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# A Comparative Study on the Double Prior for Reliability Kumaraswamy Distribution with Numerical Solution

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## Abstract:

This work, deals with Kumaraswamy distribution. Kumaraswamy (1976, 1978) showed well known probability distribution functions such as the normal, beta and log-normal but in (1980) Kumaraswamy developed a more general probability density function for double bounded random processes, which is known as Kumaraswamy's distribution. Classical maximum likelihood and Bayes methods estimator are used to estimate the unknown shape parameter (b). Reliability function are obtained using symmetric loss functions by using three types of informative priors two single priors and one double prior. In addition, a comparison is made for the performance of these estimators with respect to the numerical solution which are found using expansion method. The results showed that the reliability estimator under $R_n$ and $R_3$ is the best.

**Key words**: Bayes methods, Expansion method, Kumaraswamy distribution, Power function, Reliability function.

## Introduction:

The Kumaraswamy distribution is a family of continuous probability distribution defined on (0, 1), which has many similarities to the beta distribution, but it takes advantage of an invertible closed from cumulative distribution function. This distribution was originally proposed by Poondi Kumaraswamy (1980) (1).

The probability density and cumulative distribution function of a Kumaraswamy distribution random variable are given (2, 3, 4):

$$f(t;a,b) = ab\,t^{a-1}(1-t^a)^{b-1} \quad ; \; 0<t<1 \quad \ldots (1)$$

$$F(t;a,b) = 1-(1-t^a)^b \quad ; \quad 0<t<1 \ldots (2)$$

where $b>0$ and $a>0$ are the shape parameters. The corresponding reliability function, R(t), and failure rate function, h(t) at mission time t are given as (2, 3):

$$\left.\begin{aligned} R(t) &= (1-t^a)^b \\ h(t) &= \frac{f(t)}{R(t)} = \frac{ab\,t^{a-1}}{1-t^a} \end{aligned}\right\} \ldots (3)$$

The Kumaraswamy distribution (KD) is applicable to many natural phenomena whose outcomes have lower and upper bounds, such as the higight of individuals, scores obtained on a test, atmospheric temperatures, hydrological dated such as daily rain fall, daily stream flow, etc. (5). The Kumaraswamy distribution using different methods of estimation, some of which are Munashaker (2017) (3), Cholizadeh and et al (2011) (6) and Simbolon and et al (2017) (7). Singh and et al. (2012) (8) discussed Bayes estimators of reliability function of inverted exponential distribution using informative and non-informative priors a long with the comparison of them. Radha and Vekatesan (2013) (9), Raja and Ahmad (2014) (10) and Ronak (2017) (11) discussed the double prior of different distributions.

In this work, the informative priors two single priors and one double prior with entropy loss function are used to find the reliability function. Also, numerical method (polynomial expansion method) (12, 13) are used to estimate the reliability function R(t), in this method expanding function R(t) in terms of a set of power function as in (14) to find approximate solution of R(t), and then a comparison between the exact and all estimator using least square errors is given.

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**Maximum Likelihood Estimator (MLE):(7)**

Let random sample of size n, $\underline{t} = (t_1, t_2, \ldots, t_n)$ is drawn independently from Kumaraswamy distribution (KD) defined by equation (1). The likelihood function for the given sample is defined as:

$$L(a,b|\underline{t}) = \prod_{i=1}^{n} f(t_i|a,b) = a^n b^n e^{(a-1)\sum_{i=1}^{n}\ln(t_i)} e^{(b-1)\sum_{i=1}^{n}\ln(1-t_i^{a})} \quad \ldots (4)$$

The MLE of the unknown shape parameter b, denoted by $\hat{b}_{ML}$, assuming that the other shape parameter (a) is known yields by taking the derivative of natural log- likelihood function with respect to b and setting it equal to zero as:

$$\hat{b}_{ML} = \frac{-n}{T} \qquad \text{where} \quad T = \sum_{i=1}^{n}\ln(1-t_i^{a}) \quad \ldots(5)$$

The MLE of R(t) , based on the invariant property of the MLE is defined as:

$$\hat{R}_{ML}(t) = (1-t^a)^{\hat{b}_{ML}} \quad \ldots (6)$$

**Bayes Estimator (BE):(9, 10, 11)**

From Bayes' rule the posterior probability density function of unknown parameter b, results by combining the likelihood function $L(a,b|\underline{t})$ with the density function of the prior distribution g(b), as:

$$\pi(b|\underline{t}) = \frac{L(a,b|\underline{t})\; g(b)}{\int_b L(a,b|\underline{t})\; g(b)\, db} \quad \ldots (7)$$

