DOI: http://dx.doi.org/10.21123/bsj.2021.18.1(Suppl.).0836

Approximate Analytical Solutions of Bright Optical Soliton for Nonlinear Schrödinger Equation of Power Law Nonlinearity

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Received 10/12/2020, Accepted 11/3/2021, Published 30/3/2021

Abstract:

This paper introduces the Multistep Modified Reduced Differential Transform Method (MMRDTM). It is applied to approximate the solution for Nonlinear Schrodinger Equations (NLSEs) of power law nonlinearity. The proposed method has some advantages. An analytical approximation can be generated in a fast converging series by applying the proposed approach. On top of that, the number of computed terms is also significantly reduced. Compared to the RDTM, the nonlinear term in this method is replaced by related Adomian polynomials prior to the implementation of a multistep approach. As a consequence, only a smaller number of NLSE computed terms are required in the attained approximation. Moreover, the approximation also converges rapidly over a wide time frame. Two examples are provided for showing the ability and advantages of the proposed method to approximate the solution of the power law nonlinearity of NLSEs. For pictorial representation, graphical inputs are included to represent the solution and show the precision as well as the validity of the MMRDTM.

Key words: Adomian polynomials, Multistep approach, Multistep Modified Reduced Differential Transform Method, Nonlinear Schrodinger equations of power law nonlinearity.

Introduction:

The nonlinear nature of the system is vital especially for understanding numerous natural phenomena. Recently, nonlinear science has arisen as a prevailing field to elucidate the complexities of the challenging nature. Nonlinearity is an interesting phenomenon in nature that has been well known in a number of areas, ranging from chemical, biological, geological, fluids to solid as well as nonlinear optical processes in the form of large amplitude waves or high-intensity laser pulses. This interesting topic has developed in almost every field of science and its applications percolate across science. Generally, nonlinear phenomena are regularly modeled by nonlinear evolution equations with a broad range of high complexities in terms of various linear and nonlinear effects (1-5).

Schrödinger equations arise in many fields of physics, nonlinear optics, superconductivity, plasma physics, and quantum mechanics. On the other hand, the nonlinear Schrödinger equations (NLSEs) serve as a critical function in various areas of engineering, biological and physical science. These equations are applicable in some applied fields such as protein chemistry, plasma physics, nonlinear optics, and liquid dynamics (6). Countless partial differential equations (PDEs) and ordinary differential equations (ODEs) have been solved by using approximate analytical methods such as Differential Transform Method (DTM) and the Reduced Differential Transform Method (RDTM) (7–10).

For solving NLSEs of power law nonlinearity, Wazwaz applied the variational iteration method (11). Later, introduced the residual power series method (RPSM) and homotopy analysis transform method (HATM) to approximate the solutions of Schrödinger equation of power law nonlinearity (12). Other than that, Kilic & Inc presented dispersive optical solitons with the nonlinearity of power law governed by the Schrödinger-Hirota equation (SHe)(13). Moreover, the higher-order NLSE with the non-Kerr nonlinear term is investigated in distinct analytical and semi-analytical solutions by Khater et al. (14).

Apart from that, Ray proposed a modification on the fractional RDTM and implemented it to find solutions of fractional Korteweg-de Vries equations (15). In this approach, the adjustment embraces the substitution of the nonlinear term by relating Adomian polynomials. As a result, the solutions of the nonlinear problem can be obtained in a simpler way with reduced calculated terms. Later, El-Zahar introduced an adaptive multistep DTM to obtain the solution of singular perturbation initial-value problems. It produces a solution in a rapidly convergent series that is converging in a wide time area (16).

Multistep Modified Reduced Differential Transform Method (MMRDTM) for solving NLSEs is proposed by Che Hussin et. al (17). By using this method, the solutions of NLSE can be approximated with high accuracy. In addition, the MMRDTM also was experimentally tested to approximate the Klein-Gordon equations where the obtained results showed that the MMRDTM is an effective and precise method (18). Later, Che Hussin et al. applied the MMRDTM to obtain the approximate solution of fractional NLSEs (19). Recently, Che Hussin et al. solved the nonlinear KdV equation by using MMRDTM (20). The approximation results are obtained with a smaller number of computed terms. On top of that, the results also converge over a large time frame in a shorter time.

