Application of Groebner Bases to Study a Communication System

Alaa Jony[*]   *Shawki Al-Rashed*

Department of Mathematics, Faculty of Sciences, Damascus University of Syria
[*]Corresponding author: alaajoony@gmail.com[*], sh.shawki@googlemail.com
[*]ORCID ID: https://orcid.org/0000-0002-1200-2914[*] , https://orcid.org/0000-0002-1464-9244

Abstract:

This paper introduces a relationship between the independence of polynomials associated with the links of the network, and the Jacobian determinant of these polynomials. Also, it presents a way to simplify a given communication network through an algorithm that splits the network into subnets and reintegrates them into a network that is a general representation or model of the studied network. This model is also represented through a combination of polynomial equations and uses Groebner bases to reach a new simplified network equivalent to the given network, which may make studying the ability to solve the problem of network coding less expensive and much easier.

Keywords: Groebner bases, Network Coding, Resultant.

Introduction:

Communication is the exchange of information between individuals by different means of transmission. The simplest communication system can consist of an information source, and a receiver and the link between them is called a communication channel, which can be a wire or wireless or the air range in which the electromagnetic waves propagate between the source and the receiver.

A communications network consists of a set of source nodes. Each node generates a symbol or set of symbols taken from a finite field, as well as a set of downstream nodes, in addition to a set of internal nodes. These nodes are linked to each other through a set of channels so that each channel transmits a specific amount of data called the channel capacity.

The concept of network coding emerged as an important field of study in the research presented by Ahlswede, Cai Li, and Yeung, [1]. This paper has shown that it is possible to accomplish the network coding if and only if the coding vectors for each channel in the network are linearly independent.

Robertet al.found that it is possible to find a solution to the network coding problem using linear coding, but [3] R. Dougherty, C. Freiling, and K. Zeger proved that the linear coding was not sufficient to solve the problem, so nonlinear coding was used, and several studies have shown the

importance of this type of coding in information theory.

Network coding is an area of research created in papers from the late 1990s to early the second millennium [4] [5]. However, the concept of network cipher, and especially linear network encryption, appeared much earlier in 1978 [6]

The communication network contains a set of source nodes so that each node generates a symbol from its finite field. These symbols are called data units, and it also includes a set of downstream nodes, a set of internal nodes. These nodes are linked with each other through a group of channels so that each channel transmits a specific amount of data called channel capacity.

If the communications network contains a single source node and a set of downstream nodes that are asking for data generated in this source node, then the connection problem over this network is called a multicast transmission [7] while, it is called Intersession network coding [8] if it contains two source nodes and two downstream nodes such that each downstream node requests the symbols generated in one of the source nodes.

The coding problem is solvable if and only if all the target nodes can get the message M using only the information they received; otherwise, it is not solvable.

Open Access
Published Online First: July 2021

Baghdad Science Journal
2022, 19(1): 98-104

P-ISSN: 2078-8665
E-ISSN: 2411-7986

Definition 1: polynomial map [9]
Let K be a field. The polynomial map is a map of the form
$$F = (f_1, \dots, f_n): K^n \to K^n$$
$$(a_1, \dots, a_n) \mapsto (f_1(a_1, \dots, a_n), \dots, f_n(a_1, \dots, a_n)).$$
Where each f_i is an element of the ring $K[X] = K[x_1, \dots, x_n]$.
Linear maps are the simplest example of polynomial maps, where
$$f_i(x_1, \dots, x_n) = a_{i1}x_1 + \cdots + a_{in}x_n : a_{ij} \in K.$$

Definition 2: [9]
The Jacobian polynomial of $f(x, y), g(x, y)$ with coefficients from a field K is the determinant of the form:
$$\det J(f, g) = \begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix}.$$

Definition 3: [9]
The ideal I of a ring $\mathcal{R}$ is defined as a non-empty subset that achieves:
1- I is an additive subgroup of $\mathcal{R}$ with $(+)$.
2. Whatever $a \in I$ and $r \in \mathcal{R}$ then $r.a \in I$.

Definition 4: [9]
For $S \subseteq \mathcal{R}$ a non-empty subset, the ideal I generated by a set S has the form:
$$I = \left\{ \sum_1^m r_i s_i : r_i \in \mathcal{R}, s_i \in S \right\}.$$
If $S = \{s_1, \dots, s_m\}$ is a finite set, then the ideal I is finitely generated and write $I = \langle s_1, \dots, s_m \rangle$. Also, the ideal is generated by the set $S \subseteq \mathcal{R}$, which can be expressed as the intersection of all ideals in $\mathcal{R}$. Each of them contains the ideal I.

