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The Dominant Metric Dimension of Corona Product Graphs

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Abstract:

The metric dimension and dominating set are the concept of graph theory that can be developed in terms of the concept and its application in graph operations. One of some concepts in graph theory that combine these two concepts is *resolving dominating number*. In this paper, the definition of *resolving dominating number* is presented again as the term dominant metric dimension. The aims of this paper are to find the dominant metric dimension of some special graphs and *corona* product graphs of the connected graphs G and H, for some special graphs H. The dominant metric dimension of G is denoted by $Ddim(G)$ and the dominant metric dimension of *corona* product graph G and H is denoted by $Ddim(G \odot H)$.

Key words: Corona product graph, Dominant metric dimension, Metric dimension, Resolving dominating number.

Introduction:

The G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The vertex u is adjacent to vertex v if $uv \in E(G)$. If every vertex in $V(G) - S$ is adjacent to a vertex in S, then set $S \subseteq V(G)$ is called the dominating set of G. The dominating set of G which has minimum cardinality is called the domination number of G and denoted by $\gamma(G)$. The resolving set $W = \{w_1, w_2, w_3, \dots, w_k\} \subseteq V(G)$ is a set which $r(u|W) \neq r(v|W)$ for every pair of vertices $u, v \in V(G)$, where $r(v|W) = \{d(v, w_1), d(v, w_2), \dots, d(v, w_k)\}$ with $d(v, w_i), i = 1,2,3, \dots, k$ is the distance between v and w_i. The metric dimension of G and denoted by $dim(G)$, is defined by $dim(G) = \min\{|W|, W \textit{ is the resolving set of } G\}$ (1).

The study of metric dimension concepts and its applications have been done by Careres (2), Yero (3), Iswadi (4), Saputro (5), and Susilowati (6) for cartesian product graphs, corona product graphs, path graph and comb product graph. The concept of dominating set was studied by Gupta (7), Reni Umilasari and Darmaji (8).

The combination of dominating set and metric dimension concepts by the term resolving dominating number, has been presented in (9,10) and denoted by $\gamma_r(G)$. Henning and Oellarmann (11) developed the same definition but with a different term, namely metric locating dominating number of G, denoted by $\gamma_M(G)$. For corona product graph, Iswadi (12) has gotten the upper bound, that is $\gamma_M(G \odot H) \leq |V(G)|(\dim(H) + 1)$ if H contains a dominant vertex or $\gamma_M(G \odot H) \leq |V(G)|(\dim(K_1 + H) + 1)$ for otherwise. Susilowati (13) found the dominant metric dimension of particular classes of graphs.

This research studies the exact value of dominant metric dimension of corona product graphs. First, recall the metric dimension and the dominating set of some graphs (11-14).

Theorem 1.
1. If $G = P_m$ or $G = C_n$ with $m \geq 2$ and $n \geq 3$, then $\gamma(G) = \left\lceil \frac{|V(G)|}{3} \right\rceil$.
2. If $G = K_m$ or $G = K_{1,n-1}$ with $m \geq 1$ and $n \geq 2$, then $\gamma(G) = 1$.
3. If $G = K_{p,q}$ with $p \geq 3$ and $q \geq 3$, then $\gamma(G) = 2$.
4. $dim(G) = n - 1$ if and only if $G = K_n$.
5. $dim(G) = 1$ if and only if $G = P_n$.
6. For $n \geq 3$, then $dim(C_n) = 2$.

Dominant Metric Dimension of a Graph

Susilowati (12) rename the resolving

dominating number of G by term the dominant metric dimension of G, presented below.

Definition 1. For a connected graph $\boldsymbol{G}$ and ordered set $\boldsymbol{S} \subseteq \boldsymbol{V}(\boldsymbol{G})$, the dominant metric dimension of $\boldsymbol{G}$ denoted by $\boldsymbol{Ddim}(\boldsymbol{G})$, is defined $\boldsymbol{Ddim}(\boldsymbol{G}) = \mathbf{min}\{|\boldsymbol{S}|; \boldsymbol{S}\ \boldsymbol{is\ the}\ \mathbf{dominant\ resolving\ set\ of}\ \boldsymbol{G}\}$
Susilowati (12) found some results are presented below.

Lemma 1. Let $\boldsymbol{G}$ be a connected graph. If any ordered subset of $\boldsymbol{V}(\boldsymbol{G})$ of cardinality $\boldsymbol{k}$ isn't dominant resolving set, then set $\boldsymbol{W} \subseteq \boldsymbol{V}(\boldsymbol{G})$ with $|\boldsymbol{W}| < \boldsymbol{k}$, $\boldsymbol{W}$ isn't dominant resolving set.

Lemma 2. For a connected graph G of order n, satisfies $max\{\gamma(G), dim(G)\} \leq Ddim(G) \leq min\{\gamma(G) + dim(G), n-1\}$.

Lemma 3. Let G be a connected graph. If $W \subseteq V(G)$, then for every $v_i, v_j \in W$, $r(v_i|W) \neq r(v_j|W)$.

Theorem 2.

