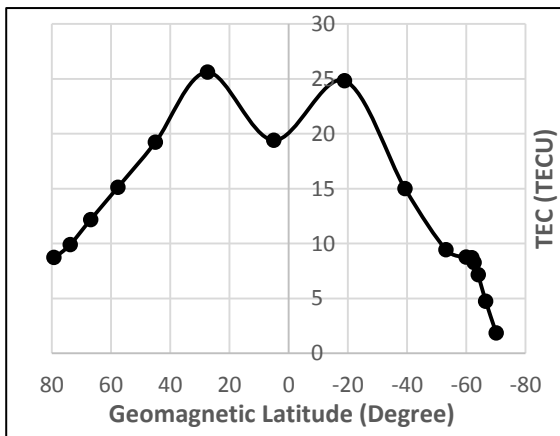
The best validation can be obtained by integrating the  $N(h)$  profile derived from the IRI2012 and NeQuick2 models from 70 to 2000 Km above the Earth surface. The smoothed sunspot number R12 is obtained from space weather services, Australian Government/ Bureau of Meteorology (15). The results of latitudinal and seasonal variations are discussed below.

**Latitudinal Variation of TEC**

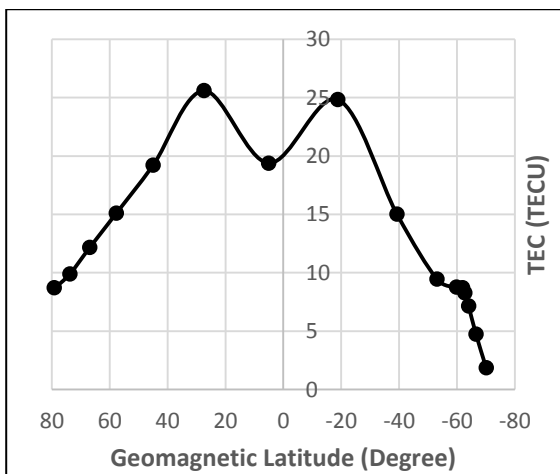
Figures 2,3,4,5,6,7 show the latitudinal variation of the TEC with the geomagnetic latitudes for the following input data  
 Longitude=44° 21' 41.3568" E  
 Year=2010, Month=6, Day=15, Local Time=12, R12=18.8



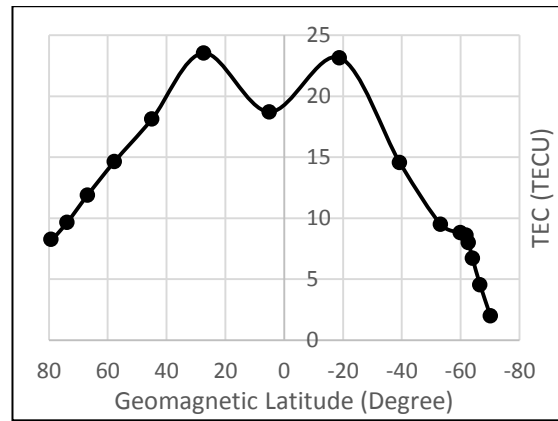
**Figure 2. The latitudinal variation of TEC using IRI2012 model.**



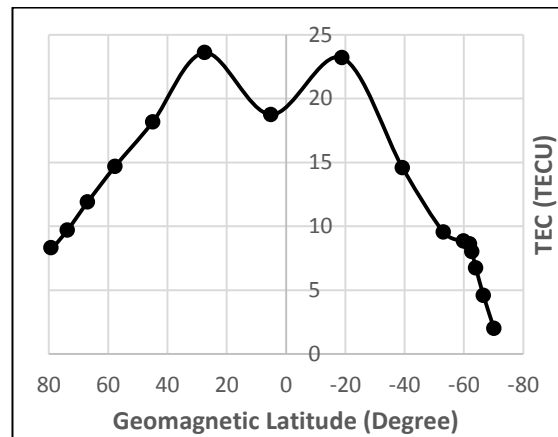
**Figure 3. The latitudinal variation of TEC using Simpson method.**



