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Numerical Solution for Linear State Space Systems using Haar Wavelets Method

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Abstract:

In this research, Haar wavelets method has been utilized to approximate a numerical solution for Linear state space systems. The solution technique is used Haar wavelet functions and Haar wavelet operational matrix with the operation vec to transform the state space system into a system of linear algebraic equations which can be resolved by MATLAB over an interval from 0 to γ. The exactness of the state variables can be enhanced by increasing the Haar wavelet resolution. The method has been applied for different examples and the simulation results have been illustrated in graphics and compared with the exact solution.

Keywords: Approximation solutions, Collocation points method, Haar wavelets, State system.

Introduction:

A state space is a mathematical model of a physical system, with involving a set of state variables interrelated by first order differential equations with zero initial conditions [1]. In this paper, the Haar wavelet basis function and Haar wavelet operational matrix are interested to approximate a system of differential equations. As of late, Haar wavelets have been related to signal and image processing in communication and physics research and have been proved to be excellent mathematical tools [2]. Compared with other wavelet functions, Haar wavelet has a few advantages. Haar wavelet is the oldest and the simplest wavelet function and it is an orthogonal function [3]. Also, its bases have compact support, which means that the Haar wavelet vanishes outside of a limited interval and enable us to display functions with sharp spikes or edges, better than other bases. The respected properties of Haar functions in numerical calculation include the sparse representation for piecewise constant function, quick conversion, and the possibility of implementing a quick algorithm in matrix [4]. Nonetheless, the advantage remains when a large matrix is involved, whereby great computer stowage space and a vast number of mathematical operations are required [5].

Operational matrix technique has received considerable attention from numerous researchers for solving dynamical system analysis [6], system identification [7], numerical computation of integral and differential equations [8], and solving systems of PDEs [9]. In addition, Hsiao and Wang [10] introduced the application of Haar wavelets to solve optimal control for linear time-varying systems. Based on Haar wavelet method, Prabakaran *et. al* [11] used Haar wavelet series method to get discrete solutions for a state space system of differential equations. Abuhamdia and Taheri [12] presented survey a wide-ranging of research on utilizing wavelets in the analysis and design of dynamic systems, and the main focus of this survey is electromechanical and mechanical systems furthermore to their controls. Karimi *et.* al [13] solved second-order linear systems with respect to a quadratic cost function using Haar wavelet. Abdul Khader and Monica [14] used Haar wavelet method to solve fractional of partial differential equations. Ali and Baleanu [15] solved system of unsteady gas-flow of four dimensional by alter the possibility of an algorithm based on collocation points and four dimensions Haar wavelet method.

In this study, Haar wavelet operational matrix of integration and Haar wavelet collocation points with the operation vec for one dimension on the interval $[0,\gamma)$ were used. The paper is organized as follows: The problem statement has been described in the second section. The formulates of the Haar wavelet method and Haar operational matrix are presented in the third part of this paper. In the fourth section, the proposed strategy to approximate the linear state space system by using Haar operational matrix, and Haar wavelet collocation points are presented. Numerical examples and discussions are shown at the end of this paper.

Problem statement

The linear state-space system can be defined as[20, 22]:

$$\dot{x}(\mathbf{t}) = \mathbf{A}\, x(\mathbf{t}) + \mathbf{B} \quad , x(0) = \mathbf{x}_0 , \qquad \dots (1)$$

Where $x(\mathbf{t}) \in R^{n_1}$ is a vector of state space, A is $n_1 \times n_1$ the system matrix, $\mathbf{B}$ is the constant vector $n_1 \times 1$ and $x(0) = \mathbf{x}_0$ is the initial condition vector of size $n_1 \times 1$.