The most widely used prior distribution of the parameter b is the gamma distribution with hyper-parameter 'α' and '$\beta$' with probability density function given by (10).

$$g_1(b) = \frac{\beta^\alpha}{\Gamma(\alpha)} b^{\alpha-1} e^{-b\beta} \quad ; \; b>0 \text{ and } \alpha,\beta>0 \quad \ldots (8)$$

The posterior distribution of the unknown parameter b of KD have been obtained by substitute equation (8) in equation (7):

$$\pi_1(b|\underline{t}) = \frac{\frac{\beta^\alpha}{\Gamma(\alpha)} b^{\alpha-1} e^{-b\beta}\, b^n e^{(b-1)\sum_{i=1}^{n}\ln(1-t_i^{a})}}{\frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty b^n e^{(b-1)\sum_{i=1}^{n}\ln(1-t_i^{a})}\, b^{\alpha-1} e^{-b\beta}\, db}$$

$$= \frac{b^{n+\alpha-1} e^{-b(\beta-T)}}{\int_0^\infty b^{n+\alpha-1} e^{-b(\beta-T)}\, db}$$

where $T = \sum_{i=1}^{n}\ln(1-t_i^{a})$

By using the transformation $y = b(\beta - T) \Rightarrow b = y/(\beta - T)$ and $db = dy/(\beta - T)$ , we obtain the final formula as:

$$\pi_1(b|\underline{t}) = \frac{b^{n+\alpha-1} e^{-b(\beta-T)} (\beta-T)^{n+\alpha}}{\Gamma(n+\alpha)} \quad \ldots(9)$$

The second prior distribution is the exponential distribution with hyper-parameter 'c' having probability density function given by (10).

$$g_2(b) = c e^{-bc} \quad ; \; b>0 \text{ and } c>0 \quad \ldots (10)$$

The posterior distribution of the unknown parameter b of KD has been obtained by combining equation (7) with equation (10) as:

$$\pi_2(b|\underline{t}) = \frac{b^n e^{(b-1)\sum_{i=1}^{n}\ln(1-t_i^{a})}\, c e^{-bc}}{\int_0^\infty b^n e^{(b-1)\sum_{i=1}^{n}\ln(1-t_i^{a})}\, c e^{-bc}\, db}$$

$$= \frac{b^n e^{-b(c-T)}}{\int_0^\infty b^n e^{-b(c-T)}\, db}$$

where $T = \sum_{i=1}^{n}\ln(1-t_i^{a})$

By using the transformation $y = b(c - T)$ $\Rightarrow$ $b = y/(c - T)$ and $db = dy/(c - T)$ , we obtain the final formula as:

$$\pi_2(b|\underline{t}) = \frac{b^n e^{-b(c-T)} (c-T)^{n+1}}{\Gamma(n+1)} \quad \ldots(11)$$

Now the third prior distribution is double (gamma-exponential) as the form (11):

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$$g_1(b).g_2(b) = g_3(b) = \frac{c\beta^{\alpha}}{\Gamma(\alpha)} b^{\alpha-1} e^{-b(\beta+c)}; \; b>0 \text{ and } \alpha,\beta,c>0 \quad \dots (12)$$

By the same procedure, we have the posterior distribution of the unknown parameter b of KD which has been obtained by combining equation (7) with equation (12) as:

$$\pi_3(b|\underline{t}) = \frac{b^n e^{(b-1)\sum_{i=1}^{n}\ln(1-t_i^a)} \frac{c\beta^{\alpha}}{\Gamma(\alpha)} b^{\alpha-1} e^{-b(\beta+c)}}{\int_0^{\infty} b^n e^{(b-1)\sum_{i=1}^{n}\ln(1-t_i^a)} \frac{c\beta^{\alpha}}{\Gamma(\alpha)} b^{\alpha-1} e^{-b(\beta+c)} db}$$

Then

$$\pi_3(b|\underline{t}) = \frac{b^{n+\alpha-1} e^{-b(\beta+c-T)} (\beta+c-T)^{n+\alpha}}{\Gamma(n+\alpha)} \quad \dots(13)$$

**Bayes Estimators under the Entropy Loss Function:**

It is well known that the Bayes estimators depend on the form of the prior distribution and the loss function assumed (15,16,17):

