ON SOME TYPES OF ALMOST–PERIODIC POINT IN Bi-TOPOLOGCAL DYNAMCS

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ABSTRACT

In this paper We introduce some new types of almost bi-periodic points in topological bitransformation groups and their effects on some types of minimaliy in topological dynamics.

INTRODUCTION

Let X be a topological space, A be any subset of X, the sets $\mathring{A}$, $\bar{A}$ and A^c denote the interior, closure and complement of A respectively. A is called **semi-open** in X if $A \subseteq \bar{\mathring{A}}$ [10] then A^c is called **semi-closed** [6] such that every open set in X is semi-open set and every closed set in X is semi-closed set . A subset N_x of X is called a **semi-neighborhood** (s.nbd) of a point $x \in X$ if there exists a semi-open set A in X s.t $x \in A \subseteq N_x$ [2] , every nbh is semi-nbh. The smallest semi-closed set containing A is called the **semi-closure** of A (scl A) [2] s.t A is semi-closed set iff A=sclA and $sclA \subseteq \bar{A}$. $p \in X$ is called a semi-limit point of A if $A \cap (U/\{p\}) \neq \emptyset$ for each semi- open set $U \subseteq X$ which is containing p [2] . A family @ of semi-open subsets of X is called **a semi-open cover** of X if X is a subset of the union of elements of @ and X is called a **semi-compact space** if every semi-open cover of X containing a finite sub cover [3], every semi-compact set is compact, X is called **locally semi- compact** space if for every $x \in X$ there exist a s.nbh Ux is compact set. If G is a topological group and $A \subseteq G$ then A is called a **left syndatic** set if there exists a compact set $K \subseteq G$ s.t. G=AK and $B \subseteq G$ is called a **right syndatic** set if G=KB [4] , is called **left semi- syndatic** set if there exists a semi-compact set $K \subseteq G$ s.t G=AK similar B is called **right semi-syndatic** set if G=KB [1] ,every semi syndatic set is syndatic. If (X, G,π) is a right topological transformation group (θ, H, X) is a left topological transformation group then (H, X, G) is called **right-left topological bitransformation** group (T.B.G) if satisfy the condition: $(hx)g=(\theta(h,x),g)\pi=\theta(h,(x,g)\pi=h(xg)$ for every $x \in X$,$h \in H$,$g \in G$.

Definition 1: Let (H, X, G) be a T.B.G and $A \subseteq X$ then A is called a **bi-invariant set under G and H** if $HAG \subseteq A$.

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Definition 2: Let (H,X,G) be a T.B.G and $x \in X$ then the set $HxG=\{hxg \mid g \in G, h \in H\}$ is called **the bi-orbit of x under G and H,** the set $\overline{HxG}$ is called **the bi-orbit closure of x under G and H** and set $(\widetilde{HxG})$ is called an **bi-orbit semi-closure of x under G and H.**

Lemma 1: Let (H,X,G) be a T.B.G, $A \subseteq X$ and $g \in G, h \in H$ then $\overline{hAg}=h\bar{A}g$.

Proof: It is obvious.

Lemma 2: Let (H,X,G) be a T.B.G, $A \subseteq X$ and A is bi-invariant set then $\bar{A}$ is bi-invariant set .

Proof: By using lemma 1.

Proposition 1: Let (H,X,G) be a T.B.G then the following statements are true:

1) If $x \in X$ then the bi-orbit of x is a smallest bi-invariant subset of X contain x.

2) If $x \in X$ and $y \in HxG$, then $HxG=HyG$.

3) If $x \in X$ then the bi-orbit closure of x is a smallest bi-invariant subset of X contain x.

4) If $x \in X$ and $y \in \overline{HxG}$ then $\overline{HxG} \subseteq \overline{HyG}$.

5) The class of all bi-orbit closure under G and H covered X.

Proof:

(1) Let HxG be the bi-orbit of x then $HHxGG=HxG$ i.e. HxG is bi-invariant set. Let $A \subset HxG$ and A is bi-invariant s.t. $x \in A$, let $a \in HxG - A$ then there exist a $g \in G$, $h \in H$ s.t. $a=hxg$ but this contradiction.