Haar wavelets

Haar wavelets $h_i(x)$ are the orthogonal set of square waves on the interval $[\gamma_1, \gamma_2)$. These wavelets are defined as:

$$h_0(x) = \begin{cases} 1, & \gamma_1 \le x < \gamma_2 , \\ 0, & \textbf{elsewhere.} \end{cases} \qquad \dots(2)$$

$$h_1(x) = \begin{cases} 1, & \gamma_1 \le x < \frac{1}{2}(\gamma_1 + \gamma_2), \\ -1, & \frac{1}{2}(\gamma_1 + \gamma_2) \le x < \gamma_2 , \\ 0, & \text{otherwise}. \end{cases} \qquad \dots(3)$$

Where other wavelets can be determined through enlarging and translating the mother wavelet $h_1(x)$; $h_i(x) = h_1(2^j x - k)$, where $i = 2^j + k$, i, j belong to $\mathrm{N} \cup \{0\}$, $j = 0,1,2,\dots,\log_2(m-1)$ and $0 \le k < 2^j$ which fulfills

$$\int_{\gamma_1}^{\gamma_2} h_i(x)\, h_l(x) dx = \begin{cases} 2^{-j}(\gamma_2 - \gamma_1), & i = l \\ 0, & i \ne l \end{cases} \qquad \dots(4)$$

Any analytic function $g(x) \in L^2([\gamma_1, \gamma_2])$ can be written to a finite of Haar sequence:

$$g_m(x) = \sum_{i=0}^{m-1} d_i h_i(x) \qquad \dots (5)$$

Where $g(x)$ is a piecewise constants, which can be written in a compacted form:

$$g(x) = \mathbf{d}_m^{\mathbf{T}}\, \mathbf{h}_m(x) \qquad \dots(6)$$

where, $\mathbf{h}_m(x) = [\, h_0(x)\ \ h_1(x)\ \dots\ h_{m-1}(x)\,]^{\mathbf{T}}$ is vector of the Haar function, m is the Haar wavelt resolution and $\mathbf{d}_m = [d_0\ \ d_1\ \ d_2\ \dots\ d_{m-1}]^{\mathbf{T}}$ is the coefficient vector which can be determined from [16]

$$d_i = \frac{2^j}{(\gamma_2 - \gamma_1)} \int_{\gamma_1}^{\gamma_2} g(x)\, h_i(x)\, dx \qquad \dots(7)$$

Where, the points of Haar collocation $x_s = \gamma_1 + \frac{\gamma_2 - \gamma_1}{2m}(2s - 1)$, $s = 1,2,3, \dots\ m-1$ [16], so the Haar function vector $\mathbf{h}_m(x)$ can be represented into matrix shape $\mathbf{H}_m$, where the elements are donated by

$$(\mathbf{H}_m)_{i,s} = h_i\,(x_s). \dots(8)$$

For example, the matrix of Haar wavelet of fourth-order $\mathbf{H}_4$ can be expressed into matrix shape in the interval of $[0, 1)$ with the collocation points from Eqn. (8) as follows:

$$\mathbf{H}_4 = \begin{bmatrix} h_0(\frac{1}{8}) & h_0(\frac{3}{8}) & h_0(\frac{5}{8}) & h_0(\frac{7}{8}) \\ h_1(\frac{1}{8}) & h_1(\frac{3}{8}) & h_1(\frac{5}{8}) & h_1(\frac{7}{8}) \\ h_2(\frac{1}{8}) & h_2(\frac{3}{8}) & h_2(\frac{5}{8}) & h_2(\frac{7}{8}) \\ h_3(\frac{1}{8}) & h_3(\frac{3}{8}) & h_3(\frac{5}{8}) & h_3(\frac{7}{8}) \end{bmatrix}, \qquad \dots(9)$$

$$\mathbf{H}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}. \qquad \dots (10)$$