Theorem 1. (Hilbert Basis Theorem): [9]
Every ideal $I \subset K[x_1, \dots, x_n]$ has a finite generating set. In other words, given an ideal I, there exists a finite collection of polynomials $g_1, \dots, g_t \in K[x_1, \dots, x_n]$ such that $I = \langle g_1, \dots, g_t \rangle$.

Definition 5: [10]
Let $f, g \in K[x_1, \dots, x_n]$ be a nonzero polynomials, fix a monomial order and let
$LT(f) = cx^\alpha$, $LT(g) = dx^\beta$ where $c, d \in K$ and $\alpha = (\alpha_1, \dots, \alpha_n), \beta = (\beta_1, \dots, \beta_n)$:
α_i, β_j are positive integers $\forall i, j = 1, \dots, n$. Let x^γ be the least common multiple of x^α, and x^β where $\gamma = (\gamma_1, \dots, \gamma_n): \gamma_i$ positive integer, then:
The *S-polynomial* of f and g is the polynomial
$$S(f, g) = \frac{x^\gamma}{LT(f)} \cdot f - \frac{x^\gamma}{LT(g)} \cdot g.$$

Example 1 [9]
Let $\begin{cases} f = x^3 y^2 - x^2 y^3 + x \\ g = 3x^4 y + y^2 \end{cases}$ be polynomials from $\mathbb{R}[x, y]$, $x >_{\text{grlex}} y$
grlex *is the* **Graded Lexicographic Order** .
Then

$LM(f) = x^3 y^2$, $LM(g) = x^4 y$ $\Rightarrow$ Lcm
$$= x^4 y^2 \Rightarrow \gamma = (4,2)$$
$$S(f, g) = \frac{x^4 y^2}{x^3 y^2} \cdot f - \frac{x^4 y^2}{3x^4 y} \cdot g = x.f - \frac{1}{3}.y.g.$$

Definition 6: [9]
Let K be a field, and let $f, h, f_1, \dots, f_s \in K[x_1, \dots, x_n]$ be polynomials where $f_i \neq 0, (1 \leq i \leq s)$ and $F = \{f_1, \dots f_s\}$, then f is reduced to h (via F), and denoted by $f \xrightarrow[+]{F} h$ if and only if there is a sequence of indexes $i_1, \dots, i_t \in \{1, \dots, s\}$ and a sequence of polynomials $h_1, \dots, h_{t-1} \in K[x_1, \dots, x_n]$ such that:
$$f \xrightarrow{f_{i1}} h_1 \xrightarrow{f_{i2}} h_2 \xrightarrow{f_{i3}} \dots \xrightarrow{f_{it}} h$$

Definition 7: [10]
Let I be an ideal in polynomial ring $[x_1, \dots, x_n]$, and $G = \{g_1, \dots, g_t\}$ be a finite subset from I then G is a Groebner basis for I if and only if
$$\langle LT(g_1), \dots, LT(g_t) \rangle = \langle LT(I) \rangle :$$
$$LT(g_i) \text{ is the Leading Term of } g_i.$$
Or, equivalently G is a Groebner basis for an ideal I if and only if the Leading Term of any element of I is divisible by one of the terms $LT(g_i)$; $i = 1, \dots, t$.

Theorem 2. (Buchberger's criterion): [9]
Let I be an ideal in $K[x_1, \dots, x_n]$ and $G = \{g_1, \dots, g_t\}$ a generated set of I then G is a Groebner basis for I if and only if $(g_i, g_j) \xrightarrow{G} 0; \forall i \neq j$.