1. If $n \geq 7$, then $Ddim(C_n) = \gamma(C_n)$.
2. If $n \geq 4$, then $Ddim(P_n) = \gamma(P_n)$.
3. If $n \geq 2$, then $Ddim(K_{1,n-1}) = n-1$.
4. If $m \geq 2$ and $n \geq 2$, then $Ddim(K_{m.n}) = dim(K_{m,n})$.
5. If $n \geq 2$, then $Ddim(K_n) = dim(K_n)$.
6. $Ddim(G) = 1$ if and only if $G \cong P_n, n = 1,2$.

On Corona Product Graph with Respect to The Dominant Metric Dimension

On graphs G and H, defined the corona product denoted by $S = G \odot H$, is a graph derived by taking one copy of G and $|V(G)|$ copy of H and connecting each vertex to the i-th of H to the i-th of G. Let $V(G) = \{v_i | i = 1,2,3,\ldots,m\}$ and $V(H) = \{v_j | j = 1,2,3,\ldots,n\}$. The i-th copy with $i = 1,2,3,\ldots,m$ of H is denoted by H_i, and has a vertex naming $V(H_i) = \{v_{ij} | j = 1,2,3,\ldots,n\}$ for every $i = 1,2,3,\ldots,m$. Furthermore, $V(G \odot H) = V(G) \cup_{i=1}^{m} V(H_i)$and
$E(G \odot H) = E(G) \cup_{i=1}^{m} E(H_i) \cup \{u_i v_{ij} | u_i \in V(G), v_{ij} \in V(H_i)\}$ (9).

The following section presents the value of $Ddim(G \odot H)$ where H is path, cycle, complete, star and complete bipartite graphs.

Lemma 4. Let P_m is a path graph with $m \geq 3$, then

$$Ddim(K_1 + P_m) = \begin{cases} \left\lceil \frac{2m+3}{5} \right\rceil, m \equiv 1 (mod\ 5)\ or\ m \equiv 3 (mod\ 5) \\ \left\lfloor \frac{2m+3}{5} \right\rfloor, m \equiv 2 (mod\ 5)\ or\ m \equiv 4 (mod\ 5) \\ \left\lfloor \frac{2m+2}{5} \right\rfloor, m \equiv 0 (mod\ 5) \end{cases}$$

Proof. Let $V(K_1) = \{u_1\}$ and $V(P_m) = \{v_i | i = 1,2,3,\ldots,m\}$ with $E(P_m) = \{v_i v_{i+1} | i = 1,2,3,\ldots,m-1\}$. There exist five cases for m: (i) $m \equiv 1\ (mod\ 5)$, (ii) $m \equiv 3\ (mod\ 5)$, (iii) $m \equiv 2\ (mod\ 5)$, (iv) $m \equiv 4\ (mod\ 5)$ and (v) $m \equiv 0\ (mod\ 5)$.

(i) For $m \equiv 1\ (mod\ 5)$. Select $W = \{v_{5i-3} | i = 1,2,3,\ldots,\left\lfloor \frac{m}{5} \right\rfloor\} \cup \{v_{5j-1} | j = 1,2,3,\ldots,\left\lfloor \frac{m}{5} \right\rfloor\} \cup \{v_m\}$, then $|W| = \left\lceil \frac{2m+3}{5} \right\rceil$. By **Lemma 3**, $r(u|W) \neq r(v|W)$ for every $u, v \in W$ with $u \neq v$. Moreover, for every $v_i \in V(K_1 + P_m)$, applies:

$$r(v_i|W) = \begin{cases} (1,2,2,2,\ldots,2,2,2,2), i = 1 \\ \left(2, \underbrace{1}_{\left\lceil \frac{i}{3} \right\rceil - th}, 2,2,\ldots,2,2,2,2\right), i \equiv 0\ (mod\ 5), i < m-1 \\ \left(2,2, \underbrace{1}_{\left(\left\lceil \frac{i}{2} \right\rceil - \left\lfloor \frac{i}{10} \right\rfloor\right) - th}, 2,\ldots,2,2,2,2\right), i \equiv 1\ (mod\ 5), i < m \\ \left(2,2,2,2,\ldots, \underbrace{1}_{\left(\left\lceil \frac{i}{3} \right\rceil + \left\lfloor \frac{i}{15} \right\rfloor\right) - th}, \underbrace{1}_{\left(\left\lceil \frac{i}{3} \right\rceil + \left\lfloor \frac{i}{15} \right\rfloor + 1\right) - th}, 2,2\right), i \equiv 3\ (mod\ 5), i < m \\ (2,2,2,2,2,2,\ldots,2,2,1,1), i = m-1 \end{cases}$$

Then W is a resolving set of $K_1 + P_m$. Since P_m is a path graph, then for every $v_{5i-4}, v_{5i-2} \in V(P_m)$ is adjacent to $v_{5i-3} \in V(P_m)$ with $i = 1,2,3,\ldots,\left\lfloor \frac{m}{5} \right\rfloor$ and for every $v_{5j-2}, v_{5j} \in V(P_m)$ is adjacent to $v_{5j-1} \in V(P_m)$ with $j = 1,2,3,\ldots,\left\lfloor \frac{m}{5} \right\rfloor$ and for