In this paper, the modification by implementing Adomian polynomials and the multistep approach are combined to execute the MMRDTM for solving NLSEs of power law nonlinearity. In addition, we apply parametrization methods for generating Adomian polynomials in which the algorithm doesn't require tedious calculations of high derivatives (21). The proposed technique has the ability to rapidly yielding a fast-convergent sequence of analytical approximation. As a result, the solutions converge in a wide time area. At the same time, the number of computed terms is also significantly reduced.

Methods:

The Development of Multistep Modified Reduced Differential Transform Method

For notation purpose, the original functions are denoted using lowercase letter such as the letter u in the function $u(x,t)$, while the transformed functions are denoted using uppercase letter such as the letter U in the function $U_k(x)$. Basically, the differential transformation of the function $u(x,t) = f(x)g(t)$ is obtained as follows (22),

$$u(x,t) = \sum_{i=0}^{\infty} F(i)x^i \sum_{j=0}^{\infty} G(j)t^j = \sum_{k=0}^{\infty} U_k\,(x)t^k$$

where $U_k(x)$ is known as the function of $u(x,t)$. The following definitions describe some fundamental properties of RDTM:

Definition 1: For an analytically and continuously differential function $u(x,t)$ with respect to time t and space variable x, the differential transformation of $u(x,t)$ is defined by,

$$U_k(x) = \left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0} \qquad (1)$$

where $U_k(x)$ is the transformed function.

Definition 2. The inverse transform of $U_k(x)$ is given by,

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k. \qquad (2)$$

By combining equations (1) and (2), the following equation is obtained,

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0} t^k. \qquad (3)$$

To signify the core features of the RDTM, consider the following nonlinear PDE,

$$Du(x,t) + Pu(x,t) + Qu(x,t) = h(x,t), \qquad (4)$$

where $u(x,\ 0) = f(x)$ is the initial condition. Note that, $D = \frac{\partial}{\partial t}$ and P is the remaining part of a linear operator. The nonlinear and inhomogeneous terms are represented as $Nu(x,t)$ and $h(x,t)$ respectively.

Based on the MMRDTM, the iteration formula can be formed as follows:

$$(k+1)U_{k+1}(x) = H_k(x) - PU_k(x) - NU_k(x). \quad (5)$$

The functions $Du(x,t), Pu(x,t), Nu(x,t)$ and $h(x,t)$ are transformed and then represented as $U_k(x), PU_k(x), NU_k(x)$ and $H_k(x)$ respectively. We have

$$U_0(x) = f(x), \qquad (6)$$

from the initial condition. Referring to Ray, the nonlinear term is denoted as follows (15),

$$Nu(x,t)=\sum_{n=0}^{\infty}A_n\big(U_0(x),U_1(x),\dots,U_n(x)\big).$$

Recently, Kataria and Vellaisamy proposed a novel method for calculating the Adomian polynomials (21),

$$A_0=N\big(U_0(x)\big),$$

$$A_n(U_0(x),U_1(x),\dots,U_n(x))=\frac{1}{2\pi}\int_{-\pi}^{\pi}N\left(\sum_{k=0}^{n}U_k(x)e^{ikx}\right)e^{-in\lambda}\,d\lambda,\quad n\geq 1.$$

It can be observed that the algorithm does not involve tedious calculations with high derivatives. By combining equations (6) and (5), the $U_k(x)$ values can be obtained through iterative calculation. Furthermore, the set of values $\{U_k(x)\}_{k=0}^{n}$ of the inverse transformation produces the approximate solution as follows,

$$u(x,t)=\sum_{k=0}^{K}U_k(x)t^k,\qquad t\in[0,T].$$

For $m=1,2,\dots,M$, divide the interval $[0,T]$ is into M subintervals $[t_{m-1},t_m]$ by equal step size $s=\frac{T}{M}$ and nodes $t_m=ms$ The following steps are used to calculate MMRDTM. Firstly, apply modified RDTM to the initial value problem of interval $[0,t_1]$. Then by using the initial conditions