We will consider the entropy loss function (ELF) to obtain our Bayes estimator.

$$L(\hat{b},b) = k\left[\frac{\hat{b}}{b} - \log(\frac{\hat{b}}{b}) - 1\right], \; k>0 \quad \dots (14)$$

$$\text{Risk} = E\left[L(\hat{b},b)\right] = E\left[k\,(\frac{\hat{b}}{b} - \log(\frac{\hat{b}}{b}) - 1)\right] = E\left[k\,\frac{\hat{b}}{b} - k\log(\hat{b}) + k\log(b) - k)\right]$$

The value of $\hat{b}$ that minimizes the risk function is obtained by setting its partial derivative with respect to $\hat{b}$ equal to zero, that is:

$$\hat{b} = \frac{1}{E(1/b)} \quad \dots(15)$$

Therefore, the Bayes estimators of b based on the ELF is:

$$\hat{R}(t;a,b) = \frac{1}{E(1/R(t))} = \frac{1}{E(R(t))^{-1}}$$

where

$$E(R(t)|\underline{t}) = \int_0^{\infty} R(t)\,\pi(b|\underline{t})\,db$$

Now, the Bayes estimators of the reliability function R(t) corresponding to $\pi_1(b|\underline{t})$ can be found as (8):

$$\hat{R}(t)_{BE_1} = E_{\pi_1}\left[(R(t))^{-1}|\underline{t}\right] = \int_b (R(t))^{-1}\pi_1(b|\underline{t})\,db$$

$$= \int_0^{\infty} (1-t^a)^{-b} \frac{b^{n+\alpha} e^{-b(\beta-T)} (\beta-T)^{n+\alpha}}{\Gamma(n+\alpha)} db$$

$$= \frac{(\beta-T)^{n+\alpha}}{\Gamma(n+\alpha)} \int_0^{\infty} b^{n+\alpha} e^{-b(\beta-T+\ln(1-t^a))} db$$

By using the transformation, $y = b(\beta - T + \ln(1-t^a))$ which implies that $b = \frac{y}{\beta - T + \ln(1-t^a)}$ and $db = \frac{dy}{\beta - T + \ln(1-t^a)}$, we get:

$$E\left[(R(t))^{-1}|\underline{t}\right] = \left[\frac{(\beta-T)}{\beta-T+\ln(1-t^a)}\right]^{n+\alpha}$$

And

$$\hat{R}_1(t) = \left[\frac{\beta-T+\ln(1-t^a)}{(\beta-T)}\right]^{n+\alpha} \quad \dots(16)$$

Similarly, the Bayes estimators of the reliability function R(t) corresponding to $\pi_2(b|\underline{t})$ and $\pi_3(b|\underline{t})$ can be found as:

$$\hat{R}(t)_{BE_2} = E_{\pi_2}\left[(R(t))^{-1} \mid \underline{t}\right] = \int_b (R(t))^{-1}\, \pi_2(b \mid \underline{t})\, db$$

$$= \int_0^{\infty} (1-t^a)^{-b}\, \frac{b^n e^{-b(c-T)} (c-T)^{n+1}}{\Gamma(n+1)}\, db$$

And

$$\hat{R}(t)_{BE_2} = \left[\frac{c - T + \ln(1-t^a)}{(c-T)}\right]^{n+1} \quad ...(17)$$

So

$$\hat{R}(t)_{BE_3} = \left[\frac{\beta - T + c + \ln(1-t^a)}{(\beta - T + c)}\right]^{n+\alpha} \quad ...(18)$$

**Estimate Reliability Function Using Expansion Method**

In this section, the reliability function is estimated using expansion method, in which R(t) is expanded as a set of known function $q_i(t)$ (14):

$$R_n(t) = \sum_{i=1}^{n} d_i\, q_i(t) \qquad 0 \le t < 1 \; ...(19)$$

where $d_i$ are expansion coefficients to be determined and $q_i(t)$ are the expansion functions to be chosen, in this work $q_i(t) = t^{i-1}$ is taken.