When the Haar wavelet matrix is defined as in Eqn. (8), then the coefficient $\mathbf{d}_m^{\mathbf{T}}$ in equations. (6) and (7) can be readily obtained as

$$\mathbf{d}_m^{\mathbf{T}} = g_m\ \ \mathbf{H}_m^{-1}, \qquad \dots(11)$$

where

$$g_m = [g(x_1)\quad g(x_2)\quad g(x_3)\quad \cdots\quad g(x_m)] \dots(12)$$

In the specific domain of $[0, \gamma)$, $h_i(x)$ can be extended in a Haar series by integration as [17]:

$$\int_0^x \mathbf{h}_m(x) dx \cong \mathbf{P}_m\, \mathbf{h}_m(x), \qquad \dots (13)$$

where $\mathbf{P}_m$ is an $m \times m$ the operational matrix of integration, which is acquired recursively by (16):

$$\mathbf{P}_m = \frac{1}{2m}\begin{bmatrix} 2m\,\mathbf{P}_{m/2} & -\gamma\,\mathbf{H}_{m/2} \\ -\gamma\,\mathbf{H}^{-1}_{m/2} & O_{m/2} \end{bmatrix},\ \mathbf{P}_1 = \begin{bmatrix} \dfrac{\gamma}{2} \end{bmatrix} \ldots(14)$$

Numerical Solution for State Space Systems using Haar Wavelet Method

The numerical solution to a linear state space system with initial conditions is following as; an approximate solution to fre by utilizing equation (5) as follow:

$$\dot{x}_\alpha(\mathbf{t}) = \sum_{i=0}^{m-1} d_{\alpha i}\, h_i(\mathbf{t}),\ \alpha = 1, 2, \cdots, n_1, \ldots(15)$$

where $d_{\alpha 0}, d_{\alpha 1}, d_{\alpha 2}, \cdots, d_{\alpha m-1}$, $\alpha = 1, 2, 3, \cdots, n_1$ are unknown parameters for the state variables,

Equation (15) can be indicated in matrix shape as following:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n_1} \end{bmatrix} = \begin{bmatrix} d_{10} & d_{11} & \cdots & d_{1m-1} \\ d_{20} & d_{21} & \cdots & d_{2m-1} \\ \vdots & \vdots & \cdots & \vdots \\ d_{n_1 0} & d_{n_1 1} & \cdots & d_{n_1 m-1} \end{bmatrix} \begin{bmatrix} h_0(\mathbf{t}) \\ h_1(t) \\ \vdots \\ h_{m-1}(\mathbf{t}) \end{bmatrix} \ldots(16)$$

This equation can be rewritten into compact form as :

$$\dot{x}(\mathbf{t}) = \mathbf{d}^{\mathbf{T}}\mathbf{h}(\mathbf{t}) \ \ldots(17)$$

where $\mathbf{d}^{\mathbf{T}}$ is unknown coefficients in matrix form $n_1 \times m$ for Haar wavelet functions; and $\mathbf{h(t)}$ is the vector of known Haar wavelet function with dimension of $m \times 1$, where $\mathbf{h}(t) = [\,h_0(t)\ h_1(t)\ h_2(t) \ldots h_{m-1}(t)\,]^{\mathbf{T}}$ and $\mathbf{T}$ is the transpose.

By integrating equation (17) with respect to $\mathbf{t}$ besides applying equation (13), $x(\mathbf{t})$ is found, which is represented into terms of Haar operational matrix and the Haar wavelet functions as

$$x(\mathbf{t}) = \int_0^{\mathbf{t}} \mathbf{d}^{\mathbf{T}}\mathbf{h}(\mathbf{t})\,\mathbf{dt} + \mathbf{x}_0 \qquad \ldots(18)$$

Thus

$$x(\mathbf{t}) = \mathbf{d}^{\mathbf{T}}\mathbf{P}\,\mathbf{h}(\mathbf{t}) + \mathbf{x}_0\,\theta^{\mathbf{T}}\,\mathbf{h}(\mathbf{t}), \ \ldots(19)$$

where $\mathbf{x}_0$ is $n_1 \times 1$ column vector of the initial conditions that is $\mathbf{x}_0 = \left[x_{01}\ x_{02}\ x_{03} \cdots x_{0n_1}\right]^{\mathbf{T}}$, and $\theta = [1,\ 0,\ 0,\ \cdots,\ 0\,]^{\mathbf{T}}$ is an $m \times 1$ vector.