Definition 8. (Resultant): [9]
The Resultant is defined with respect to n homogeneous polynomials $F_1, \dots, F_n$ in n variables, of degrees $l_1, \dots, l_n$ each polynomial being full in all its terms with literal coefficients $F_i = \sum_{|\alpha|=l_i} u_{i,\alpha} X^\alpha$ for $i = 1, \dots, n$. Then the Resultant of n given homogeneous polynomials in n variables is a unique polynomial $Res \in \mathbb{Z}[u_{i,\alpha}]$ which has the following properties:
i. The equations $F_1 = \cdots = F_n = 0$ have a non-trivial solution over $\mathbb{C}$ if and only if $Res(F_1, \dots, F_n) = 0$.
ii. $Res(x_1^{l_1}, \dots, x_n^{l_n}) = 1$.
iii. Res is irreducible, even when regarded as a polynomial in $\mathbb{C}[u_{i,\alpha}]$.
For more details about this polynomial see. [11]

A mathematical model for the transmission problem in a communication network
Here the network transmission problem is defined to be the septuplet $(G, \Sigma, C, M, S, R, F)$ where
• $G = (V, E)$ is a directed graph.
• Σ is a given alphabet.
• C is the set of capacities of communication channels in the network.

Open Access
Published Online First: July 2021

Baghdad Science Journal
2022, 19(1): 98-104

P-ISSN: 2078-8665
E-ISSN: 2411-7986

- $M = (m_1, \dots, m_n)$ message from the dimension n on the alphabet Σ.
- S is the set of sources.
- $R \subseteq V$ is the set of receivers.
- $F = (f_1, \dots, f_{|E|})$ is the set of encoding functions associated with each link.

Graph G represents a communications network where routers or computers are represented by nodes, and communication channels are represented by links.

The message M is generated in the source S and must be transferred to all target nodes in R. Also an encoding function f_e related to link $e = (v, u)$ is defined as follows:

$$f_e = \begin{cases} \Sigma^{|M|} \to \Sigma \,, if\ v = S \\ \Sigma^{|E_I(v)|} \to \Sigma \,, if\ v \neq S \end{cases}.$$

Where $E_I(v)$ is the set of links entering the node v. Put (S_i, R_j) ; $1 \leq i \leq h$, the h edge-disjoint paths from the sources to receiver R_j, $1 \leq j \leq N$. Links will be carrying linear combinations of their father node inputs, and the set $\{\alpha_k\}$ denotes the coefficients used in these linear combinations. Put ρ_i^j to refer to the symbol on the last link of the path

(S_i, R_j). Therefore, receiver R_j has to solve the following system of equations:

$$\begin{bmatrix} \rho_1^j \\ \vdots \\ \rho_h^j \end{bmatrix} = C_j \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_h \end{bmatrix}.$$

Where C_j are $h \times h$ matrices which are the *receiver transfer matrices* [12].
Note that the elements of C_j are polynomials in $\{\alpha_k\}$.

Example 2: [12]
Consider a network with two sources and three receivers, as in (Fig. 1). Note that there is two edge disjoint paths from the sources to each receiver (Fig. 1a).
Therefore, each receiver can receive the information from both sources when using the network alone. However, when all three receivers use the network at the same time, then the intersections between paths at BD and GH have to be resolved. In (Fig. 1b), the nodes linearly combine their inputs at BD and GH, and the receivers observe linear combinations of the source symbols determined by matrices C_i.
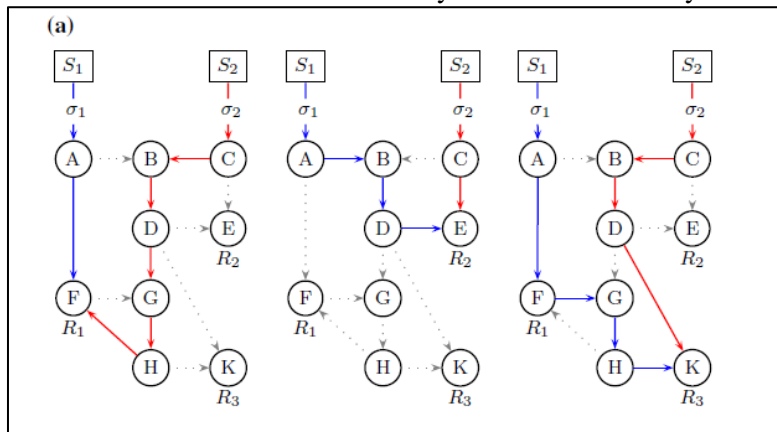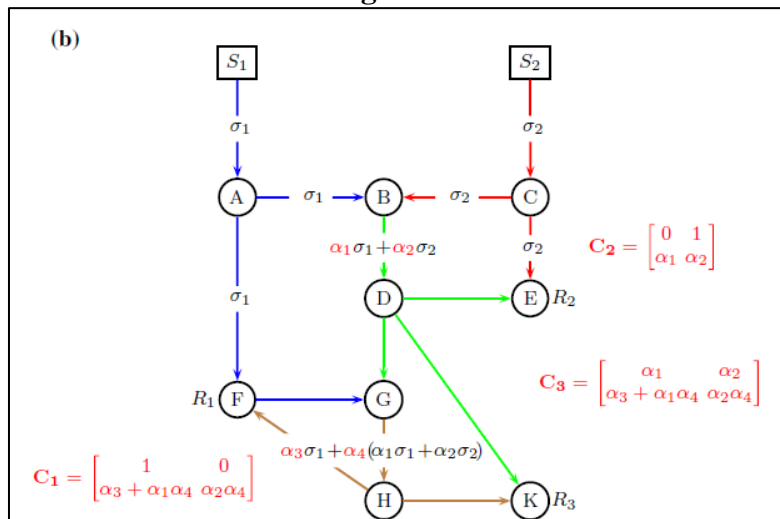