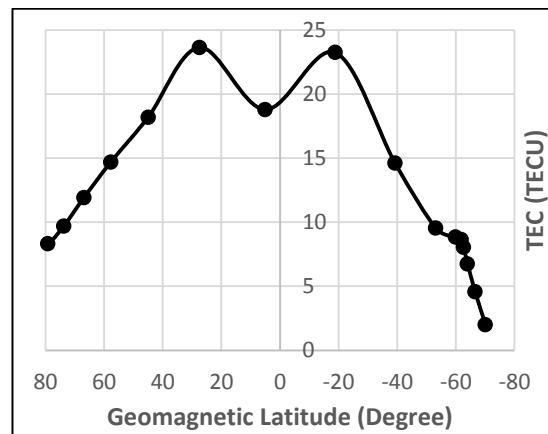
**Figure 4. The latitudinal variation of TEC using Trapezoidal method.**



**Figure 5. The latitudinal variation of TEC using NeQuick2 model.**



**Figure 6. The latitudinal variation of TEC using Simpson method.**



**Figure 7. The latitudinal variation of TEC using Trapezoidal method**

The mean absolute percentage error (MAPE) which is given by (16):

$$MAPE = \frac{100}{n} \sum_{i=1}^n \left| \frac{TEC(predicted) - TEC(estimated)}{TEC(predicted)} \right| \dots\dots 5$$

The MAPE values for both the numerical integration methods compared with both IRI2012 and NeQuick2 models are given in Table 1.

**Table 1. The MAPE values for the numerical integration methods**

IRI2012		NeQuick2	
Numerical Method			
Simpson	Trapezoidal	Simpson	Trapezoidal
0.0253	0.02273	0.0213	0.0124

From Table 1, it is shown that the results of TEC obtained using the trapezoidal method has good correspondence with the results of TEC obtained using IRI2012 and NeQuick2 models than for Simpson method. Figures (1-6) provides an important up normal (anomaly) phenomena called equatorial ionization anomaly (EIA). In the equatorial low latitudes ionosphere at F region the ionization density distribution is characterized by a trough at the equator and dual crests on other sides of the equator, are called the crests of EIA (17).

**Seasonal Variation of TEC**

Figures 8, 9, 10, 11 represent the seasonal variation of TEC using IRI2012 model compared with the results obtained using the numerical integration methods. The study includes two years (March 2010 to February 2011). Each year has three seasons, equinox (March and April, September and October), summer (May, June, July and August) and winter (November and December of current year 2010, January and February of successive year 2011). The following procedure and input data for calculating the TEC.

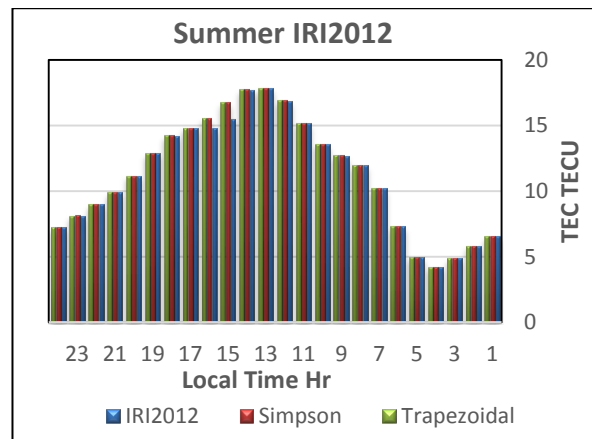
**Input Data**

Date: 15<sup>th</sup> of each month of the year 2010-2011.