Eqns. (17), and (19) can then be expressed by using the properties of the operation vec, where $vec\,(\mathbf{ACB}) = (\mathbf{A} \otimes \mathbf{B}^{\mathbf{T}})\,vec\,(\mathbf{C})$ [18], as follows:

$$\dot{x}(\mathbf{t}) = (\mathbf{I}_n \otimes \mathbf{h}^{\mathbf{T}}(\mathbf{t}))\,vec\,(\mathbf{d}) \ldots(20)$$

$$x(\mathbf{t}) = ((\mathbf{I}_n \otimes \mathbf{h}^{\mathbf{T}}(\mathbf{t}))\,\mathbf{P}^{\mathbf{T}})\,vec\,(\mathbf{d}) + (\mathbf{I}_n \otimes \mathbf{h}^{\mathbf{T}}(\mathbf{t}))\,vec\,(\mathbf{x}_0\,\theta^{\mathbf{T}}) \ldots(21)$$

where $\mathbf{I}_n$ denote $n_1 \times n_1$ and identity matrix,. In addition

$$vec(\mathbf{d}) = \left[d_{10}\ \ d_{20}\ \cdots\ d_{n_1 0}\ \ d_{11}\ \ d_{21} \cdots d_{n_1 1}\ \cdots\ d_{1m-1}\ \ d_{2m-1} \cdots d_{n_1 m-1}\right]^{\mathbf{T}}$$

is the vector of unknown Haar wavelet coefficients with dimension $nm \times 1$, and $vec(\mathbf{x}_0\ \theta^{\mathbf{T}})$ is an $n_1 m \times 1$ vector of known coefficients that can be framed as

$$vec\,(\mathbf{x}_0\,\theta^{\mathbf{T}}) = [\,x_{01}\,0\ 0\ 0\ 0 \cdots\ x_{02}\,0\ 0\ 0\ 0\ \cdots\ x_{03}\,0\ 0\ 0\ 0\ \cdots\,]^{\mathbf{T}}$$

.

Given the notation above, substituting the equations (17) and (19) into equation (1) with expanding $\mathbf{B}$ in terms of Haar approximation functions, obtain

$$\mathrm{d}^{\mathrm{T}}\,\mathrm{h(t)} = \mathrm{A}\left\{\mathrm{d}^{\mathrm{T}}\,\mathrm{P}\,\mathrm{h(t)} + \mathrm{x}_0\,\theta^{\mathrm{T}}\,\mathrm{h(t)}\right\} + \mathrm{B}\theta^{\mathrm{T}}\,\mathrm{h(t)}, \ldots(22)$$

Simplifying equation (22) and by utilizing Kronecker product properties such as $(\mathrm{A} \otimes \mathrm{C}\mathbf{B}) = (\mathrm{A} \otimes \mathrm{C})(\mathrm{A} \otimes \mathrm{B})$ [18], have

$$\begin{aligned}(\mathrm{I}_{n_1} \otimes \mathrm{h}^T(\mathrm{t}))\,vec\,(\mathrm{d}) - (\mathrm{I}_{n_1} \otimes \mathrm{h}^T(\mathrm{t}))\,(\mathrm{A} \otimes \mathrm{P}^{\mathrm{T}})\,vec\,(\mathrm{d}) \\ = (\mathrm{I}_{n_1} \otimes \mathrm{h}^T(\mathrm{t}))vec\,(\mathrm{Ax}_0\theta^{\mathrm{T}}) + (\mathrm{I}_{n_1} \otimes \mathrm{h}^T(\mathrm{t}))vec(\mathrm{B}\theta^{\mathrm{T}})\end{aligned} \quad \ldots(23)$$