$$u(x,0)=f_0(x),\qquad u_1(x,0)=f_1(x),$$

the approximate result

$$u_1(x,t)=\sum_{k=0}^{K}U_{k,1}(x)t^k,\qquad t\in[0,t_1]$$

is obtained. At each subinterval $[t_{m-1},t_m]$, the initial conditions

$$u_m(x,t_{m-1})=u_{m-1}(x,t_{m-1}),$$

$$(\partial/\partial t)u_m(x,t_{m-1})=(\partial/\partial t)u_{m-1}(x,t_{m-1})$$

are used for $m\geq 2$ and the multistep RDTM is implemented to the initial value problem on $[t_{m-1},t_m]$, where t_0 is replaced with t_{m-1}. To produce a sequence of approximate solutions $u_m(x,t)$ the step is performed and carried out repeatedly for $m=1,2,\dots,M$, such as,

$$u_m(x,t)=\sum_{k=0}^{K}U_{k,m}(x)(t-t_{m-1})^k,\qquad t\in[t_{m-1},t_m].$$

Finally, MMRDTM proposes the following solutions:

$$u(x,t)=\begin{cases}u_1(x,t),\text{for } t\in[0,t_1]\\ u_2(x,t),\text{for } t\in[t_1,t_2]\\ \vdots\\ u_M(x,t),\text{for } t\in[t_{M-1},t_M].\end{cases}$$

With better computing performance, the new algorithm MMRDTM is straightforward for all values of s. Note that, the MMRDTM reduces to the modified RDTM once the step size $s=T$.

Application of MMRDTM For Solving Nonlinear Schrödinger Equation of Power Law Nonlinearity

Consider the NLS equation of power law nonlinearity (12),

$$iu_t+u_{xx}+\gamma|u|^{2r}u=0\qquad (7)$$

subject to the initial condition,

$$u(x,0)=[2(r+1)\,\text{sech}^2(2rx)]^{1/(2r)},\quad r\geq 1.$$

The exact solution is

$$u(x,t)=[2(r+1)\,\text{sech}^2(2rx)]^{1/(2r)}e^{4it}.$$

Applying MMRDTM to (7) and using basic properties of MMRDTM, the following equation

$$U_{k+1,m}(x)=\left(\frac{I}{k+1}\right)\left(\frac{\partial^2}{\partial x^2}\big(U_{k,m}(x)\big)+\gamma(A_{k,m})\right).\qquad (8)$$

is obtained. From the initial condition, write

$$U_0(x)=f(x).\qquad (9)$$

Referring to Ray the nonlinear term is denoted as follows (15),

$$Nu(x,t)=\sum_{n=0}^{\infty}A_n\big(U_0(x),U_1(x),\dots,U_n(x)\big).$$

By replacing equation (9) into equation (8), the $U_k(x)$ values can be obtained by direct iterative computation. Then, the n-terms approximation solution is obtained from the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$ as given below,

$$u(x,t)=\sum_{k=0}^{K}U_k(x)t^k,\qquad t\in[0,T].$$

For $m=1,2,\dots,M$, divide the interval $[0,T]$ into M subintervals $[t_{m-1},t_m]$ by equal step size $s=\frac{T}{M}$ and nodes $t_m=ms$. The following steps are used to calculate MMRDTM. Firstly, apply modified RDTM to the initial value problem of interval $[0,t_1]$. Then by using the initial conditions,

$$u(x,0) = f_0(x), \qquad u_1(x,0) = f_1(x),$$

the approximate result

$$u_1(x,t) = \sum_{k=0}^{K} U_{k,1}(x)t^k, \qquad t \in [0, t_1]$$

is obtained. Then the multistep RDTM is implemented to the initial value problem on $[t_{m-1}, t_m]$, where t_0 is replaced with t_{m-1}. To produce a sequence of approximate solutions $u_m(x,t)$, the step is performed and carried out repeatedly for $m = 1,2,\dots,M$ such as,

$$u_m(x,t) = \sum_{k=0}^{K} U_{k,m}(x)(t - t_{m-1})^k, \qquad t \in [t_{m-1}, t_m].$$

Numerical Results and Discussion:

We provide the following examples to demonstrate the robustness of the proposed method. For accuracy evaluation purposes, comparisons between the solutions obtained by the MMRDTM, exact solutions, and solutions obtained by the MRDTM are made.