$v_{m-1} \in V(P_m)$ is adjacent to $v_m \in V(P_m)$. Therefore, $|W|$ is a dominant metric dimension with the following explanation, taken any $S \subseteq V(K_1 + P_m)$, with $|S| < |W|$. Let $|S| = |W| - 1$, then there exist two cases for S:

1. S does not contain u_1. Let $S = \left\{v_{5i-3} | i = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\} \cup \left\{v_{5j-1} | j = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\}$, then there exists $v_m \in V(K_1 + P_m) - S$ such that $uv_m \notin E(K_1 + P_m)$ for any $u \in S$. Furthermore, S isn't a dominating set of $K_1 + P_m$.
2. S is contains u_1. Let $S = \{u_1\} \cup \left\{v_{5i-3} | i = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\} \cup \left\{r_{5j-1} | j = 1,2,3, \dots, \left\lfloor \frac{m}{6} \right\rfloor\right\}$, then there exists $v_{m-2}, v_{m-1}, v_m \in V(K_1 + P_m) - S$, $r(v_{m-2}|S) = r(v_{m-1}|S) = r(v_m|S) = (2,2,2, \dots ,2,2)$. Furthermore, S isn't a resolving set of $K_1 + P_m$.

Therefore, S isn't a dominant resolving set of $K_1 + P_m$. By **Lemma 1**, thus, W is a dominant basis of $K_1 + P_m, m \equiv 1 \ (mod\ 5)$.

(ii) For $m \equiv 3 \ (mod\ 5)$. Select $W = \left\{v_{5i-3} | i = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\} \cup \left\{v_{5j-1} | j = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\} \cup \{v_{m-2}, v_m\}$, then $|W| = \left\lceil \frac{2m+3}{5} \right\rceil$. By **Lemma 3**, $r(u|W) \neq r(v|W)$ for every $u, v \in W$ with $u \neq v$. Moreover, for every $v_i \in V(K_1 + P_m)$, applies:

$$r(v_i|W) = \begin{cases} (1,2,2,2, \dots ,2,2,2,2), i = 1 \\ \left(2, \underbrace{1}_{\left\lceil \frac{i}{3} \right\rceil - th}, 2,2, \dots ,2,2,2,2\right), i \equiv 0 \ (mod\ 5), i < m - 3 \\ \left(2,2, \underbrace{1}_{\left(\left\lceil \frac{i}{2} \right\rceil - \left\lfloor \frac{i}{10} \right\rfloor\right) - th}, 2, \dots ,2,2,2,2\right), i \equiv 1 \ (mod\ 5), i < m \\ \left(2,2,2,2, \dots, \underbrace{1}_{\left(\left\lceil \frac{i}{3} \right\rceil + \left\lfloor \frac{i}{15} \right\rfloor\right) - th}, \underbrace{1}_{\left(\left\lceil \frac{i}{3} \right\rceil + \left\lfloor \frac{i}{15} \right\rfloor + 1\right) - th}, 2,2\right), i \equiv 3 \ (mod\ 5), i < m \\ (2,2,2,2, \dots ,2,1,1,2), i = m - 3 \\ (2,2,2,2, \dots ,2,2,1,1), i = m - 1 \end{cases}$$

Then W is a resolving set of $K_1 + P_m$. Since P_m is a path graph, then for every $v_{5i-4}, v_{5i-2} \in V(P_m)$ is adjacent to $v_{5i-3} \in V(P_m)$ with $i = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor$ and for every $v_{5j-2}, v_{5j} \in V(P_m)$ is adjacent to $v_{5j-1} \in V(P_m)$ with $j = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor$ and for $v_{m-1} \in V(P_m)$ is adjacent to $v_{m-2}, v_m \in V(P_m)$. Therefore, $|W|$ is a dominant metric dimension of $K_1 + P_m$, with the following explanation, taken any $S \subseteq V(K_1 + P_m)$, with $|S| < |W|$. Let $|S| = |W| - 1$, then there exist two cases for S:

1. S does not contain u_1, in the same manner of case (i) no.1.
2. S is contains u_1. Let $S = \{u_1\} \cup \left\{v_{5i-3} | i = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\} \cup \left\{v_{5j-1} | j = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\}$, then there exists $v_{m-2}, v_{m-1}, v_m \in V(K_1 + P_m) - S$, $r(v_{m-2}|S) = r(v_{m-1}|S) = r(v_m|S) = (2,2,2, \dots ,2,2)$.

Furthermore, S isn't a resolving set of $K_1 + P_m$.
So, S isn't a dominant resolving set of $K_1 + P_m$. By **Lemma 1**, $|W| = Ddim(K_1 + P_m)$, for $m \equiv 3 \ (mod\ 5)$.

