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An Approximate Solution for Solving Linear System of Integral Equation With Application on " Stiff" Problems

Omar M. AL-Faour *

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Abstruct

An approximate solution of the linear system of ntegral equations for both Fredholm (SFIEs) and Volterra (SVIEs) types has been derived using Taylor series expansion . The solution is essentially based on substituting for the unknown function after differentiating both sides of the systemof integral equations. It may further be used to implement the same method uses for "stiff problems".It has been shown that for "stiff problems", Runge-kutta and modified Euler methods may lead to poor results. The proposed method is convenient for computer programming using MATLB language. Finally, by using various examples, the accuracy and the stability of this method will be shown.

1. Introduction

Consider first the system of integral equation of the form :

$$u_i(x) = f_i(x) + \sum_{j} \int k_{ij}(x,t) u_j(t)\,dt, \quad i = 1,2,\ldots,n \qquad (1)$$

where $k_{ij}(x,t)$ and $f_i(x)$ are known and $u_j(t), j = 1,2,\ldots,n$ are unknown functions to be determined.

This system appears in many applications for instance: the Dirichelt- Neumann mixed boundary value problems (MBVPs) on closed surfaces in $\Re^3$ baced on an equivalent formulation of the MBVP as a system of two integral equations [1].Problem (1) was previously studied and treated using collocation method [2].

Second, we consider the "stiff" initial value problems (SIVPs) of order n which is:

$$y^{(n)}(t) + a_1 y^{(n-1)}(t) + a_2 y^{(n-2)}(t) + \ldots + a_n y(t) = f(t) \qquad (2)$$

with the initial conditions

$$y(a) = x_0,\ y'(a) = x_1, \ldots, y^{(n-1)}(a) = x_{n-1}$$

Stiffness is a property of a mathematical problem (not of the numerical solution method). Systems of equations having the stiffness proprties arise in a variety of applications. including chemical neaction kineties, guidance and control problems, electrical transmission networks, and heat and matter transfer. To determine whether or not the IVP is stiff, we shall give the definition of SIVP in section 6.

This paper is concerned with the use of Taylor series for obtaining an approximate solution of (1) and (2). A brief review of some background on the Taylor s theorem as well as the fundumental theorem is given in the following section.

* Dr.-University of Technology

2. Basic Theorems

Taylor's Theorem:

Suppose f is a real function on $[a,b]$, n is a positive integer, $f^{(n-1)}$ is continuous on $[a,b]$, $f^{(n)}(t)$ exists for every $t \in (a,b)$. Let α, β be distinct points of $[a,b]$, and define

$$P(t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(\alpha)}{k!}(t-\alpha)^k$$

Then there exists a point x between α and β such that

$$f(\beta) = P(\beta) + \frac{f^{(n)}(x)}{n!}(\beta-\alpha)^n \qquad (3)$$

In general, the theorem shows that f can be approximated by a polynomial of degree $n-1$, and that (3) allows us to estimate the error, if we know bounds on $|f^{(n)}(x)|$

Fundamental Theorem of Integral Calculus (Leibnitz Generalized Formula):

$$\frac{d}{dx}\int_{\alpha(x)}^{\beta(x)} F(x,y)\,dy = \int_{\alpha(x)}^{\beta(x)} \frac{\partial F(x,y)}{\partial x}\,dy + F(x,\beta(x))\frac{d\beta(x)}{dx} - F(x,\alpha(x))\frac{d\alpha(x)}{dx}$$

3. The Existence and Uniqueness Theorem

In this section we discuss the existence and uniqueness of the solution of eq.(1).

Definition

Let H be a Hilbert space and T a bounded operator on H. T is not necessary a linear operator. T is said to be ***a contraction operator*** if there exists a positive constant $\alpha < 1$ such that:

$$\|Tf - Tf_1\| \le \alpha \|f - f_1\|$$

for all f_1, f in H.

Theorem 1 :

Let T be a contraction operator on H. The equation

$$Tf = f$$

has a unique solution f in H, such a solution is said to be ***a fixed point*** of T (see [3] for proof).