Example 1

Consider the one-dimensional NLSE of power law nonlinearity for $\gamma = 2$ and $r = 3/2$,

$$u_t + u_{xx} + 2|u|^3 u = 0 \qquad (10)$$

subject to the initial condition

$$u(x,0) = [5\,\text{sech}^2(3x)]^{1/3}.$$

The exact solution of this equation is $[5\,\text{sech}^2(3x)]^{1/3} e^{4it}$.

By applying the MMRDTM to equation (10) and using basic properties of MMRDTM, we have

$$U_{k+1,m}(x) = \left(\frac{I}{k+1}\right)\left(\frac{\partial^2}{\partial x^2}\left(U_{k,m}(x)\right) + 2(A_{k,m})\right). \qquad (11)$$

From the initial condition, write

$$U_0(x) = [5\,\text{sech}^2(3x)]^{1/3}. \qquad (12)$$

The comparison between exact solutions and approximation of MMRDTM and MRDTM are shown in Fig. 1. Observe that, MRDTM values diverge from the exact solution. From the results, Fig. 1 shows an approximation of MMRDTM and MRDTM with $\gamma = 2$ and $r = 3/2$ for $t \in [-1,1]$ and $x \in [-5,5]$. Table 1 and 2 summarize the performance error analysis obtained by MMRDTM with $\gamma = 2$ and $r = 3/2$ with $N = 5$ for MMRDTM and MRDTM respectively. Therefore, it reveals that the solutions of the current technique for this type of NLS equation with power law nonlinearity are closed to exact solutions compared to MRDTM.

Table 1. Comparison between MMRDTM and exact solution of NLSE with power law nonlinearity

t	x	Absolute Error MMRDTM	Absolute Error MRDTM
	0.01	1.0012×10^{-9}	3.3904×10^{-3}
	0.02	1.0000×10^{-11}	3.3786×10^{-3}
0.01	0.03	1.0004×10^{-9}	3.3591×10^{-3}
	0.04	1.0004×10^{-9}	3.3320×10^{-3}
	0.05	3.0000×10^{-11}	3.2975×10^{-3}
	0.01	1.0000×10^{-9}	1.3473×10^{-2}
	0.02	0.00000	1.3412×10^{-2}
0.02	0.03	0.00000	1.3324×10^{-2}
	0.04	1.00000×10^{-9}	1.3197×10^{-2}
	0.05	1.00000×10^{-10}	1.3038×10^{-2}
	0.01	1.0050×10^{-9}	3.1549×10^{-2}
	0.02	1.0000×10^{-9}	3.1277×10^{-2}
0.03	0.03	1.0050×10^{-9}	3.0840×10^{-2}
	0.04	2.0025×10^{-9}	3.0257×10^{-2}
	0.05	1.0000×10^{-9}	2.9555×10^{-2}
	0.01	1.0198×10^{-9}	6.4569×10^{-2}
	0.02	1.0000×10^{-10}	6.3351×10^{-2}
0.04	0.03	2.0000×10^{-10}	6.1403×10^{-2}
	0.04	1.0050×10^{-9}	5.8839×10^{-2}
	0.05	1.0000×10^{-10}	5.5806×10^{-2}
	0.01	1.0000×10^{-10}	1.3140×10^{-1}
	0.02	1.0440×10^{-9}	1.2705×10^{-1}
0.05	0.03	1.0770×10^{-9}	1.2011×10^{-1}
	0.04	1.1180×10^{-9}	1.1102×10^{-1}
	0.05	2.0000×10^{-10}	1.0036×10^{-1}