Then both sides of equation (23) are multiplied with the matrix inverse $\left[\mathbf{I}_{n_1} \otimes \mathbf{h}^T(\mathbf{t})\right]^{-1}$ to remove the term of $(\mathbf{I}_{n_1} \otimes \mathbf{h}^T(\mathbf{t}))$. Thus, obtain

$$vec(\mathrm{d}) - (\mathrm{A} \otimes \mathrm{P}^{\mathrm{T}})\,vec\,(\mathrm{d}) = vec(\mathrm{Ax}_0\theta^{\mathrm{T}}) + vec(\mathrm{B}\theta^{\mathrm{T}}) \quad \ldots(24)$$

Now, equation (24) is transformed into a standard system of linear equations as follows

$$\left[\mathrm{I}_{n_1 m} - (\mathrm{A} \otimes \mathrm{P}^T)\right]\left[vec(\mathrm{d})\right] = \left[vec(\mathrm{Ax}_0\theta^{\mathrm{T}})\ +\ vec\,(\mathrm{B}\theta^{\mathrm{T}})\right] \ldots(25)$$

Equation (25) is a system of determined linear equation with $n_1 m$ unknown variables and $(n_1 m)$ equations that can solve for the unknown vector $vec(\mathbf{d})$ such in MATLAB [19]. As soon as, the result to the unknown parameters are got, these parameters to equation (19) are replaced to identify the solution $x(t)$ as follows:

$$x(\mathrm{t}) = (\mathrm{I}_{n_1} \otimes \mathrm{h}^{\mathrm{T}}(\mathrm{t})\,\mathrm{P}^{\mathrm{T}})\,vec\,(\mathrm{d}) + (\mathrm{I}_{n_1} \otimes \mathrm{h}^{\mathrm{T}}(\mathrm{t}))\,vex\,(\mathrm{x}_0\,\theta^{\mathrm{T}}) \ldots(26)$$

Numerical Examples

In this section, four examples of free linear dynamic systems are solved using the method illustrated above. The present method was applied to display the simplicity, effectiveness, and exactness of the proposed numerical method.

Example 1:

Consider the following free state space system [20, 21].

$$\begin{bmatrix} \dot{x}_1\ (\mathbf{t}) \\ \dot{x}_2\ (\mathbf{t}) \end{bmatrix} = \begin{bmatrix} \mathbf{-3} & \mathbf{4} \\ \mathbf{2} & \mathbf{-1} \end{bmatrix} \begin{bmatrix} x_1(\mathbf{t}) \\ x_2(\mathbf{t}) \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Where the initial condition and the exact solution of the state space model are $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{4e}^{t} + 2e^{-5t} - 2 \\ \mathbf{4e}^{t} - e^{-5t} - 2 \end{bmatrix}$ respectively.

By applying the Haar wavelet collocation points method described in the previous section; that is, directly transform the free state space system into the set of linear algebraic equations with $n_1 m$ equations and $n_1 m$ unknown variables that can resolve for the unknown vector $vec(\mathbf{d})$ utilizing inv() MATLAB solver, the numerical solution to this example is obtained by approximating the state space variables based on the Harr wavelet series of unknown parameters. The numerical results are found for this example as shows in Table 1, which are very close to the exact values to $m = 16$. Also, the Fig. 1 shows that even a coarse Haar wavelet resolution of $m = 32$ already yields an accurate result.

Tab1e 1. Comparison between the exact and numerical solution in Example 1 using Haar wavelets method for $m = 16$

