

DOI: <http://dx.doi.org/10.21123/bsj.2022.19.3.0528>

On New Weibull Inverse Lomax Distribution with Applications

*Jamilu Yunusa Falgore**

Sani Ibrahim Doguwa

Department of Statistics, Ahmadu Bello University Zaria-Nigeria.

Corresponding author: jyfalgore@gmail.com, sidoguwa@gmail.com

ORCID ID: <https://orcid.org/0000-0003-4762-1560>, <https://orcid.org/0000-0002-5779-2358>

Received 18/12/2020, Accepted 10/1/2021, Published Online First 20/11/2021 Published 1/6/2022



This work is licensed under a [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/).

Abstract:

In this paper, simulation studies and applications of the New Weibull-Inverse Lomax (NWIL) distribution were presented. In the simulation studies, different sample sizes ranging from 30, 50, 100, 200, 300, to 500 were considered. Also, 1,000 replications were considered for the experiment. NWIL is a fat tail distribution. Higher moments are not easily derived except with some approximations. However, the estimates have higher precisions with low variances. Finally, the usefulness of the NWIL distribution was illustrated by fitting two data sets.

Keywords: Entropy, MGF, Moments, NWIL, Simulation Studies.

Introduction:

In 2003, Kleiber and Kotz¹ mentioned that Inverse Lomax Distribution (ILD) belongs to the Beta-Type size distributions alongside with Dagum, Lomax, and Generalized Beta (GB) of the second kind as other members of the family. They postulated that ILD has applications in economics, actuarial sciences, and stochastic modeling. In 2004, Kleiber² studied Lorenz ordering relationships between order statistics from log-logistic samples of potentially different sizes. Some results extend other families including the Burr XII, Lomax, and Burr III distributions. They applied the ILD model on geophysical data, specifically on the sizes of land fires in California State of United States. Moreover, in 2013 Rahman³ studied the ILD via the Bayesian approach by drawing inferences about the unknown shape parameter of the distribution. A comparison between Bayes estimates and Maximum Likelihood estimate was made after getting the simulated and real-data results. Therefore, it is concluded that the Bayesian method of estimation leads to better outcomes as compared to Maximum Likelihood (ML) estimates. This shows the supremacy of a Bayesian approach to classical one for parameter estimation. Other references that considered the Bayesian approach are Rahman and Aslam⁴ Jan and Ahmed⁵, Rahman and Aslam⁶ and Yadav⁷. All the references discussed so far, considered only the ILD.

Recently, some generalization of ILD appeared in the literature with the sole aim of increasing the flexibility of the distribution. In 2018, Falgore et. al.⁸ extended the ILD with the Odd Generalized Exponential family by Tahir⁹ in 2015. They derived some mathematical properties of the proposed distribution such as the quantile function, moments, order statistics and asymptotic behavior of the distribution. The estimation of the proposed distribution's parameters was conducted using the method of Maximum Likelihood Estimation (MLE) procedure, and finally, they illustrated the usefulness of the proposed model by fitting the proposed distribution using a real-life data and compared its performance with comparator distributions. In 2019, Maxwell¹⁰ extended the ILD by utilizing a methodology of adding a new parameter(s) by Marshall and Olkin¹¹. They called their proposed distribution the Marshall-Olkin Inverse Lomax distribution (MO-ILD). They hope that by adding a new parameter(s), it may lead to greater flexibility in modeling various data types. The Marshall-Olkin Inverse Lomax distribution provided a better fit than the Marshall Olkin Flexible Weibull Extension Distribution, and the Marshall-Olkin exponential Weibull distribution based on log-likelihood AIC, CAIC, BIC, and HQIC values. Therefore, they concluded that the Marshall Olkin inverse Lomax distribution is the most appropriate model amongst the considered

distributions and a very competitive model for describing lifetime phenomenon. Recently in 2019, Hassan¹² and Falgore et. al.¹³ extended the ILD with the Weibull G family by Bourguignon¹⁴ in 2014. They called it Weibull Inverse Lomax (WIL). Some structural properties were derived. Such properties include moments, Reliability Characterization, Inverse moments and moments. The model parameter estimation was carried out based on censored samples of Type II. Maximum likelihood estimators are built along with confidence intervals and reliability function asymptotic population parameters. The accuracy property of maximum likelihood estimators was verified based on simulated samples. Also, the results were applied to two real data.