(2) Since $y \in HxG$ then there exist a $g \in G$ s.t. $y=hxg$ i.e. $x=h^{-1}gg^{-1}$ this means $x \in HyG$ and $HxG \subseteq HyG$ then $HxG=HyG$.

(3) Since $HxG \subseteq \overline{HxG}$ then $\overline{HxG}$ is a smallest closed set contain x and the closed of bi-invariant set is bi-invariant set then $\overline{HxG}$ is smallest closed bi-invariant set contain x.

(4) It is obvious.

(5) Let $@=\{\overline{HxG} \mid x \in X\}$ then $X=\cup HxG \subseteq \overline{HxG}=\cup @$.

Definition 3: Let (H, X, G) be a T.B.G and $M \subseteq X$ then M is called a **b_1-minimal set {b_2-minimal set}** in T.B.G if satisfy the following conditions:

(1) $M \neq \varnothing$, M is closed {semi-closed} bi-invariant set. (2) M is not containing closed {semi-closed} bi-invariant subset.

Proposition 2: Let (H, X, G) be a T.B.G, and $M \subseteq X$ then the following statements are equivalient:

(1) M is b_1-minimal set.

(2) $M \neq \varnothing$, $M=\overline{HxG}$ for every $x \in M$.

(3) $M \neq \varnothing$, M is closed and $M=\overline{HUG}$ for every nonempty open subset U of M.

Proof:

(1)→(2) Since M is b_1-minimal set then M is b-invariant set and closed, this means $HxG \subseteq M$ and $\overline{HxG} \subseteq M$. If $\overline{HxG} \neq M$ means M is not b_1-minimal set then $\overline{HxG}=M$.

(2)→(1) Since $M \neq \varnothing$ and $M=\overline{HxG}$ then M is b-invariant set, if $A \subseteq M$ closed b_1-invariant then there exist a $\in A$ s.t. $M=\overline{HaG}$, this lead to $M \subseteq A$ i.e. $M=A$.

(4)→(1) M is b_1-invariant set ($HMG=HHUGG=HUG=M$). Let $A \subset M$ and $A \neq \varnothing$, closed and bi-invariant set , then M-A is open set in M and $H(M-A)G=M$ i.e. there exist $x,y \in M$ s.t. $x \in A, y \in M-A$ and $g \in G, h \in H$ s.t. $y=hxg$ this means $y \in A$ and $M=A$.

(1)→(4) If M is b_1- minimal set and $U \in M$ non-empty open set, then $HUG \subseteq HMG=M$ and $M-HUG \subseteq M$ is closed set. If $x \in M-HUG$ and $g \in G$, $h \in H$ s.t. $hxg \in HUG$ then $H^{-1}HUGg^{-1}$ this means M is contain HUG b_1-minimal set but this contradiction then $M=HUG$.

Lemma 3: Let (H,X,G) be a T.B.G , $A,B,C \subseteq X$ are compact sets then ABC is compact subset of X.

Proof: Since θ:H×(X×G)π →X is continuous map and since, A, B, C are compact sets and θ(C×AB) =CAB is compact set under continuous map.

Remark: We can use the above proposition to b_2-minimal set with take difference of b_1-minimal.

Definition 4: Let (H,X,G) be a T.B.G and x∈X then x is called b_1-almost periodic point (b_1) if for every nbh U of x there exist a left syndatic set A⊆ G and right syndatic set B⊆H s.t. BxA⊆U.

Proposition 3: Let (H,X,G) be a T.B.G ,X is T_2 locally compact space and x∈X then x is b_1 iff HxG is b_1-minimal compact subset of X.

Proof: → Let x be a b_1 point, Ux be a nbh of x then there exist a left syndatic set A⊆G and right syndatic set B⊆G s.t. BxA⊆U_x. Since A, B are syndatic set this means there exist two compact set K⊆G, J⊆H s.t. G=AK, H=JB and HxG=JBxAK⊆JU_xK. By Lemma (3) JU_xK is compact subset of T_2-space then it is closed set. Hence, HxG⊆JU_xK and HxG is compact.

