

Development Of Six-Degree Of Freedom Strapdown Terrestrial INS Algorithm

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Abstract

Many of accurate inertial guided missile systems need to use more complex mathematical calculations and require a high speed processing to ensure the real-time operation. This will give rise to the need of developing an efficient high-speed computing system with modern inertial guided algorithms. In this work, a flexible computer simulation program for the terrestrial strapdown INS algorithm was developed and implemented for six-degree of freedom missile flight simulator. The evaluation of the algorithm is based on the accuracy of the proposed algorithm with real data.

1. Introduction

The celestial strapdown inertial navigation system is mechanized in inertial frame. This frame is widely used for spacecraft applications in which geographical information is not required [1]. But, for terrestrial navigation, the inherent time-varying relationship between the inertial and geographic frames complicates the space-stable system design [2].

Thus, the inertial frame implementation results in the most straight-forward navigation-state differential equations, which is not commonly used. The reasons for this lack of use are the difficulty in calculating gravitational forces, and the terrestrial navigation has the same

coordinate used by the GPS system, for GPS aided navigation system [3, 4].

In this work the terrestrial algorithm will be derived and implemented for six-degree of freedom. This algorithm represents a developed version of the algorithm described in [5].

The navigation equations are expressed on the basis of navigation frames, which are used in the navigation society. So, the descriptions and assumptions of the coordinate systems transformation, will introduced at Appendix A.

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2. Terrestrial Strapdown System Dynamic Equation

The differential equation of the relative quaternion between body coordinate and geographic coordinate given by [2, 6]:

$$\dot{u} = \frac{1}{2} \Omega_{ib}^b \cdot u - \frac{1}{2} \Omega_{in}^b \cdot u \dots (1)$$

Where, the angular velocity skew-symmetric matrix Ω_{in}^i and Ω_{in}^b are given by [5]:

$$\Omega_{in}^b = \begin{bmatrix} 0 & -w_D & w_E & w_N \\ w_D & 0 & -w_N & w_E \\ -w_E & w_N & 0 & w_D \\ -w_N & -w_E & -w_D & 0 \end{bmatrix} \dots \dots \dots (2)$$

$$\Omega_{in}^b = \begin{bmatrix} 0 & w_Y & -w_P & w_R \\ -w_Y & 0 & w_R & w_P \\ w_P & -w_R & 0 & w_Y \\ -w_R & -w_P & -w_Y & 0 \end{bmatrix} \dots \dots \dots (3)$$

and

$$\begin{bmatrix} w_N \\ w_E \\ w_D \end{bmatrix} = \begin{bmatrix} (|w_{ic}| + \dot{l}) \cos L \\ \dot{l} \\ -(|w_{ic}| + \dot{l}) \sin L \end{bmatrix} \dots \dots \dots (4)$$

where

$[L, l, h]$: are geodetic positions (latitude, longitude, and height)

w_R, w_P, w_Y are the body angular velocities in the body coordinate (roll, pitch, and yaw), respectively.

Body fixed coordinate to navigation coordinate (C_b^n) can be described in terms of the quaternion parameters [6]

$$C_b^n = \begin{bmatrix} a_0^2 + a_1^2 + a_2^2 + a_3^2 & 2(a_1a_2 - a_0a_3) & 2(a_1a_3 + a_0a_2) \\ 2(a_0a_3 + a_1a_2) & a_0^2 - a_1^2 - a_2^2 + a_3^2 & 2(a_2a_3 - a_0a_1) \\ 2(a_1a_3 - a_0a_2) & 2(a_0a_1 + a_2a_3) & a_0^2 - a_1^2 - a_2^2 + a_3^2 \end{bmatrix} \dots \dots \dots (5)$$

The differential equations of the vehicle position in terms of latitude, longitude, and heading can be arranged in matrix form instead of set of equations as described in [6, 7]:

$$\begin{bmatrix} \dot{L} \\ \dot{l} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} 1/(R_N + h) & 0 & 0 \\ 0 & 1/(R_N + h) \cos L & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} \dots \dots \dots (6)$$

where

$[V_N \ V_E \ V_D] = V^n$: geodetic velocity vector (north, east, and down)

R_N and R_E : are the radii of curvature in the north and east direction and given by [8]:

$$R_N = \frac{R_c}{(1 - e^2 \sin^2(L))^{1.5}} \dots \dots (7)$$

$$R_E = \frac{R_c}{\sqrt{(1 - e^2 \sin^2(L))}} \dots \dots (8)$$

and e : eccentricity (= 0.0818)

The differential equations relating the second derivative of the geodetic position and velocities can be derived as [5]:

$$\begin{bmatrix} \ddot{L} \\ \ddot{l} \\ \ddot{h} \end{bmatrix} = \begin{bmatrix} \dots \left[\frac{V_D}{(R_N + h) \cos L} \right] V_N \sin L + \frac{V_N V_D}{(R_N + h)} \\ \dots \left[\frac{V_D}{(R_N + h) \cos L} \right] V_N \sin L + \frac{V_D^2}{(R_N + h)} + 2w_x V_D \cos L \\ \dots \left[\frac{V_D^2}{(R_N + h)} - \frac{V_N^2}{(R_N + h)} \right] \dots \dots \dots \end{bmatrix} \dots \dots \dots (9)$$

where

f^b : Specific force outputs in the body coordinate = $[f_x, f_y, f_z]^T$

g_e : Gravity force applied on down direction