t	Exact Solution $x_1(\mathbf{t})$	Approximate Solution $x_1(\mathbf{t})$	Error $\lvert Exact - x_1 \rvert$	Exact Solution $x_2(\mathbf{t})$	Approximate Solution $x_2(\mathbf{t})$	Error $\lvert Exact - x_2 \rvert$
0.0625	3.7212	3.7905	0.0693	1.5264	1.5048	0.0216
0.1875	3.6081	3.6337	0.0256	2.4333	2.4365	0.0031
0.3125	3.8866	3.8984	0.0118	3.2577	3.2712	0.0135
0.4375	4.4197	4.4300	0.0103	4.0831	4.1015	0.0184
0.5625	5.1403	5.1539	0.0135	4.9602	4.9818	0.0216
0.6875	6.0192	6.0378	0.0185	5.9228	5.9476	0.0248
0.8125	7.0486	7.0729	0.0243	6.9969	7.0256	0.0287
0.9375	8.2328	8.2634	0.0306	8.2051	8.2387	0.0335
1.0625	9.5842	9.6218	0.0375	9.5695	9.6088	0.0394
1.1875	11.1208	11.1661	0.0453	11.1129	11.1593	0.0465
1.3125	12.8646	12.9188	0.0542	12.8604	12.9153	0.0549
1.4375	14.8421	14.9065	0.0644	14.8399	14.9047	0.0648
1.5625	17.0837	17.1600	0.0763	17.0825	17.1590	0.0765
1.6875	19.6242	19.7143	0.0900	19.6236	19.7138	0.0902
1.8125	22.5032	22.6093	0.1061	22.5029	22.6090	0.1062
1.9375	25.7656	25.8904	0.1248	25.7654	25.8903	0.1248

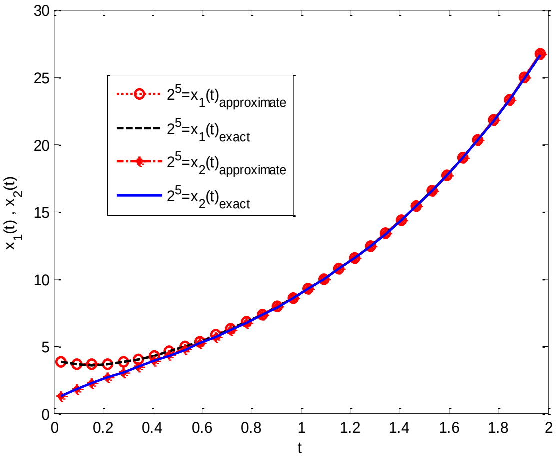

Figure 1. State space variables $x_1(\mathbf{t})$ and $x_2(\mathbf{t})$ for Haar wavelet resolutions $m = 2^5$ and $\mathbf{t} = 2$ obtained from Example 1

Examp1e 2:

Consider the following problem of homogeneous dynamic system equation [20].

$$\begin{bmatrix} \dot{x}_1\ (\mathbf{t}) \\ \dot{x}_2\ (\mathbf{t}) \end{bmatrix} = \begin{bmatrix} \mathbf{-4} & \mathbf{3} \\ \mathbf{1} & \mathbf{-2} \end{bmatrix} \begin{bmatrix} x_1(\mathbf{t}) \\ x_2(\mathbf{t}) \end{bmatrix} + \begin{bmatrix} \mathbf{6} \\ \mathbf{1} \end{bmatrix}$$

Where the initial condition and the exact solution of the free state space model are $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and

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$$\begin{bmatrix} x_1(\mathbf{t}) \\ x_2(\mathbf{t}) \end{bmatrix} = -\frac{9}{4}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\mathbf{e}^{-\mathbf{t}} - \frac{1}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix}e^{-5\mathbf{t}} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

respectively.

The numerical and exact solutions for state space variables obtained using Haar wavelet collocation points method for various resolution $m = 4, 8, 16,$ and 32 are illustrated in Figs. 2 and 3. These figures clearly show that the Haar wavelets functions come closer to the exact solutions as the resolution of Haar wavelet functions increases.