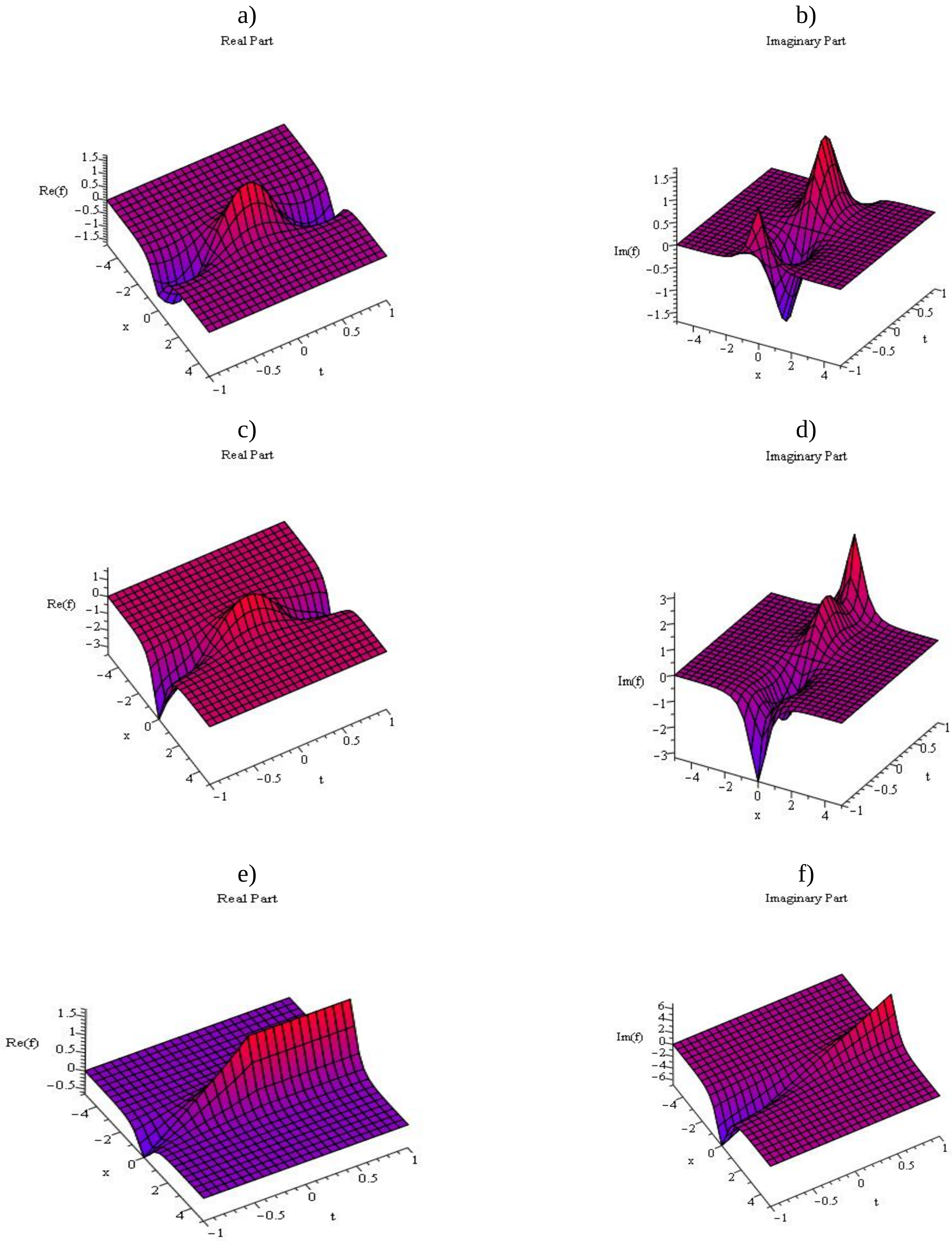

Figure 1. The surface graphs of NLS equation of power law nonlinearity for $\gamma = 2$ and $r = 3/2$, a)exact solution (Re), b)exact solution (Im), c)approximate solution with MMRDTM (Re), d) approximate solution with MMRDTM (Im), e) approximate solution with MRDTM (Re), f) approximate solution with MRDTM (Im).

Table 2. Comparison error of MMRDTM and MRDTM

t	x	Exact solution	Absolute Error MMRDTM	Absolute Error MRDTM
	0.1	1.660167981	9.4333×10^{-6}	5.6585×10^{-2}
	0.2	1.526619448	8.6752×10^{-6}	3.0265×10^{-1}
	0.3	1.345266791	7.6445×10^{-6}	1.8271×10^{-1}
	0.4	1.151063913	6.5407×10^{-6}	2.1254×10^{-1}
0.1	0.5	0.966751165	5.4936×10^{-6}	9.7276×10^{-2}
	0.6	0.803005295	4.5631×10^{-6}	3.7596×10^{-2}
	0.7	0.662757967	3.7660×10^{-6}	1.9579×10^{-2}
	0.8	0.545045162	3.0971×10^{-6}	1.0954×10^{-2}
	0.9	0.447344248	2.5418×10^{-6}	5.6815×10^{-3}
	1.0	0.366750671	2.0839×10^{-6}	2.7714×10^{-3}

Example 2

Consider the one-dimensional NLSE with power law nonlinearity for $\gamma = 2$ and $r = 2$ (12),

$$iu_t + u_{xx} + 2|u|^4 u = 0 \qquad (13)$$

subject to the initial condition,

$$u(x,0) = [6\,\mathrm{sech}^2(4x)]^{1/4}.$$

The exact solution of this equation is $[6\,\mathrm{sech}^2(4x)]^{1/4} e^{4it}$.