(iii) For $m \equiv 2 \ (mod\ 5)$. Select $W = \left\{v_{5i-3} | i = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\} \cup \left\{v_{5j-1} | j = 1,2,3, \dots, \left\lfloor \frac{m}{5} \right\rfloor\right\}$, then $|W| = \left\lfloor \frac{2m+3}{5} \right\rfloor$. By **Lemma 3**, applies $r(u|W) \neq r(v|W)$ for every $u, v \in W$ with $u \neq v$. Moreover, for every $v_i \in V(K_1 + P_m)$, applies:

$$r(v_i|W)=\begin{cases}(1,2,2,2,\dots,2,2,2,2),\ i=1\\ \left(2,\underbrace{1}_{\left\lceil\frac{i}{3}\right\rceil-th},2,2,\dots,2,2,2,2\right),\ i\equiv 0\ (mod\ 5), i<m\\ \left(2,2,\underbrace{1}_{\left(\left\lceil\frac{i}{2}\right\rceil-\left\lfloor\frac{i}{10}\right\rfloor\right)-th},2,\dots,2,2,2,2\right),\ i\equiv 1\ (mod\ 5), i<m\\ \left(2,2,2,2,\dots,\underbrace{1}_{\left(\left\lceil\frac{i}{3}\right\rceil+\left\lfloor\frac{i}{15}\right\rfloor\right)-th},\underbrace{1}_{\left(\left\lceil\frac{i}{3}\right\rceil+\left\lfloor\frac{i}{15}\right\rfloor+1\right)-th},2,2\right),\ i\equiv 3\ (mod\ 5), i<m\end{cases}$$

Then W is a resolving set of K_1+P_m. Since for every $v_{5i-4}, v_{5i-2}\in V(P_m)$ is adjacent to $v_{5i-3}\in V(P_m)$ with $i=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil$ and for every $v_{5j-2}, v_{5j}\in V(P_m)$ is adjacent to $v_{5j-1}\in V(P_m)$ with $j=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil$, so that W is a dominating set of K_1+P_m. Therefore, $|W|=Ddim(K_1+P_m)$, with the following explanation, take any $S\subseteq V(K_1+P_m)$, with $|S|<|W|$. Let $|S|=|W|-1$, then there exist two cases for S:

1. S does not contain u_1, in the same manner of case (i) no.1.
2. S is contains u_1. Let $S=\{u_1\}\cup\left\{v_{5i-3}|i=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil\right\}\cup\left\{v_{5j-1}|j=1,2,3,\dots,\left\lceil\frac{m}{6}\right\rceil\right\}$, then there exists $v_{m-3}, v_{m-2}, v_{m-1}, v_m\in V(K_1+P_m)-S$, $r(v_{m-3}|S)=r(v_{m-2}|S)=r(v_{m-1}|S)=r(v_m|S)=(2,2,2,\dots,2,2)$.

Furthermore, S isn't a resolving set of K_1+P_m. Therefore, S isn't a dominant resolving set of K_1+P_m. By **Lemma 1**, W is a dominant basis of $K_1+P_m, m\equiv 2\ (mod\ 5)$.

(iv) For $m\equiv 4\ (mod\ 5)$. Select $W=\left\{v_{5i-3}|i=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil\right\}\cup\left\{v_{5j-1}|j=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil\right\}$, then $|W|=\left\lfloor\frac{2m+3}{5}\right\rfloor$. By **Lemma 3**, $r(u|W)\neq r(v|W)$ for every $u,v\in W$ with $u\neq v$. Moreover, for every $v_i\in V(K_1+P_m)$, applies:

$$r(v_i|W)=\begin{cases}(1,2,2,2,\dots,2,2,2,2),\ i=1\\ \left(2,\underbrace{1}_{\left\lceil\frac{i}{3}\right\rceil-th},2,2,\dots,2,2,2,2\right),\ i\equiv 0\ (mod\ 5), i<m\\ \left(2,2,\underbrace{1}_{\left(\left\lceil\frac{i}{2}\right\rceil-\left\lfloor\frac{i}{10}\right\rfloor\right)-th},2,\dots,2,2,2,2\right),\ i\equiv 1\ (mod\ 5), i<m\\ \left(2,2,2,2,\dots,\underbrace{1}_{\left(\left\lceil\frac{i}{3}\right\rceil+\left\lfloor\frac{i}{15}\right\rfloor\right)-th},\underbrace{1}_{\left(\left\lceil\frac{i}{3}\right\rceil+\left\lfloor\frac{i}{15}\right\rfloor+1\right)-th},2,2\right),\ i\equiv 3\ (mod\ 5), i<m\end{cases}$$

Then W is a resolving set of K_1+P_m. Since for every $v_{5i-4}, v_{5i-2}\in V(P_m)$ is adjacent to $v_{5i-3}\in V(P_m)$ with $i=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil$ and for every $v_{5j-2}, v_{5j}\in V(P_m)$ is adjacent to $v_{5j-1}\in V(P_m)$ with $j=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil$. Therefore $|W|=Ddim(K_1+P_m)$, with the following explanation, taken any $S\subseteq V(K_1+P_m)$, with $|S|<|W|$. Let $|S|=|W|-1$, then there exist two cases for S:

1. S does not contain u_1, in the same manner of case (i) no.1.
2. S is contains u_1. Let $S=\{u_1\}\cup\left\{v_{5i-3}|i=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil\right\}\cup\left\{v_{5j-1}|j=1,2,3,\dots,\left\lceil\frac{m}{5}\right\rceil\right\}$, then there exists $v_{m-3}, v_{m-2}, v_{m-1}, v_m\in V(K_1+P_m)-S$, $r(v_{m-3}|S)=r(v_{m-2}|S)=r(v_{m-1}|S)=r(v_m|S)=(2,2,2,\dots,2,2)$.