Now in operator form eq.(1) can be written as:

$$U_m - K_m U = F_m, \quad m = 1,2,\ldots,n \qquad (4)$$

where F_m, $m = 1,2,\ldots,n$ are in H and K_m, $m = 1,2,\ldots,n$ are bounded operators with the properties that:

$$\|K_1 U_1 - K_1 V_1\| \le M_1 \|U_1 - V_1\|$$

$$\|K_2 U_2 - K_2 V_2\| \le M_2 \|U_2 - V_2\|$$

$$\vdots$$

$$\|K_m U_m - K_m V_m\| \le M_m \|U_m - V_m\|$$

Let $M = \max\{M_1, M_2, \ldots, M_m\}$, then we have

$$\|K_m U - K_m V\| \le M \|U - V\| \qquad (5)$$

where

$$K_m U = \sum_i \int k_{mi}(x,t) u_i(t)\,dt$$

we can write eq.(3) in the form

$$TU_m = U_m, \quad m = 1,2,\ldots,n$$

where $TU_m = F_m - K_m U$, $m = 1,2,\ldots,n \qquad (6)$

Theorem 2:

Equation (4) has a unique solution for all F_m, $m = 1,2,\ldots,n$, provided that K_m, $m = 1,2,\ldots,n$ are bounded operators that also satisfies (5).

Theorem (2) can be applied immediately to the linear SFIEs of the form:

$$u_i(x) = f_i(x) + \sum_{j=1}^{n} \int_a^b k_{ij}(x,t) u_j(t)\,dt, \quad i = 1,2,\ldots,n \qquad (7)$$

if $f_i(x)$, $i = 1,2,\ldots,n$ is in $L_2[a,b]$ and the integral operators are bounded.

The results apply to linear SVIEs of the form:

$$u_i(x) = f_i(x) + \sum_{j=1}^{n} \int_a^x k_{ij}(x,t) u_j(t)\,dt, \quad i = 1,2,\ldots,n \qquad (8)$$

since eq.(7) reduces to a SVIEs if $k_{ij}(x,y)=0,\ i,j=1,2,\dots,n$ for $y>x$.

Theorem 3:
Let $f_i(x)\in L_2[0,1]$, $i=1,2,\dots,n$ (we consider a finite interval and without loss of generality let it be $[0,1]$) and suppose that $k_{ij}(x,y)=0,\ i,j=1,2,\dots,n$ is continuous for $x,y\in[0,1]$ and therefore uniformly bounded. Then eq. (8) has a unique solution $u_i(x)$ $i=1,2,\dots,n$, for all $f_i(x)$, $i=1,2,\dots,n$ in $L_2[0,1]$.

4. Taylor series method for (SFIEs)

Recall eq(7), the linear SFIEs :

$$u_i(x)=f_i(x)+\sum_{j=1}^{n}\int_a^b k_{ij}(x,t)u_j(t)\,dt,\quad i=1,2,\dots,n$$

Differentiating both sides of (7) m-times with respect to x, we have

$$u_i^{(m)}(x)=f_i^{(m)}(x)+\sum_{j=1}^{n}\int_a^b\frac{\partial^m k_{ij}(x,t)}{\partial x^m}u_j(t)\,dt,\quad i=1,2,\dots,n \quad (9)$$

Then put $x=x_i$ into (9) to obtain

$$u_i^{(m)}(x_i)=f_i^{(m)}(x_i)+\sum_{j=1}^{n}\int_a^b\frac{\partial^m k_{ij}(x,t)}{\partial x^m}u_j(t)\,dt\Big|_{x=x_i},\quad i=1,2,\dots,n \quad (10)$$

Now, expand $u_j(t)$, $j=1,2,\dots,n$ in Taylor series at $t=t_0$ i.e.

$$u_j(t)=\sum_{k=0}^{N}\frac{1}{k!}u_j^{(k)}(t_0)(t-t_0)^k,\quad j=1,2,\dots,n \quad (11)$$

Finally, by substituteing eq.(11) into eq.(10), yeilds :

$$u_i^{(m)}(x_i)=f_i^{(m)}(x_i)+\sum_{j=1}^{n}\int_a^b\frac{\partial^m k_{ij}(x,t)}{\partial x^m}u_j(t)\,dt\Big|_{x=x_i}\left\{\sum_{k=0}^{N}\frac{1}{k!}u_j^{(k)}(t_0)\right\}dt,\quad i=1,2,\dots,n \quad (12)$$