Using the basic properties of MMRDTM then applying MMRDTM to equation (13), we can obtain

$$U_{k+1,m}(x) = \left(\frac{I}{k+1}\right)\left(\frac{\partial^2}{\partial x^2}\left(U_{k,m}(x)\right) + 2(A_{k,m})\right) \qquad (14)$$

From the initial condition, write

$$U_0(x) = [6\,\mathrm{sech}^2(4x)]^{1/4}. \qquad (15)$$

From the results, Fig. 2 shows an approximation of MMRDTM and MRDTM for NLSE with power law nonlinearity with $\gamma = 2$ and $r = 2$ for $t \in [-1,1]$ and $x \in [-1,1]$. Table 3 summarizes the performance error analysis obtained by MMRDTM and MRDTM when $t = 0.1$. Table 4 summarizes the performance error analysis obtained by MMRDTM, MRDTM, RPS, and HATM with $\gamma = 2$ and $r = 2$ with $N = 5$. As we can see, the errors of MRDTM, RPS, and HATM increase as time increases while MMRDTM has a minor error. The shape of MMRDTM graphs looks similar to exact solutions. Therefore, it reveals that the solutions of the current technique for this type of NLS equation with power law nonlinearity are closed to exact solutions compared to MRDTM, RPS, and HATM.