Furthermore, S isn't a resolving set of K_1+P_m. Therefore, S isn't a dominant resolving set of K_1+P_m. By **Lemma 1**, $|W|=Ddim(K_1+P_m), m\equiv 4\ (mod\ 5)$.

(v) For $m\equiv 0\ (mod\ 5)$. Select $W=\left\{v_{5i-3}|i=1,2,3,\dots,\frac{m}{5}\right\}\cup\left\{v_{5j-1}|j=1,2,3,\dots,\frac{m}{5}\right\}$, then $|W|=\left\lfloor\frac{2m+2}{5}\right\rfloor$. By **Lemma 3**, $r(u|W)\neq r(v|W)$ for every $u,v\in W$ with $u\neq v$.

Moreover, for every $v_i \in V(K_1 + P_m)$, applies:

$$r(v_i|W) = \begin{cases} (1,2,2,2,\dots,2,2,2,2), i = 1 \\ \left(2, \underbrace{1}_{\left\lceil\frac{i}{3}\right\rceil-th}, 2,2,\dots,2,2,2,2\right), i \equiv 0\ (mod\ 5), i \le m \\ \left(2,2, \underbrace{1}_{(\left\lceil\frac{i}{2}\right\rceil-\left\lfloor\frac{i}{10}\right\rfloor)-th}, 2,\dots,2,2,2,2\right), i \equiv 1\ (mod\ 5), i < m \\ \left(2,2,2,2,\dots, \underbrace{1}_{(\left\lceil\frac{i}{3}\right\rceil+\left\lfloor\frac{i}{15}\right\rfloor)-th}, \underbrace{1}_{(\left\lceil\frac{i}{3}\right\rceil+\left\lfloor\frac{i}{15}\right\rfloor+1)-th}, 2,2\right), i \equiv 3\ (mod\ 5), i < m \end{cases}$$

Then W is a resolving set of $K_1 + P_m$. Since for every $v_{5i-4}, v_{5i-2} \in V(P_m)$ is adjacent to $v_{5i-3} \in V(P_m)$ with $i = 1,2,3,\dots,\frac{m}{5}$ and for every $v_{5j-2}, v_{5j} \in V(P_m)$ is adjacent to $v_{5j-1} \in V(P_m)$ with $j = 1,2,3,\dots,\frac{m}{5}$, so that $|W|$ is a dominant metric dimension of $K_1 + P_m$, with the next explanation, take any $S \subseteq V(K_1 + P_m)$, with $|S| < |W|$. Let $|S| = |W| - 1$, then there exist two cases for S:

1. S does not contain u_1, in the same manner of case (i) no.1.
2. S is contains u_1. Let $S = \{u_1\} \cup \left\{v_{5i-3}|i = 1,2,3,\dots,\left\lfloor\frac{m}{6}\right\rfloor\right\} \cup \left\{v_{5j-1}|j = 1,2,3,\dots,\left\lfloor\frac{m}{6}\right\rfloor\right\}$, then there exists $v_{m-4}, v_{m-3}, v_{m-2}, v_{m-1}, v_m \in V(K_1 + P_m) - S$, $r(v_{m-4}|S) = r(v_{m-3}|S) = r(v_{m-2}|S) = r(v_{m-1}|S) = r(v_m|S) = (2,2,2,\dots,2,2)$. Furthermore, S isn't a resolving set of $K_1 + P_m$.

Therefore, S isn't a dominant resolving set of $K_1 + P_m$. By **Lemma 1**, $|W| = Ddim(K_1 + P_m), m \equiv 0\ (mod\ 5)$.

By explanation above, it is proven that W is a dominant basis for $K_1 + P_m$. ■

Theorem 3. If G is a connected graph then $Ddim(G \odot P_n) = |V(G)|\big(Ddim(K_1 + P_n)\big)$, for $n \ge 3$.

Proof. Let G be a connected graph with $V(G) = \{v_i|i = 1,2,3,\dots,m\}$, $m \ge 2$ and $V(P_n) = \{v_j|j = 1,2,3,\dots,n\}$, $E(P_n) = \{v_j v_{j+1}|j = 1,2,3,\dots,n-1\}$. The i-th copy of P_n with $i = 1,2,3,\dots,m$ is denoted by $P_{(n)_i}$ with $V\left(P_{(n)_i}\right) = \{v_{ij}|j = 1,2,3,\dots,n\}$, for every $i = 1,2,3,\dots,m$. Therefore

$$V(G \odot P_n) = V(G) \cup_{i=1}^{m} V\left(P_{(n)_i}\right),$$
$$E(G \odot P_n) = E(G) \cup_{i=1}^{m} E\left(P_{(n)_i}\right) \cup \left\{u_i v_{ij}|\ u_i \in V(G), v_{ij} \in V\left(P_{(n)_i}\right)\right\}.$$