Eq.(12) can be rewritten as:

$$u_i^{(m)}(x_i)=f^{(m)}(x_i)+\sum_{j=1}^{n}\sum_{k=0}^{N}T_{ij}^{mk}u_j^{(k)}(t_0),\quad i=1,2,\dots,n$$

where

$$T_{ij}^{mk}=\frac{1}{k!}\int_a^b\frac{\partial^m k_{ij}(x,t)}{\partial x^m}\Big|_{x=x_i}(t-t_0)^k\,dt,\quad i,j=1,2,\dots,n$$

solve the above system for the quantities $u_i'(x_i), u_i''(x_i),\dots,u_i^{(n)}(x_i)$, $j=1,2,\dots,n$ using Gauss elimination proceture to obtain the approximate solution to $\phi(x)$.

5. Taylor series method for (SVIEs)

In this section the linear SVIEs, eq.(8) is considerd:

$$u_i(x)=f_i(x)+\sum_{j=1}^{n}\int_a^x k_{ij}(x,t)u_j(t)\,dt,\quad i=1,2,\dots,n$$

Differentiating both sides of (8), 3-times with respect to x to get:

$$u_i'(x)=f_i'(x)+\sum_{j=1}^{n}\left\{\int_a^x\frac{\partial k_{ij}(x,t)}{\partial x}u_j(t)\,dt+k_{ij}(x,x)u_j(x)\right\},\quad i=1,2,\dots,n \quad (14)$$

$$u_i''(x)=f_i''(x)+\sum_{j=1}^{n}\left\{\int_a^x\frac{\partial^2 k_{ij}(x,t)}{\partial x^2}u_j(t)\,dt+\frac{\partial k_{ij}(x,t)}{\partial x}u_j(x)+\frac{\partial k_{ij}(x,x)}{\partial x}u_j(x)+k_{ij}(x,x)u_j'(x)\right\} \quad (15)$$

$$u_i'''(x)=f_i'''(x)+\sum_{j=1}^{n}\left\{\int_a^x\frac{\partial^3 k_{ij}(x,t)}{\partial x^3}u_j(t)\,dt+\frac{\partial}{\partial x}\left(\frac{\partial k_{ij}(x,t)}{\partial x}\right)u_j(x)+\frac{\partial k_{ij}(x,t)}{\partial x}u_j'(x)+\frac{\partial^2 k_{ij}(x,x)}{\partial x^2}u_j(x)+2\frac{\partial k_{ij}(x,x)}{\partial x}u_j'(x)+k_{ij}(x,x)u_j''(x)\right\} \quad (16)$$

Since, it is difficult to find an explicit formula for the m-th order derivative for eq. (8) as in SFIE; hence for fourth derivative, we differentiate eq.(16) and so on for fifth and higer derivative.

Now, put $x=a$ and substitute eq.(11) into eq.(8) and eqs.(14-16) to obtain:

$$u_i(a)=f_i(a),\quad i=1,2,\dots,n$$

$$u_i'(a)=f_i'(a)+\sum_{j=1}^{n}k_{ij}(a,a)u_j(a),\quad i=1,2,\dots,n$$

$$u_i''(a)=f_i''(a)+\sum_{j=1}^{n}\left\{\frac{\partial k_{ij}(x,t)}{\partial x}\Big|_{x=t=a}u_j(a)+\frac{\partial k_{ij}(x,x)}{\partial x}\Big|_{x=a}u_j(a)+k_{ij}(a,a)u_j'(a)\right\},\quad i=1,2,\dots,n$$

$$u_i'''(a) = f_i'''(a) + \sum_{j=1}^{n}\left\{\frac{\partial}{\partial x}\left(\frac{\partial k_{ij}(x,t)}{\partial x}\Big|_{t=x}\right)_{x=a} u_j(a) + \frac{\partial k_{ij}(x,t)}{\partial x}\Big|_{t=x=a} u_j'(a) + \frac{\partial^2 k_{ij}(x,x)}{\partial x^2}\Big|_{x=a} u_j(a) + 2\frac{\partial k_{ij}(x,x)}{\partial x}\Big|_{x=a} u_j'(a) + k_{ij}(a,a)u_j''(a)\right\}$$