Here, let the arbitrary points $\{t_1, t_2, \ldots, t_m\}$ in the subinterval $[t_1, t_n)$ where $m \le n$, this leads to:

$$R_n(t_j) = \sum_{i=1}^{m} d_i\, q_i(t_j) \qquad j = 1, 2, \ldots, m \; ...(20)$$

Substitute equation (6) into eq. equation (20) yields:

$$\sum_{i=1}^{m} d_i\, q_i(t_j) = (1 - t_j^a)^{\hat{b}_{ML}} \qquad j = 1, 2, \ldots, m \; ...(21)$$

Hence equation (21) is a system of (m) equations in (m) unknown $d_i$, $i = 1, 2, \ldots, m$ . This system is rewrite in matrix form as:

$$AD = B \qquad \ldots(22)$$

where

$$A = \begin{bmatrix} 1 & t_1 & \cdots & t_1^{m-1} \\ 1 & t_2 & \cdots & t_2^{m-1} \\ \vdots & & & \\ 1 & t_m & \cdots & t_m^{m-1} \end{bmatrix}, \quad D = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} (1 - t_1^a)^{\hat{b}_{ML}} \\ (1 - t_2^a)^{\hat{b}_{ML}} \\ \vdots \\ (1 - t_m^a)^{\hat{b}_{ML}} \end{bmatrix}$$

The matrix A contains (m) independent columns.

Finally, solve the above system in equation (22) for coefficients $d_i$'s using Gauss- elimination which satisfies equation (19) to find the approximate solution of R(t).

**Test Examples**

In this section, some test examples are present for different values n, a, c, β and α shown in Tables (1-9) to find the best estimated value of the reliability function using least square error. Where $\hat{b}_{ML}$ , T as in equation (5) and $t \in [0, 1)$ with $t_{i+1} = t_i + h$, i= 1, 2, …, n-1 , h=1/n and $t_1$=0.

MSE values for Bayes estimators and numerical solution of the reliability function of Kumaraswamy distribution with n=10 (sample of size), best value of m=10 (number of arbitrary points) and different values of (a), as shown in Tables (1, 2, 3).

**Table 1. Example 1 with m= n=10 and a=0.5.**

| $err_4 = \sum (R(t) - R_n(t))^2$ | | | 1.4460e-025 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1 = \sum (R(t) - R_1(t))^2$ | $err_2 = \sum (R(t) - R_2(t))^2$ | $err_3 = \sum (R(t) - R_3(t))^2$ |
| 0.7 | 0.6 | 0.5 | 0.0033 | 0.0082 | 5.8066e-004 |
| 1.3 | 1.3 | 1.5 | 0.0078 | 0.0025 | 4.8494e-004 |
| 1.9 | 1.9 | 2.5 | 0.0150 | 5.3437e-004 | 4.0579e-004 |
| 2.6 | 2.5 | 3.5 | 0.0235 | 0.0024 | 3.4910e-004 |
| 3.2 | 3.1 | 4.5 | 0.0327 | 0.0072 | 2.9956e-004 |
| 3.8 | 3.8 | 5.5 | 0.0399 | 0.0146 | 2.6314e-004 |

**Table 2. Example 2 with m= n=10 and a=1.**

| $err_4=\sum(R(t)-R_n(t))^2$ | | | 8.1708e-032 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum(R(t)-R_1(t))^2$ | $err_2=\sum(R(t)-R_2(t))^2$ | $err_3=\sum(R(t)-R_3(t))^2$ |
| 0.4 | 0.4 | 0.5 | 0.0026 | 0.0074 | 6.5472e-004 |
| 0.8 | 0.8 | 1.5 | 0.0067 | 0.0023 | 5.3945e-004 |
| 1.2 | 1.2 | 2.5 | 0.0122 | 5.9284e-004 | 4.5263e-004 |
| 1.6 | 1.6 | 3.5 | 0.0185 | 0.0018 | 3.8578e-004 |
| 2 | 2 | 4.5 | 0.0253 | 0.0056 | 3.3335e-004 |
| 2.4 | 2.4 | 5.5 | 0.0322 | 0.0117 | 2.9156e-004 |

**Table 3. Example 3 with m= n=10 and a=1.5.**

| $err_4=\sum(R(t)-R_n(t))^2$ | | | 8.6887e-029 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum(R(t)-R_1(t))^2$ | $err_2=\sum(R(t)-R_2(t))^2$ | $err_3=\sum(R(t)-R_3(t))^2$ |
| 0.3 | 0.3 | 0.5 | 0.0021 | 0.0060 | 5.8066e-004 |
| 0.6 | 0.6 | 1.5 | 0.0052 | 0.0018 | 4.8494e-004 |
| 0.9 | 0.8 | 2.5 | 0.0118 | 5.e-004 | 4.0579e-004 |
| 1.2 | 1.1 | 3.5 | 0.0171 | 0.0024 | 3.4910e-004 |
| 1.5 | 1.4 | 4.5 | 0.0225 | 0.0072 | 2.9956e-004 |
| 1.8 | 1.7 | 5.5 | 0.0281 | 0.0146 | 2.6314e-004 |

MSE values for Bayes estimators and numerical solution of the reliability function of Kumaraswamy distribution with n=25 (sample of size), best value of m=25 (number of arbitrary points) and different values of (a) , as shown in the following Tables (4, 5, 6).