Let B be a dominant basis of $K_1 + P_n$, B_i is a dominant basis of $K_1 + P_{(n)_i}$ then $|B_i| = |B|$, $i = 1,2,3,\dots,m$. Therefore, $|W|$ is a dominant metric dimension of $G \odot P_n$, with the next explanation. Taken any $S \subseteq V(G \odot P_n)$ with $|S| < |W|$. Let $|S| = |W| - 1$, then there exists i such that S contains a maximum of $|B_i| - 1$ element of $K_1 + P_{(n)_i}$. Since B_i is a dominant basis of $K_1 + P_{(n)_i}$, then there exists $u, v \in V(K_1 + P_{(n)_i})$ such that $r(u|S) = r(v|S)$ or there exists vertex in $K_1 + P_{(n)_i}$ isn't adjacent with at least one vertex in S. Furthermore S isn't a resolving set or S isn't a dominating set of $G \odot P_n$. By **Lemma 1**, $Ddim(G \odot P_n) = |W| = |V(G)|Ddim(K_1 + P_n)$ for $n \ge 3$. Thus, it's proven that if $n \ge 3$, then $Ddim(G \odot P_n) = |V(G)|Ddim(K_1 + P_n)$. ■

Lemma 5. Let C_m is a cycle graph with $m \ge 3$, then

$$Ddim(K_1 + C_m) = \begin{cases} \left\lceil\frac{2m+3}{5}\right\rceil, m \equiv 1(mod\ 5)\ or\ m \equiv 3(mod\ 5) \\ \left\lfloor\frac{2m+3}{5}\right\rfloor, m \equiv 2(mod\ 5)\ or\ m \equiv 4(mod\ 5) \\ \left\lfloor\frac{2m+2}{5}\right\rfloor, m \equiv 0(mod\ 5) \end{cases}$$

Theorem 4. If G is a connected graph, then $Ddim(G \odot C_n) = |V(G)|\big(Ddim(K_1 + C_n)\big)$ for $n \ge 3$.

Theorem 5. If G is a connected graph, then $Ddim\big(G \odot K_{p,q}\big) = |V(G)|Ddim\big(K_{p,q}\big)$, for $p \ge 2$ and $q \ge 2$

Proof. Let G be a connected graph with $V(G) = \{v_i|i = 1,2,3,\dots,m\}, m \ge 2$ and $V\big(K_{p,q}\big) = V_p \cup V_q$ with $V_p = \{a_i|i = 1,2,3,\dots,p\}$ and $V_q = \{b_j|j = 1,2,3,\dots,q\}$. The i-th copy of $K_{p,q}$ with $i = 1,2,3,\dots,m$ is denoted by $K_{(p,q)_i}$ with

$V\left(K_{(p,q)_i}\right) = \{v_{ij}|j = 1,2,3,\dots,p\} \cup \{v_{ik}|k = 1,2,3,\dots,q\}$ for every $i = 1,2,3,\dots,m$, $V(G\odot K_{p,q}) = V(G)\cup_{i=1}^{m} V\left(K_{(p,q)_i}\right)$, $E(G\odot K_{p,q}) = E(G)\cup_{i=1}^{m} E\left(K_{(p,q)_i}\right)\cup_{i=1}^{m} \left\{u_i a_{ij}|u_i \in V(G), a_{ij} \in V\left(K_{(p,q)_i}\right)\right\}$. Let B is a dominant basis of $K_{p,q}$, B_i is a dominant basis of $K_{(p,q)_i}$, then for every $i = 1,2,3,\dots,m$, $|B_i| = |B|$. Select $W = \cup_{i=1}^{m} B_i$ with $B_i = \{j = 1,2,3,\dots,p-1\} \cup \{k = 1,2,3,\dots,q-1\}$ for every $i = 1,2,3,\dots,m$. By **Lemma 3**, $r(u|W) \neq r(v|W)$ for every $u, v \in W$ with $u \neq v$. Moreover, take any two vertex in $V(G\odot K_{p,q}) - W$. From every possibility, it will be shown that the representation of each vertex is different with respect to W.