$$i = 1,2,\ldots,n$$

Solve the above system for the quantities $u_j'(a), u_j''(a)$ and $u_j'''(a); j = 1,2,\ldots,n$ using forward substitution .Then these values are substituted in eq. (9) to obtain the solution to $u(x)$:

$$u_i(x) = u_i(a) + u_i'(a)(x-a) + \frac{1}{2!}u_i''(a)(x-a)^2 + \frac{1}{3!}u_i'''(a)(x-a)^3 , \qquad i = 1,2,\ldots,n$$

6. Stiff Initial – Value Problems (SIVPs)

Definition:

A linear system $Y' = AY + Bf$ is said to be ***stiff*** if:

1. $\lambda_i < 0,\ for\ all\ i = 0,1,\ldots,n.$
2. $\max|\lambda_i| / \min|\lambda_i| >> 1$

where A is a real constant matrix of n – dimensions and $\lambda_i, i = 1,2,\ldots,n$ are the eigenvalues of the matrix A.

The ratio of $\max|\lambda_i| / \min|\lambda_i|$ is termed the ***stiffness ratio***.

Higher-order real scalar differential equations can be studied by a corresponding first-order real vector differential equation, which is a special case of the general first-order vector differential system. Therefore, eq.(2) can be written as a first-order differential system by introducing the variables $y_1, y_2, \ldots, y_n$ uses the following assumption:

$$\begin{aligned} y &= y_1 \\ y_1' &= y_2 \\ y_2' &= y_3 \\ &\vdots \\ y_{n-1}' &= y_n \\ y_n' &= -a_n y_1 - a_{n-1} y_2 - \ldots - a_1 y_n + f(t) \end{aligned} \qquad (17)$$

with the initial conditions

$y_i(a) = x_i, i = 1,2,\ldots,n$

or in matrix notation

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ \vdots \\ y_n' \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & & \vdots \\ -a_n & -a_{n-1} & -a_{n-2} & -a_{n-3} & \cdots & -a_1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} f(t) \qquad (18)$$

with the initial conditions : $y_i(a) = x_i, i = 1,2,\ldots,n$

or

$$Y' = AY + Bf$$

Here the vector $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}$ and the matrices

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & & \vdots \\ -a_n & -a_{n-1} & -a_{n-2} & -a_{n-3} & \cdots & -a_1 \end{pmatrix}$$ and $$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$ are related as above.

The solution of the nonhomogeneous equation (18) with initial data $(x_1, x_2, \ldots, x_n)$ at $t = 0$ is then

$$\begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{pmatrix} = e^{tA}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \int_0^t e^{(t-s)A}\begin{pmatrix} 0 \\ 0 \\ \vdots \\ f(s) \end{pmatrix} ds$$

where e^{tA} is a fundamental matrix solution of the homogeneous system.

Now, the first-order differential system (17) can be reduce to system of integral equation of Volterra type (SVIE). By integrating both sides of (17) over $[0,t]$ we can easily verify the formula for the solution is:

$$y_1(t) = x_1 + \int_0^t y_2(s)ds$$

$$y_2(t) = x_2 + \int_0^t y_3(s)ds$$

$$\vdots \qquad (19)$$

$$y_n(t) = x_n - a_{n1}\int_0^t y_1(s)ds - a_{n2}\int_0^t y_2(s)ds - \cdots$$

$$- a_{nn}\int_0^t y_n(s)ds + \int_0^t f(s)ds$$

Then Taylor sereis method can be applied to find an approximate solution for (19) as in section 5.

7. Numerical Results

Here we present the results of applying the Taylor series expansion discussed to three different problems.