**Table 4. Example 4 with m= n=25 and a=0.5.**

| $err_4=\sum(R(t)-R_n(t))^2$ | | | 2.5491e-009 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum(R(t)-R_1(t))^2$ | $err_2=\sum(R(t)-R_2(t))^2$ | $err_3=\sum(R(t)-R_3(t))^2$ |
| 0.7 | 0.7 | 0.5 | 0.0012 | 0.0038 | 2.4051e-004 |
| 1.4 | 1.4 | 1.5 | 0.0036 | 0.0011 | 2.2237e-004 |
| 2.1 | 2.1 | 2.5 | 0.0073 | 2.3105e-004 | 2.0711e-004 |
| 2.8 | 2.8 | 3.5 | 0.0120 | 0.0011 | 1.9424e-004 |
| 3.5 | 3.4 | 4.5 | 0.0187 | 0.0035 | 1.8153e-004 |
| 4.2 | 4.1 | 5.5 | 0.0252 | 0.0075 | 1.6823e-004 |

**Table 5. Example 5 with m= n=25 and a=1.**

| $err_4=\sum(R(t)-R_n(t))^2$ | | | 3.2762e-017 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum(R(t)-R_1(t))^2$ | $err_2=\sum(R(t)-R_2(t))^2$ | $err_3=\sum(R(t)-R_3(t))^2$ |
| 0.5 | 0.4 | 0.5 | 0.0012 | 0.0028 | 2.5673e-004 |
| 0.9 | 0.9 | 1.5 | 0.0030 | 9.8799e-004 | 2.3740e-004 |
| 1.4 | 1.3 | 2.5 | 0.0065 | 2.4873e-004 | 2.2018e-004 |
| 1.8 | 1.8 | 3.5 | 0.0099 | 8.4949e-004 | 2.0477e-004 |
| 2.3 | 2.2 | 4.5 | 0.0152 | 0.0030 | 1.9092e-004 |
| 2.7 | 2.7 | 5.5 | 0.0197 | 0.0058 | 1.7843e-004 |

**Table 6. Example 6 with m= n=25 and a=1.5.**

| $err_4=\sum(R(t)-R_n(t))^2$ | | | 5.1936e-013 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum(R(t)-R_1(t))^2$ | $err_2=\sum(R(t)-R_2(t))^2$ | $err_3=\sum(R(t)-R_3(t))^2$ |
| 0.4 | 0.3 | 0.5 | 0.0010 | 0.0022 | 2.4352e-004 |
| 0.7 | 0.6 | 1.5 | 0.0031 | 7.4394e-004 | 2.3509e-004 |
| 1 | 1 | 2.5 | 0.0051 | 2.3374e-004 | 2.0864e-004 |
| 1.4 | 1.3 | 3.5 | 0.0088 | 9.6730e-004 | 1.9517e-004 |
| 1.7 | 1.6 | 4.5 | 0.0133 | 0.0025 | 1.8856e-004 |
| 2 | 2 | 5.5 | 0.0166 | 0.0049 | 1.6921e-004 |

MSE values for Bayes estimators and numerical solution of the reliability function of Kumaraswamy distribution with n=50 (sample of size), best value of m (number of arbitrary points), m=24, m=40 and m= 27 of examples 7, 8 and 9 respectively and different values of (a), as shown in the following Tables (7, 8, 9).