The motivation behind this research is to explore the work of Falgore¹⁵ et. al. by conducting simulation study to assess the consistency of the parameter estimates and provide some applications to real-world datasets. The simulation studies are to check the desirable properties of the estimates like Unbiasedness, Consistency, Sufficiency, and Efficiency. While the application will be based on the datasets used by the New Weibull G family by Tahir¹⁶ et. al. in 2016.

The probability density function (pdf) and cumulative distribution function (CDF) of ILD are given by the following equations as defined by Yadav¹⁷ in 2016 as:

$$g(x; \gamma, \lambda) = \frac{\gamma\lambda}{x^2} \left(\frac{x+\gamma}{x} \right)^{-(1+\lambda)} \quad (1)$$

$$G(x; \gamma, \lambda) = \left(1 + \frac{\gamma}{x} \right)^{-\lambda} \quad (2)$$

where $x > 0$, $\gamma > 0$ and $\lambda > 0$ are the scale and shape parameters respectively.

Let $g(x; \omega)$ and $G(x; \omega)$ denote the (pdf) and (CDF) of a baseline model with parameter vector ω . Based on this CDF, (16) replaced the argument $W[G(x)] = -\log[G(x; \omega)]$ and define the CDF and the pdf of the New Weibull-G family by:

$$F(x; \alpha, \beta, \omega) = 1 - \int_0^{-\log[G(x; \omega)]} \alpha \beta v^{\beta-1} \exp - \alpha v^\beta dv$$

$$= \exp\{-\alpha\{-\log[G(x; \omega)]\}^\beta\} \quad (3)$$

where $G(x; \omega)$ is any baseline CDF which depends on a parameter vector ω .

$$f(x; \alpha, \beta, \omega) = \alpha \beta \frac{g(x; \omega)}{G(x; \omega)} \{-\log[G(x; \omega)]\}^{\beta-1} \times \exp\{-\alpha\{-\log[G(x; \omega)]\}^\beta\} \quad (4)$$

The additional parameters caused by the New Weibull generator are only needed to increase distribution flexibility.

The pdf, CDF, and quantile function of the NWIL distribution as defined by Falgore¹⁵ et. al. (15) are given below:

$$f(x; \theta) = \alpha \beta \gamma \lambda x^{-2} \left(1 + \frac{\gamma}{x} \right)^{-1} \left\{ -\log \left[\left(1 + \frac{\gamma}{x} \right)^{-\lambda} \right] \right\}^{\beta-1} \times \exp \left\{ -\alpha \left\{ -\log \left[\left(1 + \frac{\gamma}{x} \right)^{-\lambda} \right] \right\}^\beta \right\} \quad (5)$$

$$F(x; \beta, \alpha, \lambda, \gamma) = \exp \left\{ -\alpha \left\{ -\log \left[\left(1 + \frac{\gamma}{x} \right)^{-\lambda} \right] \right\}^\beta \right\} \quad (6)$$

$$x = \frac{\gamma}{\left[\exp \left\{ -\left[-\frac{\log(u)}{\alpha} \right]^\beta \right\} \right]^{-\frac{1}{\lambda}} - 1} \quad (7)$$

where $\theta = (\alpha, \beta, \gamma, \lambda)$ and α and γ are the scale parameters while β and λ are the shape parameters, respectively.

Nevertheless, some of the NWIL distribution's statistical properties and derivations such as, entropies, Moments, Moment Generating Functions, distribution of Order Statistics and estimations were given in Falgore¹⁵ et. al.

Methods

In this section, simulation procedures and Goodness of fit criteria are to be explained.

Simulation Studies

Monte Carlo is a process or methodology that uses replicated trials (sampling) that are generated using random numbers in computer programs like R, as given in Sambridge¹⁸ in the year 2002. However, in 1997 Mooney¹⁹ highlighted five (5) steps on how to do a Monte Carlo simulation study as follows:

1. Clearly state the pseudo-population that can be used in generating random samples usually by writing code in a specific method.
2. Sample from the population of interest (depending on your objective).

3. Estimate the parameter of interest from the sample and keep it in a vector.
4. Repeat the previous steps i.e 2 and 3 N-times (N is the number of replications).
5. Create a relative frequency distribution of resulting values that is a Monte Carlo approximation of the distribution of samples under the conditions defined by the pseudo-population and the procedures of sampling.

Based on the above procedure, we carry out our simulation studies as explained below.