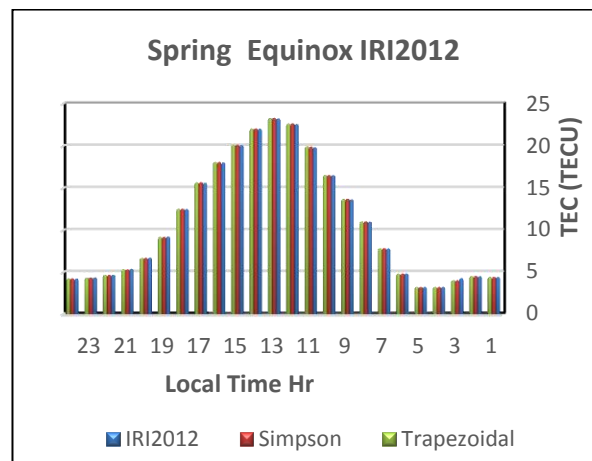
Location: Baghdad City

**Procedure**

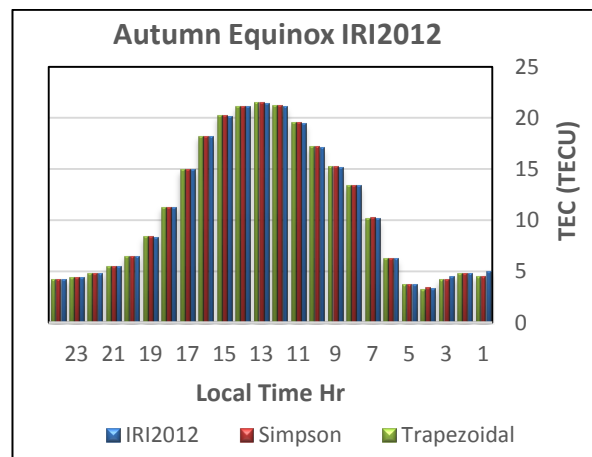
1. Calculate the hourly variation of TEC using numerical methods.
2. Estimate the average of TEC for each hour of the month of the season.
3. The result of step 2 provides the seasonal mean of the TEC.
4. The comparison of the results of the TEC of step 3 with the TEC obtained from IRI2012 model.
5. The estimation of MAPE for both numerical methods.



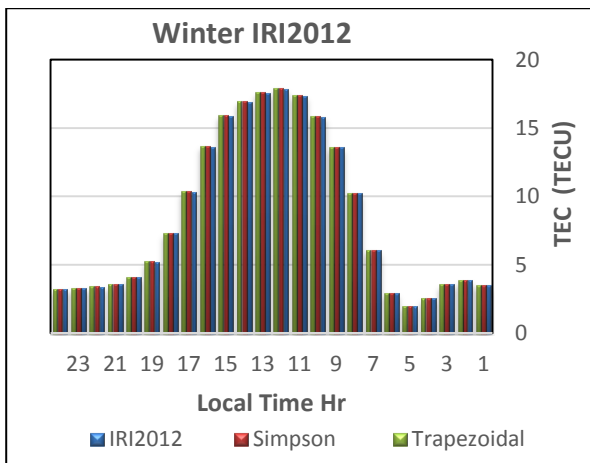
**Figure 8. The seasonal mean variation of TEC for March and April months.**



**Figure 9 The seasonal mean variation of TEC for May, June, July and August months**



**Figure 10. The seasonal mean variation of TEC for September and October months.**



**Figure 11. The seasonal mean variation of TEC for November, December 2010, and January and February 2011.**

Figures 8, 9, 10, 11 present three important facts, first, the local time variation of total electron content (TEC), in general, has a maximum value at daytime hours and decreases at nighttime hours. The maximum value occurs in the local time interval (10-16 hr) for each season approximately. Secondly, the seasonal variation of TEC for spring equinox (March and April months) has the largest value compared with other seasons. Numerically, the values of TEC for spring, summer, autumn and winter seasons equal to 23.317 TECU at 13 hr, 17.896 TECU at 12 hrs, 21.521 TECU at 13 hr and 17.936 TECU at 12 hrs respectively. The largest values were for spring and autumn equinoxes came from the dense ionosphere at these times. Finally, both numerical integratin methods have mean absolute percent error as shown in Tables 2 and 3.