Example 1:

Given $f_1(x)$ and $f_2(x)$, we wish to find $\phi_1(x)$ and $\phi_2(x)$ so that

$$\phi_1(x) = f_1(x) + \int_0^x e^{x-t}\phi_2(t)dt$$

$$\phi_2(x) = f_2(x) + \int_0^x (1+xt)\phi_1(t)dt$$

with the functions

$f_1(x) = \sin x - xe^{x}$ and

$f_2(x) = e^{x} + \cos x - x\sin x + x \cos x - 1$

the exact solution of this problem is:

$\phi_1(x) = \sin x$, $\phi_2(x) = e^{x}$

when applying Taylor's method, we have the approximate solution

$$\phi_1(x) = x - \frac{x^3}{3!} \quad \text{and} \quad \phi_2(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

Table (1) gives a comparison of our results and exact solution.

x	$\phi_1(x)$		$\phi_2(x)$	
	Taylor	Exact	Taylor	Exact
0	0	0	1	1
0.1	0.099833	0.099833	1.105167	1.105171
0.2	0.198667	0.198669	1.221333	1.221403
0.3	0.295500	0.295520	1.349500	1.349859
0.4	0.389333	0.389418	1.490667	1.491825
0.5	0.479167	0.479425	1.645833	1.648721
0.6	0.564000	0.564425	1.816000	1.822119
0.7	0.642833	0.644218	2.002167	2.013753
0.8	0.714667	0.717356	2.205333	2.225541
0.9	0.778500	0.783327	2.426500	2.459603
1	0.833333	0.841471	2.666667	2.718282
$L.S.E$	$9.958*10^{[illegible]}$		$6.595*10^{[illegible]}$	

Table (1)

Example 2 :

Here we apply the method to the example of SFIEs:

$$u_1(x) = e^{-x} - \frac{1}{4}\sin x + \int_0^1 \sin x\, u_1(t)dt$$

$$u_2(x) = x - x^3 - e^{x} + \int_0^x e^{x-t} u_1(t)dt$$

the exact solution is : $u_1(x) = e^{-x}$, $u_2(x) = x - x^3$

The solution of $u_1(x)$ and $u_2(x)$ for $0 \le x \le 1$ is required. Using the method derived in section 4, the Taylor series solutions of $u_i(x), i = 1,2$ for $m = 3$ are obtained as shown in Table 2. Again, the results obtained by the exact solution are also listed in Table 2 for comparison.

x	$u_1(x)$		$u_2(x)$	
	Taylor	Exact	Taylor	Exact
0	1	1	0	0
0.1	1.105171	0.904837	0.099000	0.099000
0.2	1.221333	0.818730	0.192000	0.192000
0.3	1.349500	0.740818	0.273000	0.273000
0.4	1.490667	0.670320	0.336000	0.336000
0.5	1.645833	0.606531	0.375000	0.375000
0.6	1.816000	0.548812	0.384000	0.384000
0.7	2.002167	0.496585	0.357000	0.357000
0.8	2.205333	0.449329	0.288000	0.288000
0.9	2.426500	0.406570	0.171000	0.171000
1	2.666667	0.367879	0	0
$L.S.E$	$6.595*10^{[illegible]}$		0.000000	

Table(2)

Example 3:

As a third example, we studied the following " Stiff " problem

$$y'' + 1001y' + 1000y = 0, \quad x \in [0,1], \quad y(0) = 1 \quad and$$
$$y'(0) = -1$$

which can be written as a system of first order differential equations

$$\left.\begin{array}{ll} y_1' = y_2 & y_1(0)=1 \\ y_2' = -1001y_2 - 1000y_1 & y_2(0)=-1 \end{array}\right\} \quad (20)$$

or in matrix notation

$$Y' = AY$$

where $Y' = (y_1' \; y_2')$ and the coefficient matrix $A = \begin{pmatrix} 0 & 1 \\ -1000 & -1001 \end{pmatrix}$, with the vector solution $Y = (y_1 \; y_2)^T$. The exact solution of this problem is:

$$y_1(x) = e^{-x}, \; and \; y_2(x) = -e^{-x}$$

By integrating both sides of eq.(18) over $[0,x]$, we obtain the following SVIEs :

$$y_1(x) = 1 + \int_0^x y_2(t)dt$$

$$y_2(x) = -1 - 1001\int_0^x y_2(t)dt - 1001\int_0^x y_1(t)dt$$

The stiff problem (20) has been solved using Rung-Kutta and modified Euler methods, these methods lead to a rapid deteriorotion of the accuracy of the solution. We obtained comparable accuracy using Taylors method (see Tables 3 and 4).