1. For specific parameter values i.e $\theta = (\alpha, \beta, \gamma, \lambda)^1$, we simulated a random sample of size n from the NWIL distribution using equation (7).
2. We then estimated the parameters of the NWIL distribution by the method of Maximum Likelihood.
3. Perform 1,000 replications of steps 1 through 2.
4. For each of the four (4) parameters of the NWIL, we computed the estimate, bias, variance, and Mean Squared Error (MSE) from the 1,000 parameter estimates. The statistics are given by:

$$\hat{\theta} = \frac{1}{1000} \sum_{i=1}^{1000} \hat{\theta}_i, \quad \text{Bias}(\hat{\theta}) = \frac{1}{1000} \sum_{i=1}^{1000} (\hat{\theta}_i - \theta), \quad V(\hat{\theta}) = \frac{\sum_{i=1}^{1000} (\hat{\theta}_i - \theta)^2}{1000}$$

$$MSE(\hat{\theta}) = V(\hat{\theta}) + (\text{Bias}(\hat{\theta}))^2$$

where $\hat{\theta}_i = (\hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i, \hat{\lambda}_i)$ is the MLE for the i^{th} replication ($i = 1, 2, \dots, 1000$).

($n = 30, 50, 100, 200, 300, 500$). These estimations are conducted for sample sizes n ranging from 30, 50, 100, 200, 300, and 500. The true values of the parameters used in the simulation are $\alpha = 1, \beta = 0.5, \gamma = 1$ and $\lambda = 0.7$.

Discussion of the Simulation Results

The estimates of the unknown parameters are relatively good as they are approaching the true parameter values as n increases (see Table 1). Moreover, the bias of the estimates of the first parameter (α) decreases when n is small but started increasing as the sample size increases. Similarly, the bias of the estimates of the second parameter (β) behaves as the first one. However, the bias of the third estimates (γ) reduces as the sample size (n) increases. Lastly, the bias of the fourth estimates (λ) behaves as α and β . Generally, the variances are very small for all the estimates. The MSEs of α and β generally reduces as n increases. While that of γ almost maintain its value as n increases. According to Walther²⁰ in 2005, Small variance shows high precision, and MSE as a measure of accuracy is good when the value is small. Also, small bias⁽⁸⁾ indicates a highly accurate estimator.

Table1. Simulation Results: Estimates, Bias, Variance, and MSE for 1,000 iterations.

sample sizes (n)	Estimates	Bias	Variance	MSE
n=30	$\hat{\alpha} = 0.9482$	-0.0518	0	0.0028
	$\hat{\beta} = 0.0241$	-0.4759	0.0002	0.2267
	$\hat{\gamma} = 0.0131$	0.1313	0	0.0002
	$\hat{\lambda} = 0.7189$	0.0189	0	0.0004
n=50	$\hat{\alpha} = 0.9546$	-0.0454	0	0.0021
	$\hat{\beta} = 0.0288$	-0.4712	0.003	0.2223
	$\hat{\gamma} = 1.0143$	0.0143	0	0.0002
	$\hat{\lambda} = 0.7269$	0.0269	0	0.0008
n=100	$\hat{\alpha} = 0.9635$	-0.3653	0	0.2184
	$\hat{\beta} = 0.0332$	-0.4669	0.0004	0.2184
	$\hat{\gamma} = 1.0163$	0.0163	0	0.0003
	$\hat{\lambda} = 0.7379$	0.0379	0	0.0015
n=200	$\hat{\alpha} = 0.9665$	-0.0335	0	0.0011
	$\hat{\beta} = 0.0389$	-0.461	0.0005	0.2131
	$\hat{\gamma} = 1.0196$	0.0196	0	0.0004

	$\hat{\lambda} = 0.7516$	0.0516	0	0.0027
n=300	$\hat{\alpha} = 0.9719$	-0.028	0	0.0008
	$\hat{\beta} = 0.0304$	-0.4697	0.0003	0.2209
	$\hat{\gamma} = 1.0205$	0.0205	0	0.0004
	$\hat{\lambda} = 0.7571$	0.0571	0	0.0033
n=500	$\hat{\alpha} = 0.9776$	-0.0224	1.2367	0.0005
	$\hat{\beta} = 0.0401$	-0.4599	0.0006	0.2121
	$\hat{\gamma} = 1.0199$	0.0199	0	0.0004
	$\hat{\lambda} = 0.7585$	0.0583	0	0.0035

Shapes of the NWIL distribution

The following are the plots of the pdfs, hazard rate functions, and CDF (Fig. 1) according to Falgore¹⁵ et. al. . The plots in Fig. 2 are other indications of the fat tail properties of the NWIL. The hazard rate functions in Fig. 3 exhibits large skewness. For the graphs below, the letters a,b,c,d stands for the α , β , γ , and λ respectively.