**Table 2. The MAPE values for the numerical integration methods for spring and autumn months**

Season			
Spring Equinox		Autumn Equinox	
Simpson	Trapezoidal	Simpson	Trapezoidal
0.01093	0.01015	0.008254	0.0066297

**Table 3. The MAPE values for the numerical integration methods for summer and winter months.**

Season			
Summer		Winter	
Simpson	Trapezoidal	Simpson	Trapezoidal
0.04682	0.04548	0.01253	0.01231

From Tables 2 and 3, it has been noticed that the numerical integration method called trapezoidal method is more accurate than the Simpson method. The smallest MAPE occurred for autumn equinoxes with 0.008254 and 0.006629 for

Simpson and Trapezoidal methods respectively, where the largest took place at summer months with 0.046828 and 0.04548 for Simpson and Trapezoidal methods respectively.

From the above results, it is shown that the latitudinal variation of TEC for this study obey to the anomaly called the Equatorial Ionization Anomaly (EIA) which has two crests about the geomagnetic equator. The seasonal variation of TEC provides a large values at spring and autumn equinoxes than for summer and winter seasons. Also, the Trapezoidal method has the best results than for the Simpson method for calculating the TEC for both latitudinal and seasonal variations.

**Conflicts of Interest: None.**

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## حساب و تحليل المحتوى الالكتروني الكلي فوق دوائر عرض و فصول مختلفة باستخدام الطرق العديدة

علي حسين نعمه

قسم علوم الجو، كلية العلوم، الجامعة المستنصرية، بغداد، العراق

### الخلاصة:

يتبين من البحث في الايونوسفير ان حساب المحتوى الالكتروني الكلي TEC للايونوسفير هو عامل مهم لنظام الملاحة الجغرافية. في هذه الدراسة تم حساب TEC فوق مدينة بغداد، العراق، مستخدماً مساهمة طريقتين عدديتين هما طريقة سمبسون المركبة و طريقة شبه المنحرف المركبة. TEC حسب استخدام التكامل الخطي للكثافة الالكترونية المستحصلة بواسطة إنموذجي IRI2012 و NeQuick2 من ارتفاع 70 الى 2000 Km فوق سطح الارض. الساعه من اليوم و اليوم من السنة و R12 اختبرت كمدخلات لطرق الحساب اخذين بنظر الاعتبار التغيرات الجغرافية و اليومية و الفصلية ل TEC. نتائج تغير TEC مع خطوط العرض بينت شذوذ يسمى بشذوذ التأين الاستوائي و الذي يظهر فيه قمتين ل TEC حول الاستواء الجيومغناطيسي. معدل نسبة الخطا المطلق MAPE للنتائج المستحصلة من طرق التحليل العددي مستخدماً النموذجين اعلاه كانت كالتالي 0.0253 و 0.02273 ، 0.0213 و 0.0124 على التوالي. اما نتائج التغير الفصلي فقد اوضحت ان قيم TEC للاعتدالين الربيعي و الخريفي اكبر من القيم الصيفية و الشتوية. بلغت قيم MAPE للربيع 0.01093 و 0.01015 لطريقتي سمبسون وشبه المنحرف على التوالي. اما لفصول الخريف و الصيف و الشتاء فقد بلغت 0.005825 و 0.006629 و 0.04682 و 0.0454 ، 0.01253 و 0.01231 لطريقتي سمبسون وشبه المنحرف على التوالي.

**الكلمات المفتاحية:** الكثافة الالكترونية، GNSS، نظام التحديد العالمي، الايونوسفير، أنموذج IRI2012، أنموذج NeQuick2.