If y∈HxG then HyG⊆HxG i.e. y∈JU_xK this means there exist u∈U_x ,k∈K and j∈J s.t. y=juk and u= $j^{-1}yk^{-1}$ ∈U_x then HyG∩U_x≠∅ ,hence, x is a limit point of HyG and x∈HyG then HxG is b_1-minimal compact subset of X.

←Let HxG be a b_1-minimal compact subset of X . Let U_x be a nbh of x in X. Since HxG⊆HxG⊆HUG then there exist a finite sets E⊆G,F⊆H s.t. HxG⊆FUE. Hence, HUG is an open cover HxG then its contain a finite subcover FUE.

For every g∈G and h∈H there exist a u∈U,f∈F and e∈E s.t. hxg=fue.

Let A={g|hxg∈U}, B={h|hxg∈U} we prove A and B are right, left syndatic sets respectively. Since hxg=fue this leads to $f^{-1}hxge^{-1}$=u then $f^{-1}hxge^{-1}$ ∈U and g∈AE and h∈FB then G=AE, H=FB. Since E and F are finite set then A is left syndatic set and B is right syndatic set.

Then x is b_1-almost periodic point.

Definition 5: Let (H, X, G) be a T.B.G and x∈X then x is called b_2-almost periodic point (b_2) if for every nbh U of x there exist a left semi-syndatic set A⊆ G and right semi-syndatic set B⊆H s.t. BxA⊆U.

Proposition 4: Let (H, X, G) be a T.B.G, X is T_2 locally semi-compact space and x∈X is b_2 point then scl(HxG) is b_2-minimal semi-compact subset of X .

Proof: Similarly of first part of proposition 3 by supposing x∈X is a b_2 point and using the statement "every nbh is semi-nbh and change compact set by semi-compact".

Proposition 5: Let (H, X, G) be a T.B.G, and scl(HxG)is b_2-minimal semi-compact subset of X ,then x∈X is b_2 .

Proof: Similarly of second part of proposition 3.

Definition 6: Let (H, X, G) be a T.B.G and x∈X then x is called b_3-almost periodic point (b_3) if for every nbh U of x there exist a left semi-syndatic set A⊆ G and right syndatic set B⊆H s.t. BxA⊆U.

Proposition 6: Let (H, X, G) be a T.B.G, X is T_2 locally compact space and x∈X is b_3 point then HxG is b_1-minimal compact subset of X.

Proof: Similarly of first parts of proposition 3 by use the statement in introduction "every semi-syndatic set is syndatic".

Proposition 7: Let (H, X, G) be a T.B.G, and scl(HxG)is b_2-minimal semi-compact subset of X ,then x∈X is b_3 .

Proof: Similarly of second part of proposition 3 by use the statement "every open set is semi-open".

The definition of b_4 point (b_4-almost periodic point) is similar of b_3 point by suppose A is left syndatic set and B is right semi-syndatic set and we can satisfy propositions 6,7 by use it.

Definition 7: Let (H, X, G) be a T.B.G and $x \in X$ then x is called b_5-almost periodic point (b_5) if for every semi-nbh U of x there exist a left semi-syndatic set $A \subseteq G$ and right semi-syndatic set $B \subseteq H$ s.t. $BxA \subseteq U$.

Proposition 8: Let (H, X, G) be a T.B.G, X is T_2 locally semi-compact space and $x \in X$ is b_5 point then scl(HxG) is b_2-minimal semi-compact subset of X .

Proof: Similarly of first parts of proposition 3.

Proposition 9: Let (H, X, G) be a T.B.G, and scl(HxG)is b_2-minimal semi-compact subset of X ,then $x \in X$ is b_5 .