**Table 7. Example 7 with n=50, m=24 and a=0.5.**

| $err_4=\sum(R(t)-R_n(t))^2$ | | | 2.1797e-005 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum(R(t)-R_1(t))^2$ | $err_2=\sum(R(t)-R_2(t))^2$ | $err_3=\sum(R(t)-R_3(t))^2$ |
| 0.7 | 0.7 | 0.5 | 6.1058e-004 | 0.0020 | 1.2235e-004 |
| 1.5 | 1.4 | 1.5 | 0.0020 | 4.9794e-004 | 1.1597e-004 |
| 2.2 | 2.1 | 2.5 | 0.0042 | 1.1904e-004 | 1.1158e-004 |
| 2.9 | 2.9 | 3.5 | 0.0066 | 5.7312e-004 | 1.0894e-004 |
| 3.6 | 3.6 | 4.5 | 0.0101 | 0.0018 | 1.0339e-004 |
| 4.3 | 4.3 | 5.5 | 0.0143 | 0.0039 | 9.9919e-005 |

**Table 8. Example 8 with n=50, m=40 and a=1.**

| $err_4=\sum (R(t)-R_n(t))^2$ | | | 1.2690e-012 | | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum (R(t)-R_1(t))^2$ | $err_2=\sum (R(t)-R_2(t))^2$ | $err_3=\sum (R(t)-R_3(t))^2$ |
| 0.5 | 0.5 | 0.5 | 4.6344e-004 | 0.0015 | 1.2971e-004 |
| 1 | 0.9 | 1.5 | 0.0017 | 4.1367e-004 | 1.2151e-004 |
| 1.4 | 1.4 | 2.5 | 0.0033 | 1.2495e-004 | 1.1921e-004 |
| 1.9 | 1.9 | 3.5 | 0.0054 | 4.7699e-004 | 1.1303e-004 |
| 2.4 | 2.3 | 4.5 | 0.0087 | 0.0016 | 1.0926e-004 |
| 2.9 | 2.8 | 5.5 | 0.0117 | 0.0035 | 1.0605e-004 |

**Table 9. Example 9 with n=50, m=27 and a=1.5.**

| $err_4=\sum (R(t)-R_n(t))^2$ | | | | 2.1395e-009 | |
|---|---|---|---|---|---|
| c | β | α | $err_1=\sum (R(t)-R_1(t))^2$ | $err_2=\sum (R(t)-R_2(t))^2$ | $err_3=\sum (R(t)-R_3(t))^2$ |
| 0.4 | 0.3 | 0.5 | 5.4419e-004 | 0.0012 | 1.2041e-004 |
| 0.7 | 0.7 | 1.5 | 0.0014 | 4.4393e-004 | 1.1624e-004 |
| 1.1 | 1 | 2.5 | 0.0030 | 1.1982e-004 | 1.1243e-004 |
| 1.4 | 1.4 | 3.5 | 0.0047 | 3.7673e-004 | 1.0896e-004 |
| 1.8 | 1.7 | 4.5 | 0.0075 | 0.0014 | 1.0579e-004 |
| 2.1 | 2.1 | 5.5 | 0.0099 | 0.0026 | 1.0290e-004 |

## Conclusions:

- 1 to 9 Tables, the best estimation of reliability is ($R_3$) at a = 1/2, n=50, β=c=4.3 and α=5.5 as well as approaching to ($R_n$).
- The numerical method gives best estimate from $R_1$ , $R_2$ and $R_3$ for all different values of n, a, c, β and α show in above Tables (1-9).
- The sample sizes (n) if increases, the estimation methods approach to the maximum likelihood estimation R(t).

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# دراسة مقارنة على دالة اولية ثنائية لمعولية توزيع كوارسوامي مع الحل العددي

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**الخلاصة:**

هذا العمل ، يتعامل مع توزيع كوارسوامي. لقد اظهر كوارسوامي في ( 1976، 1978) ان دوال توزيع الاحتمالات المعروفة جيدًا مثل الطبيعي، بيتا و اللوغاريتم الطبيعي ولكن في (1980) طور كوارسوامي دالة الكثافة الاحتمالية الأكثر عمومية لعمليات عشوائية مزدوجة الحدود ، والتي تعرف باسم توزيع كوارسوامي. يتم استخدام الطرائق الكلاسيكية وطرق تقدير بايز لتقدير معلمة الشكل غير المعروفة (b). يتم الحصول على دالة المعولية باستخدام دوال الخسارة المتماثلة باستخدام ثلاثة أنواع من الدوال الاولية المعلوماتية اثنان منها دوال اولية مفردة والاخرى دالة اولية ثنائية. بالإضافة إلى ذلك ، يتم إجراء مقارنة حول أداء هذه المقدرات مع الحل العددي الذي تم ايجاده باستخدام طريقة التوسيع. أظهرت النتائج أن مقدّر المعولية تحت $R_n$ و $R_3$ هو الأفضل.

**الكلمات المفتاحية:** طرق بايز، طريقة التوسيع، توزيع كوارسوامي، دالة القوة، دالة المعولية.