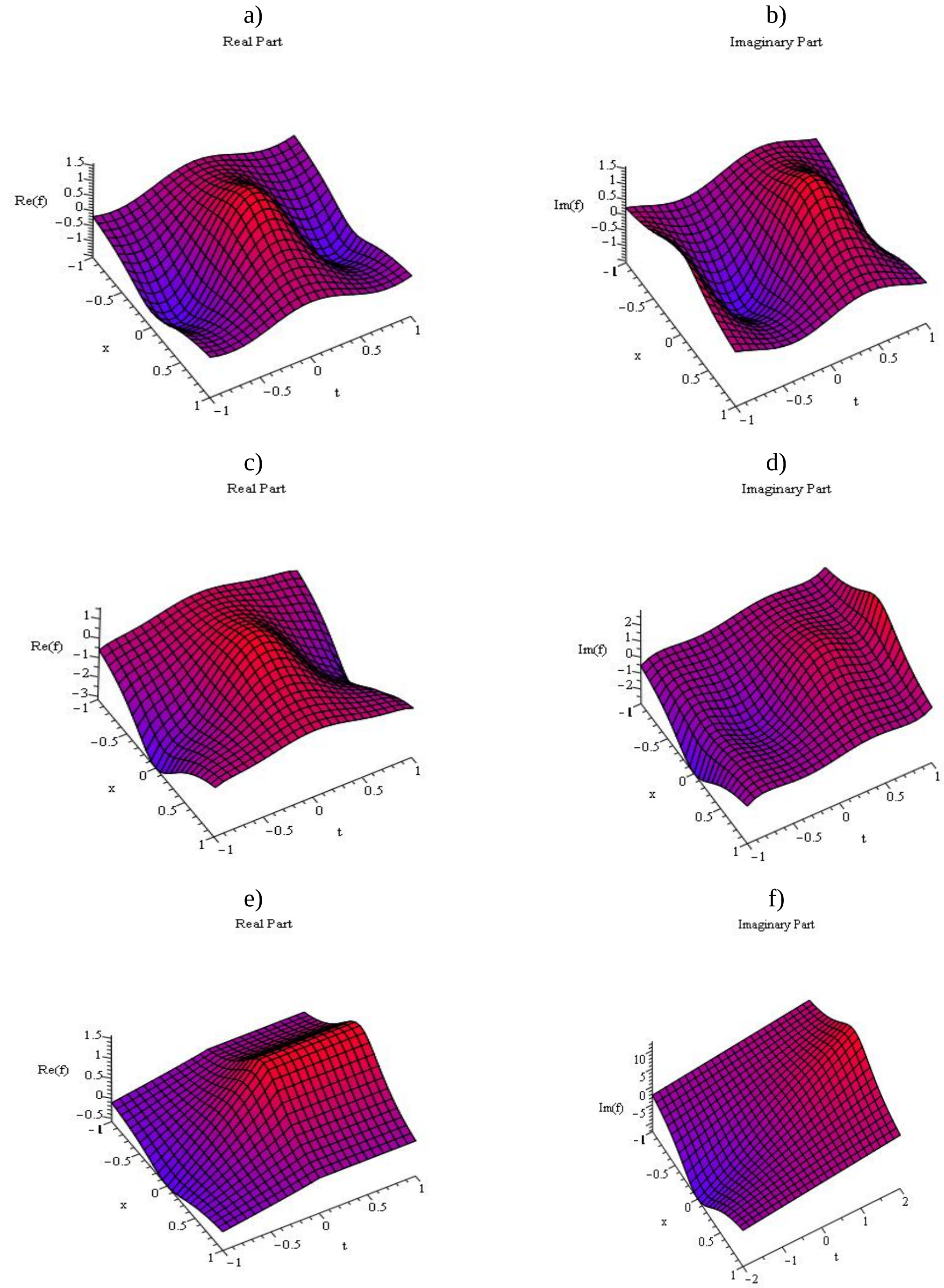

Figure 2. The surface graphs of NLSE with power law nonlinearity for $\gamma = 2$ and $r = 2$, a)exact solution (Re), b)exact solution (Im), c)approximate solution with MMRDTM (Re), d) approximate solution with MMRDTM (Im), e) approximate solution with MRDTM (Re), f) approximate solution with MRDTM (Im)

Table 3. Comparison error of MMRDTM and MRDTM

t	x	Exact solution	Absolute Error MMRDTM	Absolute Error MRDTM
0.1	0.1	1.505256212	8.5530×10^{-6}	5.4178
	0.2	1.353323050	7.6895×10^{-6}	1.7509
	0.3	1.163107591	6.6088×10^{-6}	1.3865
	0.4	0.9748582442	5.5390×10^{-6}	4.6482×10^{-1}
	0.5	0.8068951808	4.5849×10^{-6}	2.1323×10^{-1}
	0.6	0.6639260037	3.7722×10^{-6}	1.2904×10^{-1}
	0.7	0.5448024298	3.0953×10^{-6}	5.9217×10^{-2}
	0.8	0.4464996636	2.5371×10^{-6}	2.3977×10^{-2}
	0.9	0.3657300829	2.0780×10^{-6}	9.2022×10^{-3}
	1.0	0.2994959915	1.7017×10^{-6}	3.4502×10^{-3}

Table 4. Comparison between MMRDTM and exact solution of NLSE with power law nonlinearity