Figure 1.a

Figure 1.b

The main theorem in network coding [12]
Theorem 3:

Consider a directed graph without circles with unit-capacity edges, h unit-rate information sources and N receivers, such that there are h edge-disjoint paths from the sources to all receivers. Then there exists a multicast transmission scheme over a large enough finite field F_q , in which intermediate network nodes linearly combine their incoming information symbols over F_q, that delivers the information from the sources simultaneously to each receiver at a rate equal to h.

An equivalent expression of the main theorem:
The source S_i transmits symbol σ_i , which is an element of some finite field F_q. Since each node can linearly combine its inputs, each network link carries a linear combination of its father node inputs. So, links carry linear combinations of source symbols σ_i, and a receiver can recapture the source information if the h links it observes carry independent linear combinations of the σ_i.

Theorems

This paragraph presents our contributions to find the necessary and sufficient conditions under which the nodes can combine their inputs which guarantee the ability to solve the problem of multicast transmission.

Theorem 4:
Let K be a finite field, and let $f_1, \dots, f_n \in K[x_1, \dots, x_n]$ be of positive degrees in variables $x_1, \dots, x_{n-1}$, then the following are equivalent:
1. $\det(J(f_1, .., f_n)) = 0$.
2. There is a nonzero polynomial $h(u_1, \dots, u_n) \in K[u_1, \dots, u_n]$ such that $h(f_1, \dots, f_n) = 0$.

Proof
$2 \to 1$

Let $h(u_1, \dots, u_n)$ be of smallest possible positive degree so that $h(f_1, \dots, f_n) = 0$. Then calculating partial derivatives of $h(f_1, \dots, f_n)$ gives that

$$\frac{\partial}{\partial x_1} h(f_1, \dots, f_n) = \frac{\partial h}{\partial u_1}\frac{\partial f_1}{\partial x_1} + \cdots + \frac{\partial h}{\partial u_n}\frac{\partial f_n}{\partial x_1},$$
$$\vdots$$
$$\frac{\partial}{\partial x_n} h(f_1, \dots, f_n) = \frac{\partial h}{\partial u_1}\frac{\partial f_1}{\partial x_n} + \cdots + \frac{\partial f_n}{\partial x_n}\frac{\partial h}{\partial u_n},$$
the matrix form of this system is

$$\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_1}{\partial x_n} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \begin{pmatrix} \frac{\partial h}{\partial u_1} \\ \vdots \\ \frac{\partial h}{\partial u_n} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Then the minimality property implies that
$\frac{\partial h}{\partial u_i}(f_1, \dots, f_n) \neq 0$ $for\ some$ $1 \leq i \leq n$, thus $\det(J(f_1, .., f_n)) = 0$.

1 → 2
By contradiction, for $i = 1, \dots, n$ the resultant $(x_i, f_1, \dots, f_n) = 0$.
Where
$Res_{x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n}(f_1 - u_1, \dots, f_n - u_n) = R(x_i, u_1, \dots, u_n)$

$$= R_{k_i}(u_1, \dots, u_n)x_i^{k_i} + \cdots + R_0(u_1, \dots, u_n).$$
See that there exist polynomials $B_i(x_i, u_1, \dots, u_n)$ of smallest possible positive degrees in $x_1, \dots, x_n$ respectively, so that $B_1(x_1, f_1, \dots, f_n) = \cdots = B_n(x_n, f_1, \dots, f_n) = 0$ then