i. For $u_i, u_j \in V(G\odot K_{p,q}) - W$ with $i \neq j$, $d(u_j, s) = d(u_j, u_i) + d(u_i, s)$, for some $s \in B_i$. Since $B_i \subseteq W$ then $r(u_i|W) \neq r(u_j|W)$.
ii. For $a_{it}, a_{jr} \in V(G\odot K_{p,q}) - W$ with $i \neq j$, $d(a_{it}, u_j) = d(a_{it}, u_i) + d(u_i, u_j)$ and $d(a_{jr}, u_j) = 1$. As the result, $r(a_{it}|B_j) \neq r(a_{jr}|B_j)$. Moreover, since $B_j \subseteq W$ then $r(a_{it}|W) \neq r(a_{jr}|W)$. The same reason for $b_{it}, b_{jr} \in V(G\odot K_{p,q}) - W$.
iii. For $a_{it}, b_{ir} \in V(G\odot K_{p,q}) - W$. Since B_i is basis dominant of $K_{(p,q)_i}$ then $r(a_{it}|B_i) \neq r(b_{ir}|B_i)$. Moreover, since $B_i \subseteq W$ then $r(a_{it}|W) \neq r(b_{ir}|W)$. The same reason for $a_{it}, b_{jr} \in V(G\odot K_{p,q}) - W$.
iv. For $u_i, a_{it} \in V(G\odot K_{p,q}) - W$, $d(a_{it}, s) = d(a_{it}, u_i) + d(u_i, u_j)$ for every $s \in B_j$. Since $B_j \subseteq W$ then $r(u_i|W) \neq r(b_{iu}|W)$. The same reason for $u_i, b_{ir} \in V(G\odot K_{p,q}) - W$.
v. For $u_i, a_{jt} \in V(G\odot K_{p,q}) - W$ with $i \neq j$, $d(a_{jt}, s) = d(a_{jt}, u_j,) + d(u_j, u_i) + d(u_i, s)$, for every $s \in B_i$. Moreover, since $B_i \subseteq W$ then $r(u_i|W) \neq r(a_{jt}|W)$. The same reason for $u_i, b_{jr} \in V(G\odot K_{p,q}) - W$ with $i \neq j$.

By the explanation above, then $W = \cup_{i=1}^{m} B_i$ is a resolving set of $G\odot K_{p,q}$. Moreover, for every $s \in G\odot K_{p,q} - W$ can be seen that s is adjacent to some elements of W, as described below.

i. $s = u_i \in V(G)$ is adjacent to $a_{i1}, b_{i1} \in W$ for every $i = 1,2,3,\dots,m$.
ii. $s = a_{i(p-1)} \in V\left(K_{(p,q)_i}\right)$ is adjacent to $b_{i1} \in W$ for every $i = 1,2,3,\dots,m$.
iii. $s = b_{i(q-1)} \in V\left(K_{(p,q)_i}\right)$ is adjacent to $a_{i1} \in W$ for every $i = 1,2,3,\dots,m$.

It can be concluded that W is a dominating set of $G\odot K_{p,q}$. Thus, $W = \cup_{i=1}^{m} B_i$ is a dominant resolving set of $G\odot K_{p,q}$. Moreover, $|W|$ is a dominant metric dimension of $G\odot K_{p,q}$, with the next explanation. Taken any $S \subseteq V(G\odot K_{p,q})$ with $|S| < |W|$. Let $|S| = |W| - 1$, then there exists i such that S contains a maximum of $|B_i| - 1$ element of $K_{(p,q)_i}$. Since B_i is a dominant basis of $K_{(p,q)_i}$, then there are $u, v \in V(K_{(p,q)_i})$ such that have $r(u|S) = r(v|S)$, furthermore S isn't a resolving set of $G\odot K_{p,q}$. By **Lemma 1**, $Ddim(G\odot K_{p,q}) = |W| = |V(G)|Ddim(K_{p,q})$ for $p \geq 2$ and $q \geq 2$. Thus, it's proven that if $p \geq 2$ and $q \geq 2$, then $Ddim(G\odot K_{p,q}) = |V(G)|Ddim(K_{p,q})$. ■

Theorem 6. If G is a connected graph, then
$$Ddim(G\odot K_n) = |V(G)|Ddim(K_n), \text{ for } n \geq 2,$$
Proof. For a connected graph G with $V(G) = \{u_i|i = 1,2,3,\dots,m\}$, $m \geq 2$, $V(K_n) = \{v_j|j = 1,2,3,\dots,n\}$ and $E(K_n) = \{i, j = 1,2,3,\dots,n, i \neq j\}$. The i-th copy of K_n is denoted by $K_{(n)_i}$ with $V(K_{n_i}) = \{v_{ij}|j = 1,2,3,\dots,n\}$ for every $i = 1,2,3,\dots,m$, $V(G\odot K_n) = V(G)\cup_{i=1}^{m} V\left(K_{(n)_i}\right)$, $E(G\odot K_n) = E(G)\cup_{i=1}^{m} E\left(K_{(n)_i}\right)\cup_{i=1}^{m} \left\{u_i v_{ij}|u_i \in V(G), v_{ij} \in V\left(K_{(n)_i}\right)\right\}$. Let B is a dominant basis of K_n, B_i is a dominant basis of $K_{(n)_i}$, then for every $i = 1,2,3,\dots,m$, $|B_i| = |B|$. Select $W = \cup_{i=1}^{m} B_i$ with $B_i = \{v_{ij}|j = 1,2,3,\dots,n-1\}$ for every $i = 1,2,3,\dots,m$ then $|W| = n - 1$. By **Lemma 3**, $r(u|W) \neq r(v|W)$ for every $u, v \in W$ with $u \neq v$. Moreover, take any two vertex in $V(G\odot K_n) - W$. From every possibility, it will be shown that the representation of each vertex is different with respect to W.