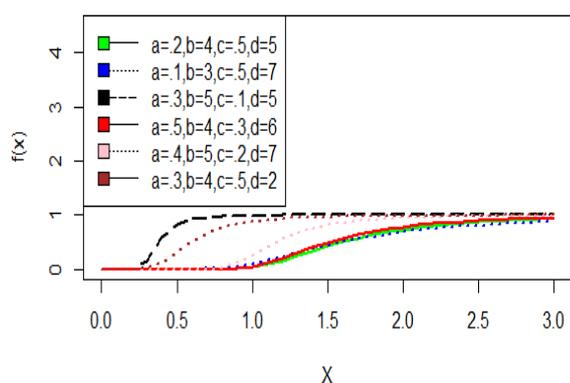


Figure 1. The CDF of NWIL at various parameter values.

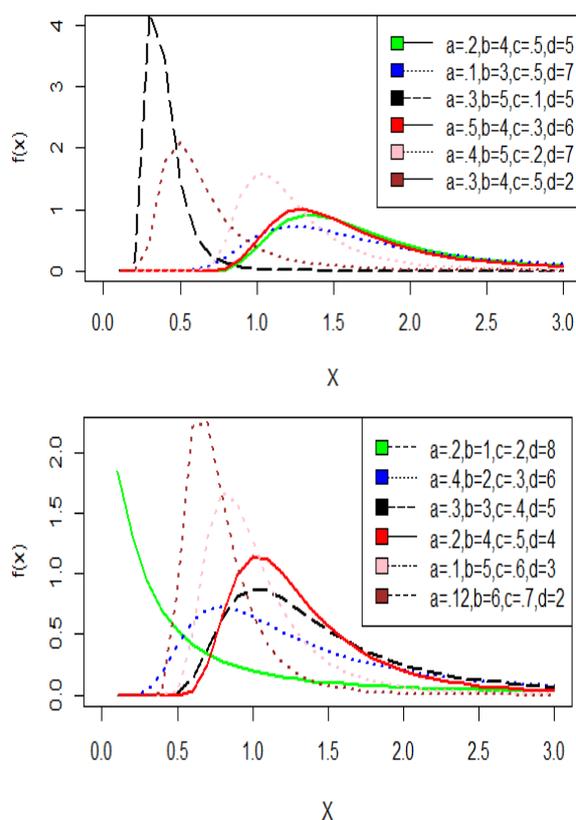


Figure 2. The Densities of NWIL at different values of parameter

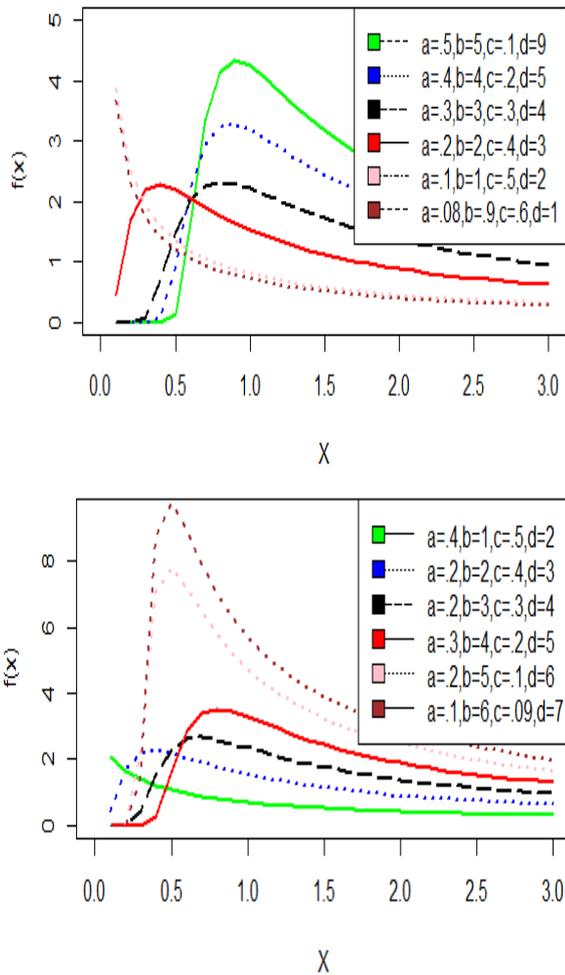


Figure 3. The hazard functions of NWIL distribution at various parameter values

Results and Discussion:

Data Set 1

Here, the applicability of the NWIL distribution to the breaking strength of 100 Yarn as reported by Duncan²¹ is presented. The summary statistics for this data is presented in Table 2. The plots of the Data set 1 are presented in Fig. 4. Throughout this section, a maxLik package developed by Henningsen²² in R Software by Team R²³ in 2014 was used. The parameters of each model were estimated by the method of ML Estimation (MLE) using the Simulated ANNealing (SANN) method. The goodness of fit statistics used in comparing the performances are the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). However, Smaller values of the AIC and BIC statistics indicate better model fittings. Throughout the analysis, we used negative log-likelihood (-ll) value to derive the AIC and BIC by using the following relations:

$$AIC = -2(-ll) + 2R \tag{9}$$

And

$$BIC = -2(-ll) + R \log(n) \tag{10}$$

Where R is the parameter number and n is the sample size.