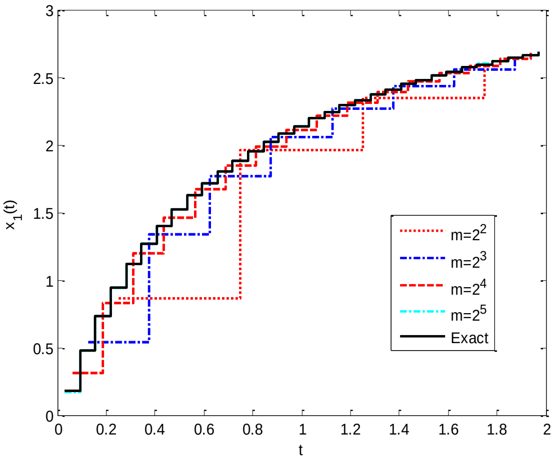

Figure 2. State space variable and exact solution to $x_1(\mathbf{t})$ for Haar wavelet resolutions $m = 4, 8, 16, 32$ and $\mathbf{t} = 2$ obtained from Example 2.

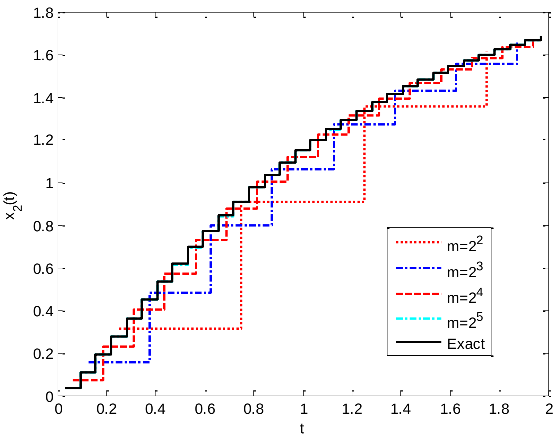

Figure 3. State space variable and exact solution to $x_2(\mathbf{t})$ for Haar wavelet resolutions $m = 4, 8, 16, 32$ and $\mathbf{t} = 2$ obtained from Example 2

Examp1e 3:

Consider the problem as the following [22].

$$\begin{bmatrix} \dot{x}_1(\mathbf{t}) \\ \dot{x}_2(\mathbf{t}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{bmatrix}\begin{bmatrix} x_1(\mathbf{t}) \\ x_2(\mathbf{t}) \end{bmatrix}$$

Where the initial condition $x(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$

Figures 4 and 5 present the graphical representations of the numerical solution for different resolutions of Haar wavelets approximations functions for $m = 4, 8, 16,$ for state variables $x_1(\mathbf{t})$ and $x_2(\mathbf{t})$. These figures clearly show that the Haar wavelets approximation functions converges to the lowest error as the resolution of Haar wavelet functions increases.

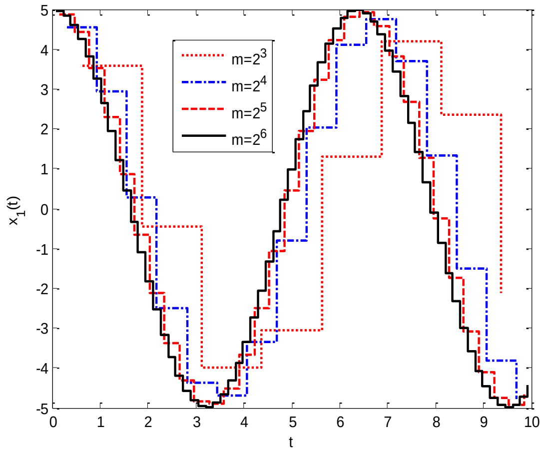

Figure 4. State space variable $x_1(\mathbf{t})$ for Haar wavelet resolutions $m = 8, 16, 32, 64$ and $\mathbf{t} = 10$ obtained from Example 3.

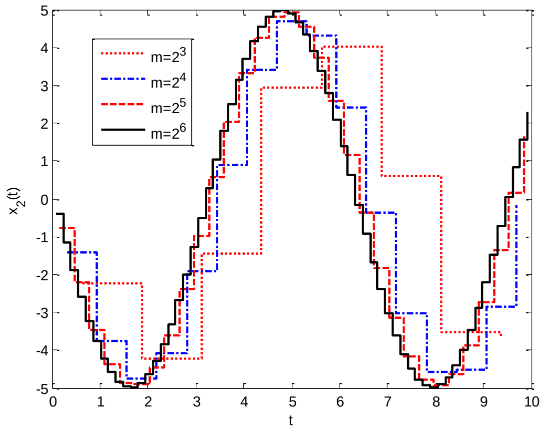

Figure 5. State space variable $x_2(\mathbf{t})$ for Haar wavelet resolutions $m = 8, 16, 32, 64$ and $\mathbf{t} = 10$ obtained from Example 3.