i. For $u_i, u_j \in V(G\odot K_n) - W$ with $i \neq j$. Since $d(u_j, v) = d(u_j, u_i) + d(u_i, v)$, then for every $v \in B_i$, $d(v, u_i) \neq d(v, u_j)$. Moreover, since $B_i \subseteq W$ then $r(u_i|W) \neq r(u_j|W)$.
ii. For $v_{iq}, v_{jq} \in V(G\odot K_n) - W$ with $i \neq j$. Since $d(v_{jq}, v) = d(v_{jq}, v_j) + d(v_j, v_i) + d(v_i, v_{iq})$, then for every $v \in B_i$ imply $d(v, v_{iq}) \neq d(v, v_{jq})$. Moreover, since $B_i \subseteq W$ then $r(v_{iq}|W) \neq r(v_{jq}|W)$
iii. For $u_i, v_{iq} \in V(G\odot K_n) - W$. There exists two possibility, $r(u_i|B_i) = r(v_{iq}|B_i)$ or $r(u_i|B_i) \neq r(v_{iq}|B_i)$. For $r(u_i|B_i) \neq r(v_{iq}|B_i)$, since $B_i \subseteq W$ then $r(u_i|W) \neq r(v_{iq}|W)$. For $r(u_i|B_i) = r(v_{iq}|B_i)$ since for $i \neq j$,

$d(u_i, u_j) \neq d(v_{iq}, u_j)$ then for every $v \in B_j$, $d(v, u_i) \neq d(v, v_{iq})$. Moreover, since $B_j \subseteq W$ then $r(u_i|W) \neq r(v_{iq}|W)$.

iv. For $u_i, v_{jq} \in V(G \odot K_n) - W$ with $i \neq j$. Using the same reason of case (ii), $r(u_i|W) \neq r(v_{jq}|W)$.

By the explanation above, then $W = \bigcup_{i=1}^{m} B_i$ is a resolving set of $G \odot K_n$. Moreover, since for every $u_i \in V(G)$ with $i = 1,2,3,\ldots,m$ is adjacent to $v_{i1} \in W$ with $i = 1,2,3,\ldots,m$ and for every $v_{iq} \in V\left(K_{(n)_i}\right)$ with $i = 1,2,3,\ldots,m$ is adjacent to $v_{i1} \in W$ with $i = 1,2,3,\ldots,m$, then W is a dominating set of $G \odot K_n$. Thus, $W = \bigcup_{i=1}^{m} B_i$ is a dominant resolving set of $G \odot K_n$. Moreover, $|W|$ is dominant metric dimension of $G \odot K_n$, with the next explanation. Taken any $S \subseteq V(G \odot K_n)$ with $|S| < |W|$. Let $|S| = |W| - 1$, then there exists i such that S contains a maximum of $|B_i| - 1$ element of $K_{(n)_i}$. Since B_i is a dominant basis of $K_{(n)_i}$, then there exist $u, v \in K_{(n)_i}$ such that $r(u|S) = r(v|S)$. Furthermore S isn't a resolving set of $G \odot K_n$. By **Lemma 1,** $Ddim(G \odot K_n) = |W| = |V(G)|Ddim(K_n)$ for $n \geq 2$. Thus, it's proven that if $n \geq 2$, then $Ddim(G \odot K_n) = |V(G)|Ddim(K_n)$. ■

Corollary 1. If G is a connected graph, then $Ddim(G \odot K_{1,n-1}) = |V(G)|Ddim(K_{1,n-1})$, for $n \geq 4$

Conclusion:

In this paper, it was found the dominant metric dimension of corona product graph of a connected graph G and H, for H is path graph, cycle graph, complete bipartite graph, complete graph, and star graph, described as below:

a. For H are path and cycle graphs, the dominant metric dimension of corona product of G and H depends on $|V(G)|$ and $Ddim(K_1 + H)$.
b. For H are complete bipartite and complete graphs, the dominant metric dimension of corona product of G and H depends on $|V(G)|$ and $Ddim(H)$.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Universitas Airlangga.

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البعد المتري السائد لرسومات منتجات كورونا

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الخلاصة:

البعد المتري والمجموعة المسيطرة هما مفهوم نظرية الرسم البياني الذي يمكن تطويره من حيث المفهوم وتطبيقه في عمليات الرسم البياني. ان حل الرقم المهيمن هو أحد المفاهيم في نظرية الرسم البياني التي تجمع بين هذين المفهومين . في هذه الورقة ، يتم تقديم تعريف حل الرقم المسيطر مرة أخرى كمصطلح البعد المتري السائد. حيث تهدف هذه الورقة إلى إيجاد البعد المتري السائد لبعض الرسوم البيانية الخاصة ورسومات حاصل ضرب الاكليل للرسوم البيانية المتصلة ، ولبعض الرسوم البيانية الخاصة. يُشار إلى البعد المتري السائد لـ ويتم الإشارة إلى البعد المتري السائد في الرسم البياني للمنتج الإكليل G و H بواسطة .

الكلمات المفتاحية: الرسم البياني لمنتج كورونا ، البعد المتري السائد ، البعد المتري ، حل الرقم المسيطر