Table 2. Summary Statistics of the first data set

Min.	Max.	Median	Mean	S.D.	Sk.	Kur.
62	138	99	99.43	12.46	0.64	5.76

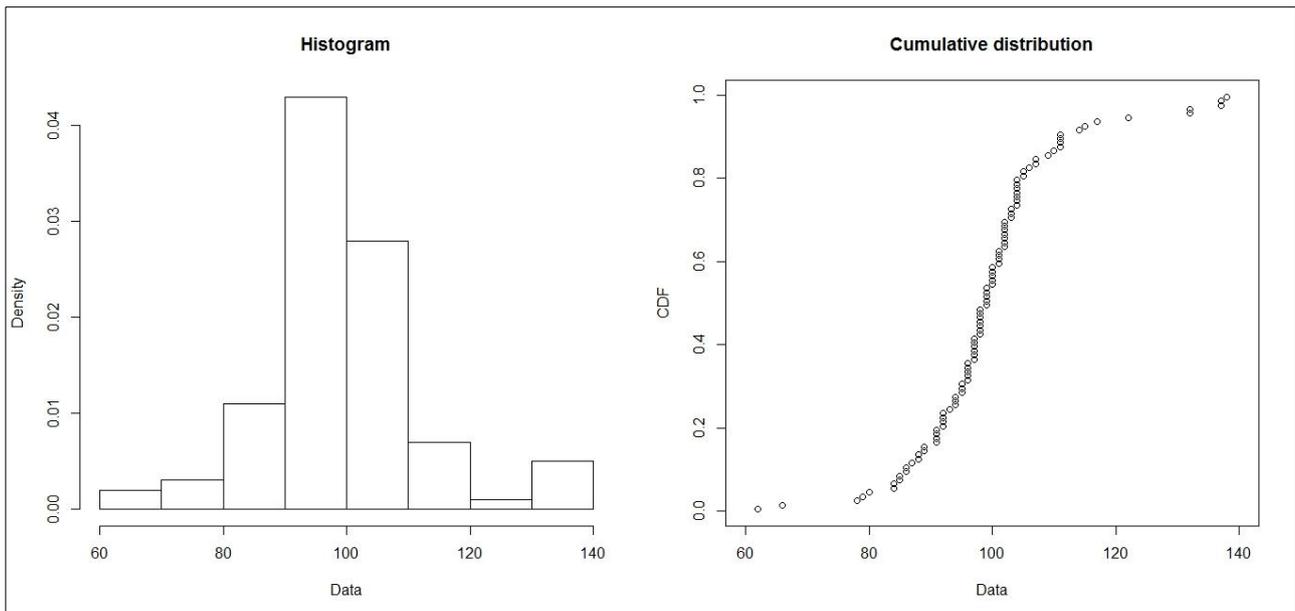


Figure 4. Histogram and cumulative plots for Data set 1

The New Weibull-Inverse Lomax (NWIL) distribution defined by Falgore¹⁵ et. al. was fitted. Its fit is also compared with the Weibull-Lomax distribution by⁹ Odd generalized exponential Inverse Lomax distribution by⁸ logistic Lomax distribution by Zubair²⁴ Inverse Lomax distribution, as in³ Weibull Log-logistic by Tahir (16), and Weibull exponential by Oguntunde²⁵ et. al. with the pdfs given below (Table 3):

$$f(x; \theta)_{WL} = \frac{\gamma\lambda\alpha}{\beta} \left[1 + \left(\frac{x}{\beta}\right)\right]^{\lambda\alpha-1} \left\{1 - \left[\frac{1+x}{\beta}\right]^{-\alpha}\right\}^{\lambda-1} \exp\left\{-\alpha \left[\left(\frac{1+x}{\beta}\right)^{\alpha} - 1\right]^{\lambda}\right\}$$