x	$y_1(x)$			
	Taylor	Rung-Kutta	M.Euler	Exact
0	1	1	1	1
0.1	0.904833	0.904838	0.905	0.904837
0.2	0.818667	0.818731	2.045275	0.818730
0.3	0.740500	0.832877	6.219681*10^[illegible]	0.740818
0.4	0.669333	3.685541*10^[illegible]	3.255044*10^[illegible]	0.670320
0.5	0.604167	1.476020*10^[illegible]	1.676755*10^[illegible]	0.606531
0.6	0.544000	5.913424*10^[illegible]	8.637381*10^[illegible]	0.548812
0.7	0.487833	2.368268*10^[illegible]	4.449132*10^[illegible]	0.496585
0.8	0.434667	9.481293*10^[illegible]	2.291962*10^[illegible]	0.449329
0.9	0.383500	3.797161*10^[illegible]	1.180647*10^[illegible]	0.406570
1	0.333333	1.526727*10^[illegible]	6.081809*10^[illegible]	0.367879
L.S.E	4.524*10^[illegible]	1.521*10^[illegible]	6.082*10^[illegible]	

Table (3)

x	$y_2(x)$			
	Taylor	Rung-Kutta	M.Euler	Exact
0	−1	−1	−1	−1
0.1	−1.105171	−0.904838	−1.155	−0.904837
0.2	−1.221333	−0.818754	−1.290084*10^[illegible]	−0.818730
0.3	−0.740500	−92.799271	−6.641330*10^[illegible]	−0.740818
0.4	−0.669333	−3.685531*10^[illegible]	3.421115*10^[illegible]	−0.670320
0.5	−0.604167	−1.476020*10^[illegible]	7.762302*10^[illegible]	0.606531
0.6	−0.544000	−5.913424*10^[illegible]	9.078058*10^[illegible]	−0.548812
0.7	−0.487833	2.368268*10^[illegible]	4.676335*10^[illegible]	−0.496585
0.8	−0.434667	−9.481293*10^[illegible]	2.408897*10^[illegible]	−0.449329
0.9	−0.383500	−3.797164*10^[illegible]	1.240883*10^[illegible]	−0.406570
1	−0.333333	−1.526727*10^[illegible]	−6.302100*10^[illegible]	−0.367879
L.S.E	4.524*10^[illegible]	1.521*10^[illegible]	6.398*10^[illegible]	

Table(4)

8. Conclusion

A method of using Taylor series expansion has been presented for solving linear SFIEs and SVIEs as well as an important class of differential equations called "stiff" initial value problem.

In practice, we conclude that:

- The solution obtained by Taylor series expansion is given by a function , and not only at some points as in Runge-Kutta and modified Euler methods.
- Numerical computations of Taylor's method are simple and the convergence s satisfactory.
- If we attempt to increase the interval, an unsuccessful doubling of the interval being followed immediately when using Runge-Kutta and modified Euler methods.
- The difficulties that arise in attempting to obtain a numerical approximation to the solution of a "stiff" problem is that numerical stability. We overcome this stability limitation when using Taylor series expansion.

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which can be written as a system of first order differential equations

$$\left.\begin{array}{ll} y_1' = y_2 & y_1(0)=1 \\ y_2' = -1001y_2 - 1000y_1 & y_2(0)=-1 \end{array}\right\} \quad (20)$$

or in matrix notation

$$Y' = AY$$

where $Y' = \left(y_1' \quad y_2'\right)$ and the coefficient matrix $A = \begin{pmatrix} 0 & 1 \\ -1000 & -1001 \end{pmatrix}$, with the vector solution $Y = (y_1 \quad y_2)^T$. The exact solution of this problem is:

$$y_1(x) = e^{-x}, \quad and \quad y_2(x) = -e^{-x}$$

By integrating both sides of eq.(18) over $[0,x]$, we obtain the following SVIEs :

$$y_1(x) = 1 + \int_0^x y_2(t)dt$$

$$y_2(x) = -1 - 1001\int_0^x y_2(t)dt - 1001\int_0^x y_1(t)dt$$

The stiff problem (20) has been solved using Rung-Kutta and modified Euler methods, these methods lead to a rapid deteriorotion of the accuracy of the solution. We obtained comparable accuracy using Taylor's method (see Tables 3 and 4).