$$\begin{pmatrix} \frac{\partial B_1}{\partial u_1}(x_1, f_1, \dots, f_n) & \cdots & \frac{\partial B_1}{\partial u_n}(x_1, f_1, \dots, f_n) \\ \vdots & \ddots & \vdots \\ \frac{\partial B_n}{\partial u_1}(x_n, f_1, \dots, f_n) & \cdots & \frac{\partial B_n}{\partial u_n}(x_n, f_1, \dots, f_n) \end{pmatrix} \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_1}{\partial x_n} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix},$$

$$= \begin{pmatrix} -\frac{\partial B_1}{\partial x_1}(x_1, f_1, \dots, f_n) & & 0 \\ & \ddots & \\ 0 & & -\frac{\partial B_n}{\partial x_n}(x_n, f_1, \dots, f_n) \end{pmatrix}.$$

But the minimality property implies that $\prod_i \frac{\partial B_i}{\partial x_i}(x_i, f_1, \dots, f_n) \neq 0$, and thus $\det(J(f_1, .., f_n)) \neq 0$.

Theorem 5:
Let K be a finite field, and $f_1, \dots, f_n \in K[x_1, \dots, x_n]$ of positive degrees in variables $x_1, \dots, x_{n-1}$ Then the polynomials $f_1, \dots, f_n$ are algebraically independent over the field K if $\det(J(f_1, .., f_n)) \neq 0$.

Proof
Since $\det J(f_1, \dots, f_n) \neq 0$ the polynomials that define the map $F = (f_1, \dots, f_n)$ are algebraically independent over K . Because in the power series ring since the Jacobian determinant does not vanish, the opposite of F exists and is uniquely determined, i.e. there is $G \in K[[X]]^n; X = (x_1, \dots, x_n)$ where $G = (g_1, \dots, g_n): g_i \in K[[X]]$ for $1 \leq i \leq n$: $f_i(G) = x_i$. For every $1 \leq i \leq n$, which means that G is the opposite of F (not necessarily polynomial in the general case).
To prove the independence, suppose as in (theorem 4) the existence of $h \in K[X]$ such that $h(f_1, \dots, f_n) = 0$. Then $0 = h(f_1(G), \dots, f_n(G)) =$

Open Access
Published Online First: July 2021

Baghdad Science Journal
2022, 19(1): 98-104

P-ISSN: 2078-8665
E-ISSN: 2411-7986

$h(x_1, \dots, x_n) = h,$ which means that the polynomials $f_1, \dots, f_n$ are algebraically independent of K.

Simplify communications network

The source sends a copy of the data. It generates to each of the downstream nodes. (Fig. 2) shows the transmission of the symbols b_1, b_2 from the source nodes to the target node R_1, R_2
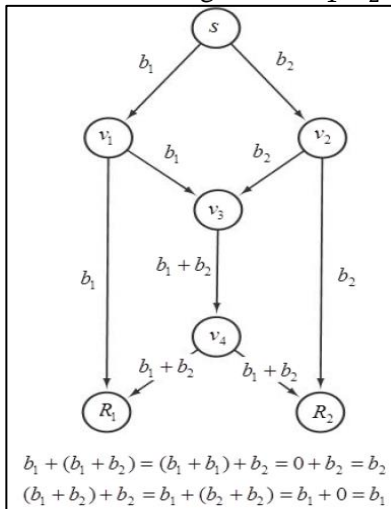

$b_1 + (b_1 + b_2) = (b_1 + b_1) + b_2 = 0 + b_2 = b_2$
$(b_1 + b_2) + b_2 = b_1 + (b_2 + b_2) = b_1 + 0 = b_1$

Figure 2

Construction algorithm

1- Choose the channels so that the same amount of data will flow through them

2- Form partial networks so that the flow through the channels of each sub network is the same amount of data

3- Represent every partial network by two nodes, and a channel whose capacity is the available capacity of the channels of the considered sub-network

4- Call the node at the beginning of the channel, a distributor (HOP).

Note: assume that every HOP distributor receives messages from the source node and can process and forward the messages.

5- Represent the set of channels in the original network that connects a node from one subnet to a node from another subnet, with a channel from the node in the first subnet to the distributor in the second subnet (and that is in the new network).