Examp1e 4:

Consider the problem as the following [22].

$$\begin{bmatrix} \dot{x}_1(\mathbf{t}) \\ \dot{x}_2(\mathbf{t}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{-2} \end{bmatrix}\begin{bmatrix} x_1(\mathbf{t}) \\ x_2(\mathbf{t}) \end{bmatrix}$$

Where the initial condition $x(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$

The numerical results for state space variables $x_1(\mathbf{t})$ and $x_2(\mathbf{t})$ with different values of Haar wavelet resolutions of $m=8,16,32,$ and 64 that are obtained from Example 4 are illustrated in Fig. 6.

These figures clearly show that the Haar wavelets approximation functions converges to the correct solutions as the resolution of Haar wavelet functions increases.

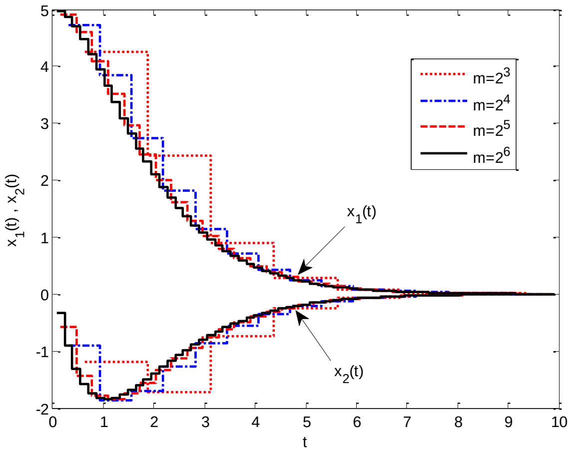

Figure 6. State space variables $x_1(\mathbf{t}), x_2(\mathbf{t})$ for Haar wavelet resolutions $m=8,16,32,64$ and $\mathbf{t}=10$ obtained from Example 4.

Conc1usion:

The proposed approach employs the free state space variables over an interval from 0 to γ using Haar wavelet functions and Haar wavelet operational matrix with the operation $vec(\,)$ to transform the state space system into a system of linear algebraic equations which can be readily resolved via MATLAB. The proposed method is simple and it has been tested for free linear state space system in two-dimensional state space. As shown in all figures, the exactness of the state variables can be enhanced by increasing the Haar wavelet resolution.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors' contributions statement:

Title of manuscript: Numerical Solution for Linear State Space Systems using Haar Wavelets Method.

The authorship of the title above certify that they have participated in different roles as follows: W. Swaidan conception, acquisition of data, analysis, proofreading and revision. H. Swaidan Ali design, interpretation of data, drafting the MS.

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الحل العددي لأنظمة فضاء الحالة باستخدام طريقة موجات هار ويفلت

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الخلاصة:

في هذا البحث، تم استخدام طريقة الموبجات الشعرية لإيجاد حل تقريبي لأنظمة فضاء الحالة الخطية. وان تقنية الحل هي تحويل أنظمة فضاء الحالة الخطية إلى نظام من المعادلات الخطية للفاصل الزمني من 0 إلى γ. كما يمكن تعزيز دقة متغيرات الحالة عن طريق زيادة دقة موجات هار ويفلت. تم تطبيق الطريقة المقترحة لأمثلة مختلفة وتم توضيح نتائج المحاكاة بالرسوم البيانية ومقارنتها بالحل الدقيق.

الكلمات المفتاحية: حلول تقريبية، طريقة نقاط التجميع، موجات هار ويفلت، نظام الحالة.