x	$y_1(x)$			
	Taylor	Rung-Kutta	M.Euler	Exact
0	1	1	1	1
0.1	0.904833	0.904838	0.905	0.904837
0.2	0.818667	0.818731	2.045275	0.818730
0.3	0.740500	0.832877	6.319681*10^[illegible]	0.740818
0.4	0.669333	3.685541*10^[illegible]	3.255044*10^[illegible]	0.670320
0.5	0.604167	1.476020*10^[illegible]	1.676755*10^[illegible]	0.606531
0.6	0.544000	5.913424*10^[illegible]	8.637381*10^[illegible]	0.548812
0.7	0.487833	2.368268*10^[illegible]	4.449332*10^[illegible]	0.496585
0.8	0.434667	9.481293*10^[illegible]	2.291962*10^[illegible]	0.449329
0.9	0.383500	3.797161*10^[illegible]	1.180647*10^[illegible]	0.406570
1	0.333333	1.520727*10^[illegible]	6.081809*10^[illegible]	0.367879
LSE	4.524*10^[illegible]	1.521*10^[illegible]	6.082*10^[illegible]	

Table (3)

x	$y_2(x)$			
	Taylor	Rung-Kutta	M.Euler	Exact
0	−1	−1	−1	−1
0.1	−1.105171	−0.904838	−1.155	−0.904837
0.2	−1.221333	−0.818754	−1.290084*10^[illegible]	−0.818730
0.3	−0.740500	−92.799271	−6.641330*10^[illegible]	−0.740818
0.4	−0.669333	−3.685531*10^[illegible]	3.421115*10^[illegible]	−0.670320
0.5	−0.604167	−1.476020*10^[illegible]	7.762302*10^[illegible]	0.606531
0.6	−0.544000	−5.913424*10^[illegible]	9.078058*10^[illegible]	−0.548812
0.7	−0.487833	−2.368268*10^[illegible]	4.676335*10^[illegible]	−0.496585
0.8	0.434667	−9.481293*10^[illegible]	2.408897*10^[illegible]	−0.449329
0.9	−0.383500	−3.797164*10^[illegible]	1.240883*10^[illegible]	−0.406570
1	−0.333333	−1.520727*10^[illegible]	−6.392100*10^[illegible]	−0.367879
LSE	4.524*10^[illegible]	1.521*10^[illegible]	6.398*10^[illegible]	

Table(4)

8. Conclusion

A method of using Taylor series expansion has been presented for solving linear SFIEs and SVIEs as well as an important class of differential equations called "stiff" initial value problem.

In practice, we conclude that:

- The solution obtained by Taylor series expansion is given by a function , and not only at some points as in Runge-Kutta and modified Euler methods.
- Numerical computations of Taylor's method are simple and the convergence s satisfactory.
- If we attempt to increase the interval, an unsuccessful doubling of the interval being followed immediately when using Runge-Kutta and modified Euler methods.
- The difficulties that arise in attempting to obtain a numerical approximation to the solution of a "stiff" problem is that numerical stability. We overcome this stability limitation when using Taylor series expansion.

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حل تقريبي لمنظومة معادلات تكاملية خطية مع تطبيق على المسائل الصلبة

عمر محمد الفاعور

الجامعة التكنولوجية

الخلاصة

تم اشتقاق حل تقريبي لمنظومة معادلات تكاملية خطية لنوعي فريدهولم و فولتيرا باستخدام متسلسلة تيلر, وذلك بتعويض متسلسلة تيلر بدل الدالة المجهولة بعد اشتقاق طرفي المنظومة الخطية. طبقت هذه الطريقة لأيجاد حل تقريبي جيد للمسائل الصلبة (stiff problems) وتمت مقارنة نتائج طريقة تيلر مع طريقتي رنكة-كوتة واويلر المطورة بالنسبة الى هذه المسائل. استخدمت لغة MATLAB للبرمجة وباستخدام امثلة متنوعة امكن ملاحظة دقة واستقرارية هذه الطريقة.