(11)

$$f(x; \theta)_{OGEL} = \alpha\beta\gamma\lambda \left(\frac{1+\beta}{x}\right)^{\alpha-1} \exp\left\{\frac{\lambda}{1 - \left(\frac{\beta}{x} + 1\right)^{\alpha}}\right\} \left\{1 - \exp\left\{\frac{\lambda}{1 - \left(\frac{\beta}{x} + 1\right)^{\alpha}}\right\}\right\}^{\alpha-1} \left[x^2 \left[\left(\frac{\beta}{x} + 1\right)^{\alpha} - 1\right]^2\right]^{-1}$$

(12)

$$f(x; \alpha, \beta, \gamma)_{LLO} = \frac{\alpha\gamma}{\beta} \left(1 + \frac{\gamma}{\beta}\right)^{-1} \left\{\log\left(\frac{1+\gamma}{\beta}\right)\right\}^{-\lambda-1} \left[1 + \left\{\log\left(1 + \frac{\gamma}{\beta}\right)\right\}^{-\lambda-1}\right]^{-2}$$

(13)

$$f(x; \gamma, \lambda)_{IL} = \frac{\gamma\lambda}{x^2} \left(1 + \frac{\gamma}{x}\right)^{-(1+\lambda)}$$

(14)

$$f(x; \theta)_{WLL} = \alpha\beta\gamma\lambda^{-1} \left(\frac{x}{\lambda}\right)^{\gamma-1} \frac{\left(1 + \left(\frac{x}{\lambda}\right)^{\gamma}\right)^{-2}}{1 - \left(1 + \left(\frac{x}{\lambda}\right)^{\gamma}\right)^{-1}} \left[-\log\left[1 - \left(1 + \left(\frac{x}{\lambda}\right)^{\gamma}\right)^{-1}\right]\right]^{\beta-1} \exp\left\{-\alpha \left[-\log\left[1 - \left(1 + \left(\frac{x}{\lambda}\right)^{\gamma}\right)^{-1}\right]\right]^{\beta}\right\}$$

(15)

$$f(x; \theta)_{WE} = \alpha\beta(\gamma \exp(-\gamma x)) \frac{(1 - \exp(-\gamma x))^{\beta-1}}{(\exp(-\gamma x))^{\beta-1}} \times \exp\left\{-\alpha \left[\frac{(1 - \exp(-\gamma x))^{\beta}}{\exp(-\gamma x)}\right]^{\beta}\right\}$$

(16)

for $0 < x < \infty$ and α, β, γ and $\lambda > 0$.

Table 3. MLE, -LL, AICs, and BICs for the first data set fitted to NWIL and other models

Model	Estimates	ll	AIC	BIC
NWIL	26.94, 0.24, 9.08, 19.54	356.75	721.5	731.92
WLL	0.66, 25.59, 97.75	383.59	773.18	785.6
OGEILD	24.2, 28.2, 24.49, 9.17	395.59	799.77	810.2
LLO	0.45, 12.35, 19.59	411.56	829.12	836.94
WE	0.66, 4.03, 0.01	413.4	832.8	845.22
WL	0.01, 2.27, 1.01, 2.34	495.67	999.34	1009.76
IL	4.93, 20.09	562.37	1128.75	1133.95

Data Set 2

The second set of real-world data corresponds to the survival times (in days) recorded by²⁶ of 72 guinea pigs infected with Virulent Tubercle Bacilli. The summary statistics for this Data set is presented in Table 4. The plots of the data is presented in Fig. 5. The NWIL distribution alongside Weibull-Weibull by¹⁷ (Table 5) was fitted, Odd Generalized exponential Inverse Lomax distribution by⁸ and Inverse Lomax distribution with the pdf:

$$f_{ww}(x; \theta) = \alpha\beta x^{-1} \left\{-\log\left(\frac{x}{\gamma}\right)\right\}^{\beta-1} \exp\left\{-\alpha \left[-\log\left(\frac{x}{\gamma}\right)\right]^{\beta}\right\}$$

(17)

other pdfs are defined earlier.