t	x	Absolute Error MMRDTM	Absolute Error MRDTM	Absolute Error RPS (12)	Absolute Error HATM (12)
0.01	0.01	1.000×10^{-11}	7.368×10^{-3}	3.126×10^{-4}	2.605×10^{-2}
	0.02	2.000×10^{-11}	7.322×10^{-3}	3.118×10^{-4}	1.042×10^{-1}
	0.03	1.000×10^{-9}	7.246×10^{-3}	3.105×10^{-4}	2.345×10^{-1}
	0.04	1.000×10^{-9}	7.143×10^{-3}	3.088×10^{-4}	4.168×10^{-1}
	0.05	0.000	7.016×10^{-3}	3.065×10^{-4}	6.514×10^{-1}
0.02	0.01	1.044×10^{-9}	3.195×10^{-2}	1.248×10^{-3}	2.532×10^{-2}
	0.02	1.044×10^{-9}	3.127×10^{-2}	1.245×10^{-3}	1.013×10^{-1}
	0.03	4.000×10^{-10}	3.021×10^{-2}	1.240×10^{-3}	2.279×10^{-1}
	0.04	4.000×10^{-10}	2.886×10^{-2}	1.233×10^{-3}	4.052×10^{-1}
	0.05	4.000×10^{-10}	2.735×10^{-2}	1.224×10^{-3}	6.332×10^{-1}
0.03	0.01	1.000×10^{-9}	1.092×10^{-1}	2.801×10^{-3}	2.414×10^{-2}
	0.02	0.000	1.025×10^{-1}	2.794×10^{-3}	9.657×10^{-2}
	0.03	0.000	9.205×10^{-2}	2.783×10^{-3}	2.173×10^{-1}
	0.04	1.005×10^{-9}	7.909×10^{-2}	2.767×10^{-3}	3.863×10^{-1}
	0.05	1.000×10^{-10}	6.490×10^{-2}	2.747×10^{-3}	6.037×10^{-1}
0.04	0.01	7.000×10^{-10}	3.875×10^{-1}	4.959×10^{-3}	2.254×10^{-2}
	0.02	6.000×10^{-10}	3.500×10^{-1}	4.947×10^{-3}	9.016×10^{-2}
	0.03	5.000×10^{-10}	2.923×10^{-1}	4.928×10^{-3}	2.029×10^{-1}
	0.04	1.118×10^{-9}	2.208×10^{-1}	4.899×10^{-3}	3.607×10^{-1}
	0.05	6.000×10^{-10}	1.436×10^{-1}	4.864×10^{-3}	5.636×10^{-1}
0.05	0.01	0.000	1.270	7.710×10^{-3}	2.056×10^{-2}
	0.02	1.000×10^{-10}	1.126	7.691×10^{-3}	8.226×10^{-2}
	0.03	1.000×10^{-10}	9.039×10^{-1}	7.661×10^{-3}	1.851×10^{-1}
	0.04	1.019×10^{-9}	6.290×10^{-1}	7.618×10^{-3}	3.291×10^{-1}
	0.05	6.000×10^{-10}	3.321×10^{-1}	7.562×10^{-3}	5.142×10^{-1}

Conclusion:

The series of solutions of NLSEs of power law nonlinearity using MMRDTM is successfully applied in this paper. We compared the obtained solutions with exact solutions and solutions obtained by MRDTM. The modification is done by replacing the nonlinear term with its Adomian polynomials and a multi-step approach was adapted. The obtained results verified that the approximate solutions of NLSEs of power law nonlinearity are obtained with high accuracy. As a conclusion, the MMRDTM is more effective, consistent, and precise than the MRDTM in obtaining an analytic approximate solution for these types of equations. All computations in this paper had been carried out by using Maple 13.

Acknowledgment:

This research is financially supported by the University of Malaysia Sabah and the Malaysian, Ministry of Higher Education.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Malaysia Sabah.

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الحلول التحليلية التقريبية للسيلتون البصري اللامع لمعادلة شرودنغر غير الخطية لقانون القوة اللاخطية

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الخلاصة:

يتناول البحث طريقة التحويل التفاضلي المنخفض المعدل متعدد الخطوات.(MMRDTM) . تطبق هذه الطريقة لتقريب حل معادلات شرودنجر غير الخطية (NLSES) لقانون القوة اللاخطية. الطريقة المقترحة لها بعض المزايا. من الممكن تعميم التقريب التحليلي كسلسلة متقاربة بسرعة من خلال تطبيق الطريقة المقترحة. علاوة على ذلك ، فإن عدد الحدود المحسوبة تم تقليله بشكل كبير. مقارنةً بـ RDTM، كما تم استبدال االحد غير الخطي في هذه الطريقة بمتعددة حدود Adomain ذات الصلة قبل تنفيذ طريقة متعدد الخطوات. نتيجة لذلك ، تم حساب عدد أقل من الحدود ل NLSE المطلوبة للتقريب. علاوة على ذلك ، فإن التقريب أيضًا يتقارب بسرعة خلال إطار زمني واسع. تم تقديم مثالين لإظهار قدرة ومزايا الطريقة المقترحة لتقريب حل قانون القوة اللاخطية لـ .NLSES بالنسبة للتمثيل التصويري ، تم تضمين المدخلات الرسومية لتمثيل الحل وإظهار الدقة وفاعلية MMRDTM كذلك.

الكلمات المفتاحية : متعدد حدود Adomian ، طريقة متعددة الخطوات، طريقة التحويل التفاضلي المنخفض المعدل متعدد الخطوات، معادلات شرودنجر اللاخطية لقانون القوة اللاخطية.