Proof: Similarly of second part of proposition 3 by suppose scl(HxG) be a b_2-minimal semi-compact subset of X and let U_x be a semi-nbh of x in X

Definition 8: Let (H, X, G) be a T.B.G and $x \in X$ then x is called b_6-almost periodic point (b_6) if for every semi-nbh U of x there exist a left syndatic set $A \subseteq G$ and right syndatic set $B \subseteq H$ s.t. $BxA \subseteq U$.

Proposition 10: Let (H, X, G) be a T.B.G, X is T_2 locally semi-compact space and $x \in X$ is b_6 point then scl(HxG) is b_2-minimal semi-compact subset of X .

Proof: Let x be a b_6 point, Ux be a semi-nbh semi-compact of x then there exist a left syndatic set $A \subseteq G$ and right syndatic set $B \subseteq G$ s.t. $BxA \subseteq U_x$. Since A, B are syndatic set this means there exist two compact set $K \subseteq G$, $J \subseteq H$ s.t. G=AK, H=JB and $HxG=JBxAK \subseteq JU_xK$. By Lemma (3) JU_xK is compact subset of T_2-space then it is closed set. Hence, $scl(HxG) \subseteq JU_xK$ and scl(HxG) is semi-compact.

If $y \in scl(HxG)$ then $scl(HyG) \subseteq scl(HxG)$ i.e. $y \in JU_xK$ this means there exist $u \in U_x$,$k \in K$ and $j \in J$ s.t. y=juk and $u= j^{-1}yk^{-1} \in U_x$ then $HyG \cap U_x \neq \emptyset$,hence, x is a semi-limit points of HyG and $x \in scl(HyG)$ then scl(HxG) is b_2-minimal semi-compact subset of X.

Proposition 11: Let (H, X, G) be a T.B.G, and scl(HxG)is b_2-minimal semi-compact subset of X ,then $x \in X$ is b_6 .

Proof: Let scl(HxG) be a b_2-minimal semi-compact subset of X . Let U_x be a semi-nbh of x in X.

Since $HxG \subseteq scl(HxG) \subseteq HUG$ then there exist a finite sets $E \subseteq G, F \subseteq H$ s.t. $HxG \subseteq FUE$. Hence, HUG is an semi-open cover of scl(HxG) then its contain a finite subcover FUE.

For every $g \in G$ and $h \in H$ there exist a $u \in U, f \in F$ and $e \in E$ s.t. hxg=fue.Let $A=\{g|hxg \in U\}$, $B=\{h|hxg \in U\}$ we prove A and B are right, left semi-syndatic sets respectively. Since hxg=fue this leads to $f^{-1}hxge^{-1}=u$ then $f^{-1}hxge^{-1} \in U$ and $g \in AE$ and $h \in FB$ then G=AE, H=FB. Since E and F are finite set then A is left semi -syndatic set and B is right semi syndatic set.

Then x is b_6-almost periodic point.

Definition 9: Let (H, X, G) be a T.B.G and $x \in X$ then x is called b_7-almost periodic point (b_7) if for every semi-nbh U of x there exist a left semi-syndatic set $A \subseteq G$ and right syndatic set $B \subseteq H$ s.t. $BxA \subseteq U$.

Proposition 12: Let (H, X, G) be a T.B.G, X is T_2 locally semi-compact space and $x \in X$ is b_7 point then scl(HxG) is b_2-minimal semi-compact subset of X .

Proof: Similarly of first part of proposition10 by suppose A is syndatic set.

Proposition 13: Let (H, X, G) be a T.B.G, and scl(HxG)is b_2-minimal semi-compact subset of X ,then $x\in X$ is b_7 .

Proof: Similarly of second part of proposition 11.

The definition of b_8 point (b_8-almost periodic point) is similar of b_7 point by suppose A is left syndatic set and B is right semi-syndatic set and we can satisfy propositions 12.13 by use it.

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في بعض أنواع النقاط الدورية تقريباً في زمر التحويلات التبولوجية الثنائية

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الملخص

يقدم البحث بعض الانوع الجديدة من النقاط الدورية تقريبا في الزمر ثنائية التحويل التبولوجي وتأثيرها على أنواع الاصغرية في الديناميكية. التبولوجية.