Table 4. Summary Statistics of the Second data set

Min.	Max.	Median	Mean	S.D.	Sk.	Kur.
10	555	145	175.4	104.19	1.37	5.16

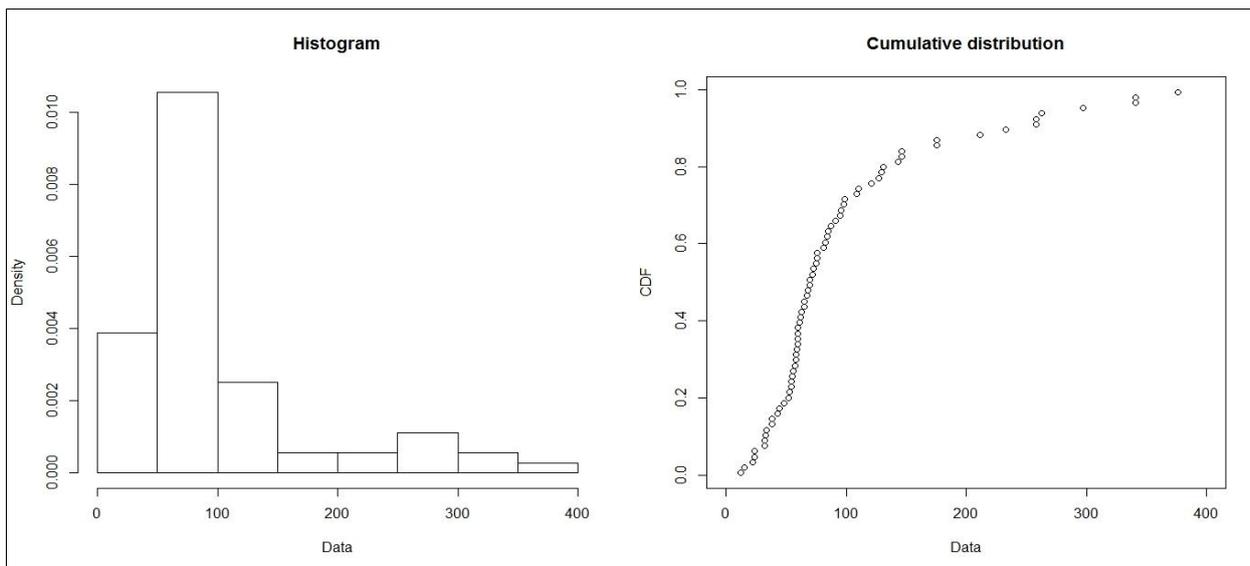


Figure 5. Histogram and cumulative plot for Data set 2

Table 5. MLE, -LL, AIC and BIC for the second data set fitted to NWIL and other models

Model	Estimates	ll	AIC	BIC
NWIL	20.64, 0.25, 11.51, 12.71	308.14	624.29	633.39
WW	2.66, 0.69, 0.03	390.23	786.47	797.57
OGEILD	3.64, 0.02, 0.55, 2.45	425.77	859.53	868.65
IL	9.18, 13.32	450.23	904.46	909.01

Concluding Remarks

A simulation study based on NWIL by Falgore¹⁵ et. al. in 2019 was performed. The results indicated that the ML estimates were consistent most of the times. The distribution was fitted to two data sets. The New Weibull-Inverse Lomax distribution (NWIL) was fitted to two datasets. Alongside the comparator distributions, the NWIL distribution outperformed its comparators, as shown in Table 3 and Table 4. For the first dataset, NWIL was compared with the Weibull Log Logistics which was fitted by Tahir¹⁶ et. al. in 2016 using the same data set. It is clear from Table 5 that the NWIL distribution performed much better than the New Weibull-Weibull distribution. We can therefore conclude that based on these two datasets the NWIL distribution outperformed its comparators.

Authors' declaration:

- There are no conflicts of interest, and all the Figures and Tables in the manuscript are derived from this study.
- Ethical Clearance: The project was approved by the local ethical committee in University of Ahmadu Bello.

Authors' contributions statement:

Falgore and Doguwa conceived of the idea. Falgore conducted the literature review and the

simulation study and wrote the first draft of the paper. Doguwa checked the results and edited the paper, and the reviewers provided some suggestions which improved the quality of the work.

References:

1. Kleiber C, Kotz S. Statistical size distributions in economics and actuarial sciences. John Wiley & Sons, Inc., Hoboken, New Jersey. Wiley Interscience; 2003:188-189.
2. Kleiber C. Lorenz ordering of order statistics from log-logistic and related distributions. JSPI. 2004;120(1-2):13-19.
3. Rahman J, Aslam M, Ali S. Estimation and prediction of inverse Lomax model via a Bayesian approach. CJASR. 2013;2(3):43-56.
4. Rahman J, Aslam M. Interval prediction of future order statistics in the two-component mixture inverse Lomax model: a bayesian approach. AJMMS. 2014;33(3):216-227.
5. Jan U, Ahmad SP. Bayesian analysis of inverse Lomax distribution using approximation techniques. MTM. 2017;7(7):1-12.
6. Rahman J, Aslam M. On the estimation of two-component mixture inverse Lomax model via a Bayesian approach. IJSAEM. 2017;8(1):99-109.
7. Yadav AS, Singh SK, Singh U. Bayesian estimation for inverse Lomax distribution under the progressive type-II censoring scheme. IJSAEM. 2019;10(5):905-917.