Application 1:

Applying the algorithm to the previous network in (Fig. 2), gives the network that is shown in (Fig. 3).
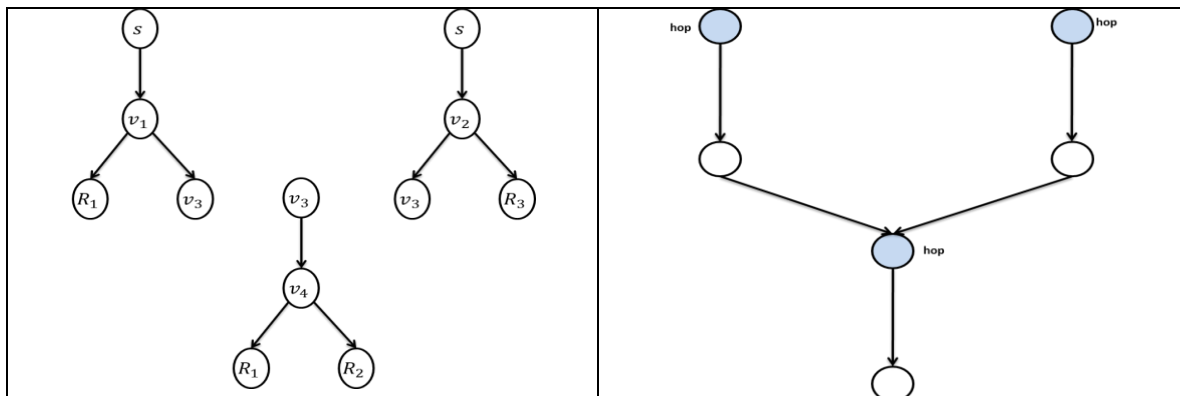

Figure 3

Representation of the new network by a set of polynomial equations:

This paragraph introduces an algorithm for representing the network through a set of polynomial equations:

1- For each node v_i put a variable x_i.

2- Each HOP (denote it by H_i) has a polynomial form

$$h_i = x_1^{a_1} x_2^{a_2} \dots x_n^{a_n} - x_1^{b_1} x_2^{b_2} \dots x_n^{b_n} \quad : n = |V|.$$

Where

$$a_i = \begin{cases} w\big((v_i, H_j)\big) & if \ (v_i, H_j) \in E \\ 0 & Otherwise \end{cases}$$

$$b_i = \begin{cases} w\big((H_j, v_i)\big) & if \ (H_j, v_i) \in E \\ 0 & Otherwise \end{cases}$$

$w\big((v_i, H_j)\big)$ Link weight representing the capacity of the channel (v_i, H_j).

Application 2:

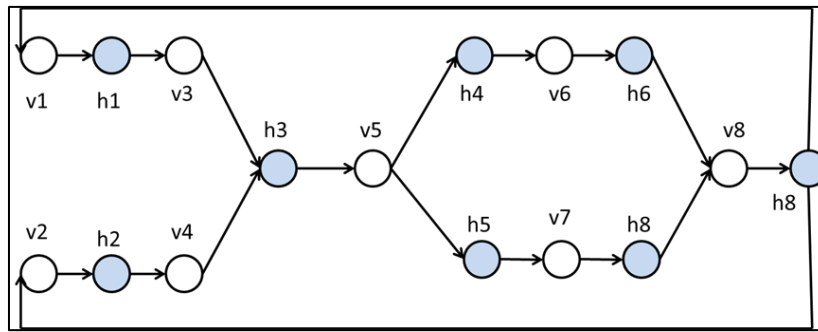Consider the network as in (Fig. 4)

Figure 4

The set of polynomials that express the distributor node are as follows

$$F = \{x_1 - x_3, x_2 - x_4, x_3x_4 - x_5, x_5 - x_6, x_5 - x_7, x_6 - x_8, x_7 - x_8, x_8 - x_1x_2\}.$$

Calculating the corresponding Groebner basis gives the set

$$G = \{x_1 - x_3, x_2 - x_4, x_3x_4 - x_8, x_5 - x_8, x_6 - x_8, x_7 - x_8\}.$$

(Theorem 6 proves that this basis always has this shape).

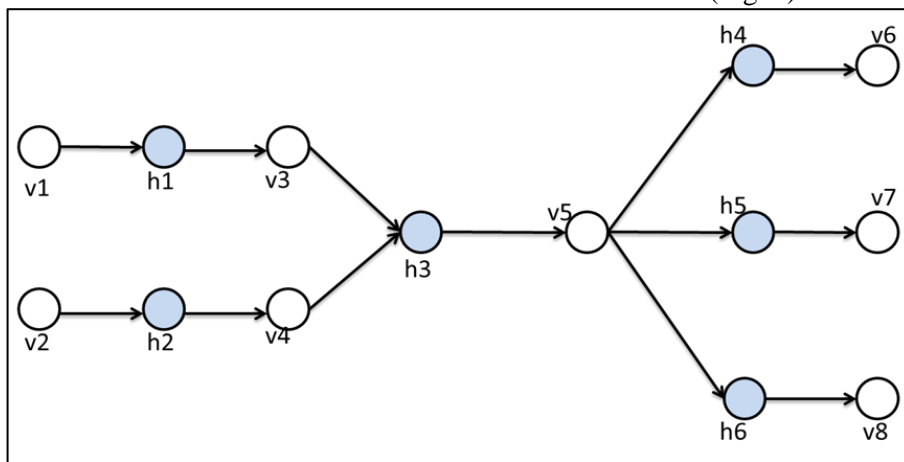The new network corresponding to the Groebner basis will be as in (Fig. 5).

Figure 5

Theorem 6:

Assuming that F is a set of polynomials of the form $f_i = m_i^1 - m_i^2$ where m_i^1, m_i^2 are monomials, then the polynomials in Groebner basis of F for some monomials order will have the same form.

Proof:

This proof shows that the polynomials obtained by computing Groebner basis for F are of the form $f_i = m_i^1 - m_i^2$ where m_i^1, m_i^2 are monomials. Lets take the S - polynomials of the pairs f_i, f_j

$$S(f_1, f_2) = u_1.(m_i^1 - m_i^2) - u_2.(m_j^1 - m_j^2).$$

Where $u_1.m_i^1 = u_2.m_j^2$ then $S(f_1, f_2) = -u_1.m_i^1 + u_2.m_j^2$.

Which is either a zero polynomial or from the desired form, also the reduction maintains this form, this is a direct consequence because reduction through polynomials of the desired form is similar to the computation of S - polynomials where $u_1 = 1$. This ends the proof.

The main result

Theorem 7:

It is now possible to define the equivalence between two communication networks as follows. The communication network N_1 is equivalent to the network N_2 if and only if the ideal I_1 generated by polynomials that define the distributor nodes in N_1, equal to I_2 which is the ideal generated by the polynomials that define the distributor nodes in N_2.

Proof - Depending on the properties of Groebner basis

Assuming G_1 is the reduced Groebner basis corresponding to the ideal I_1 is generated by polynomials that define the distributor nodes of the network N_1. And G_2 be the reduced Groebner basis corresponding to the ideal I_2 generated by polynomials that define the distributor nodes of the network N_2.

According to the properties of Groebner basis [3] the ideals I_1 and I_2 are equal if and only if $G_1 = G_2$.

Thus, the equivalence between communications networks can be easily studied.

Conclusion:

This paper introduces a very important application to the solvability problem of network coding using tools from algebraic geometry by building a simplified network using Groebner basis.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Damascus University.

Authors' contributions statement:

AL Jony: did the study within the scope of a PhD research project at Damascus University
SH Rashed: Supervisor of this work, Scientific audit.

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تطبيق قواعد غروبنر في دراسة نظام اتصالات

شـوقي الراشد   علاء جوني

قسم الرياضيات، كلية العلوم، جامعة دمشق، سوريا

الخلاصة:

قمنا في هذا البحث بإيجاد علاقة بين استقلالية متعددات الحدود المرافقة لإضلاع الشبكة وبين انعدام محدد اليعقوبي. قمنا أيضاً بتقديم طريقة لتبسيط شبكة اتصالات معطاة عبر خوارزمية تقوم بتجزئة الشبكة إلى شبكات جزئية وإعادة دمج هذه الشبكات للوصول إلى شبكة هي عبارة عن تمثيل أو نموذج عام للشبكة قيد الدراسة. كما قمنا بتمثيل هذا النموذج عبر جملة معادلات متعددات حدود والاستفادة من قواعد غروبنر للوصول إلى شبكة جديدة مبسطة مكافئة للشبكة المعطاة مما يجعل دراسة قابلية حل مسألة ترميز الشبكة أقل كلفة.

الكلمات المفتاحية: قواعد غروبنر، ترميز الشبكه، الناتج.