8. Falgore JY, Aliyu Y, Umar AA, Abdullahi UK. Odd generalized exponential-inverse Lomax distribution: properties and application. JNAMP.2018; 47:147–156.
9. Tahir MH, Cordeiro GM, Alizadeh M, Mansoor M, Zubair M, Hamedani GG. The odd generalized exponential family of distributions with applications. JSDA. 2015;2(1):1.
10. Maxwell O, Chukwu AU, Oyamakin OS, Khaleel MA. The Marshall-Olkin Inverse Lomax Distribution (MO-ILD) with Application on Cancer Stem Cell. JAMCS. 2019:1-12.
11. Marshall AW, Olkin I. A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. Biometrika. 1997;84(3):641-652.
12. Hassan AS, Mohamed RE. Weibull Inverse Lomax Distribution. PJSOR. 2019;15(3):587-603.
13. Falgore JY, Doguwa SI, Isah A. The Weibull-Inverse Lomax (WIL) distribution with Application on Bladder Cancer. BBIJ. 2019a;8(6):195–202. DOI: 10.15406/bbij.2019.08.00289.
14. Bourguignon M, Silva RB, Cordeiro GM. The Weibull-G family of probability distributions. JDS. 2014;12(1):53-68.
15. Falgore JY, Doduwa SI, Isah A. On the Properties of New Weibull-Inverse Lomax Distribution. TNAMP.2019b;(1):89-96.
16. Tahir MH, Zubair M, Mansoor M, Cordeiro GM, Alizadehk M, Hamedani GG. A new Weibull-G family of distributions. HJMS.2016;45(2):629-647.
17. Yadav AS, Singh SK, Singh U. On hybrid censored inverse Lomax distribution: Application to the survival data. Statistica. 2016;76(2):185-203.
18. Sambridge M, Mosegaard K. Monte Carlo methods in geophysical inverse problems. RG. 2002;40(3):1009.DOI:10.1029/2000RG000089.
19. Mooney CZ. Monte Carlo simulation. Sage Publications; 1997.
20. Walther BA, Moore JL. The concepts of bias, precision and accuracy, and their use in testing the performance of species richness estimators, with a literature review of estimator performance. Ecography. 2005;28(6):815-829.
21. Duncan AJ. Quality Control and Industrial Statistics, fourth edition (Irwin- Homewood, 1974).
22. Henningsen A, Toomet O. maxLik: A package for maximum likelihood estimation in R. CS. 2011;26(3):443-458.
23. Team RC. R: A Language and Environment for Statistical Computing <http://www.R-project.org>. 2014.
24. Zubair M, Cordeiro GM, Tahir MH, Mahmood M, Mansoor M. A Study of Logistic-Lomax Distribution and Its Applications. JPSS. 2017:29.
25. Oguntunde PE, Balogun OS, Okagbue HI, Bishop SA. The Weibull-exponential distribution: Its properties and applications. JAS. 2015;15(11):1305-1311.
26. Bjerkedal T. Acquisition of Resistance in Guinea Pies infected with Different Doses of Virulent Tubercle Bacilli. AJH. 1960;72(1):130-48.

الجدید ویبل العکسی لوماکس توزیع مع التطبيقات

سانی ابراهیم

جمیل یونس فالگو

جامعة أحمدو بیلو زاریا، نيجيريا

الخلاصة:

في هذه الورقة، نقدم دراسات وتطبيقات محاكاة لتوزيع Weibull-Inverse Lomax (NWIL) الجديد. في دراسات المحاكاة، درسنا أحجام عينات مختلفة تتراوح بين 30 و 50 و 100 و 200 و 300 و 500 لقد درسنا أيضاً 1000 تكرار للتجربة. لا يتبع التحيز والخطأ التربيعي (MSE) قانون الأعداد الكبيرة سواء كان توزيع ذيل الدهون. لا يتم اشتقاق لحظات أعلى إلا مع بعض التقريبية. ومع ذلك، فإن التقديرات لديها دقة أعلى مع تباينات منخفضة. أخيراً، أثبتنا فائدة توزيع NWIL من خلال تركيب مجموعتي بيانات.

الكلمات المفتاحية: Entropy، توزيع Weibull-Inverse Lomax الجديد، اللحظات، وظائف توليد اللحظات، دراسات المحاكاة.