# Periods for Transversal Coincidence Maps on Compact Manifolds With a given Cohomology ( Homology )

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## ABSTRACT

Let $M$ be a compact connected smooth manifold such that its rational cohomology ( homology ) is $H^j(M;Q) \approx Q$ ( $H_j(M;Q) \approx Q$ ) if $j \in J \cap \{0\}$ $H^j(M;Q) \approx \{0\}$ ( $H_j(M;Q) \approx \{0\}$ ) otherwise , were $J$ is a subset of the set of natural numbers $N$ with cardinal 1 or 2 . A $C^1$ maps $f$ , $g$ : $M \to M$ is called transversal coincidence maps if for all $m \in N$ the graph of $f^m$ intersects transversally the graph of $g^m$ at each point $(x, f^m(x) = g^m(x))$ such that $x$ is a coincidence point of $f^m$ and $g^m$.

This paper study the set of periods of $f$ and $g$ by using the Lefschetz coincidence numbers for periodic coincidence points .


## INTRODUCTION

Let $M$ be a compact connected manifold of dimension $n$ , let $f$ , $g$ : $M \to M$ be two continuous maps . a coincidence point of $f$ and $g$ is a point $x$ of $M$ such that $f(x) = g(x)$ . The point $x \in M$ is periodic coincidence point with period $m$ if $f^m(x) = g^m(x)$ but $f^k(x) \neq g^k(x)$ for all $k = 1, \ldots, m-1$, ( see [7,8] ) . Let *per(f,g)* denote set of all periods of periodic coincidence point of $f$ and $g$ .

For each $p \geq 0$ define the endomorphism $\theta_p : H_p(M;Q) \to H_p(M;Q)$ of the rational homology groups , as the composition round the following square

$$\begin{array}{ccc} H_p(M;Q) & \xrightarrow{f_{*p}} & H_p(M;Q) \\ \approx D_p \downarrow & \theta_p & \downarrow \approx D_p \\ H^{n-p}(M;Q) & \xleftarrow[g^{*n-p}]{} & H^{n-p}(M;Q) \end{array}$$

where $D_p$ is Poincare duality isomorphism or its inverse . Then the Lefschetz coincidence number $L_{f,g}$ of $f$ and $g$ is defined by

$$L_{f,g} = \sum_{0 \leq p \leq n} (-1)^p \, Trace\, \theta_p .$$

By Lefschetz coincidence point theorem if $L_{f,g} \neq 0$ then $f$ and $g$ have coincidence point ( see [ 4] ) . It is not true (in general ) that if $L_{f^m,g^m} \neq 0$ then $f$ and $g$ have periodic coincidence points of period $m$ . It could have periodic

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coincidence points with period some proper division of $m$. Therefore ,we will use the Lefschetz coincidence numbers for periodic coincidence points introduced in [1] , more précis-ely , for every $m \in N$ define the Lefschetz coincidence numbers of per-iod $m$ ,$l(f^m, g^m)$ as follows

$$l(f^m, g^m) = \sum_{r/m} \mu(r) L_{f^{\frac{m}{r}}, g^{\frac{m}{r}}}$$

where $\sum_{r/m}$ denotes the sum over all positive divisor $r$ of $m$, and $\mu$ is the Moebius function defined by

$$\mu(m) = \begin{cases} 1 & if \quad m=1, \\ 0 & if \quad k^2 | m \ for\ some\ k \in N, \\ (-1)^r & if \quad m = p_1 \ldots p_r\ distinct\ prime\ factors \end{cases}$$

Acording to the inversion formula (see for instance [ 6 ] ).

$$L_{f^m, g^m} = \sum_{r/m} l(f^r, g^r).$$

If $l(f^m, g^m) \neq 0$ then $f$ and $g$ have a periodic coincidence point of period $m$ ( see [ 2] ) . This is almost the case when $f$ and $g$ are transversal maps.

A $C^1$ maps $f$ and $g : M \to M$ defined on a compact $C^1$ smooth mani-fold is called transversal coincidence maps if $f(M) \subset Int(M)$ and $g(M) \subset Int(M)$ and if for all $m \in N$ at each point $x$ such that $f^m(x) = g^m(x)$ , we have $\det(df^m(x) - dg^m(x)) \neq 0$, i.e. 1 is not an eigenvalue of $df^m(x) - dg^m(x)$, ( were $df^m(x)$ and $dg^m(x)$ are the derivative of $f^m$ and $g^m$ at $x$ respecti-vely ) .From [3 ] it is note that if $f$ and $g$ are transversal coincidence maps then for all $m \in N$ the graph of $f^m$ intersects transversally the graph of $g^m$ at each point ( $x$, $f^m(x) = g^m(x)$) . Since for transversal coincidence maps $f$ and $g$ the coincidence points of $f^m$ and $g^m$ are isolated and $M$ is compact then the number of coincidence points of $f^m$ and $g^m$ is finite for every $m \in N$. The following theorem is important in this paper and it was proven in [ 2 ] .

***Theorem (1) :-***

***Let $f, g : M \to M$ be a transversal coincidence maps on a compact conne-cted manifold $M$ of dimension $n$ . Suppose that $l(f^m, g^m) \neq 0$ for some $m \in N$.***

***a) If $m$ is odd then $m \in per(f, g)$ .***

***b) If $m$ is even then $\{m62, m\} \cup per(f, g) \neq \varnothing$.***

The results in theorem (1) are in general difficult to apply because of the computation of $l(f^m, g^m)$ . If the rational cohomology (homology)groups are simple then the computation of $l(f^m, g^m)$ become easier . This paper deal with transversal coincidence maps on a compact manifold $M$ with rational cohomology (homology) .

$$\begin{array}{ll} H^j(M;Q) \approx Q\ (H_j(M;Q) \approx Q) & if\ j \in J \cup \{0\}; \\ H^j(M;Q) \approx \{0\}\ (H_j(M;Q) \approx \{0\}) & otherwise \end{array} \quad (1)$$

Here $J$ is a subset of the set of natural numbers $N$ with cardinality 1 or 2.

Transversal coincidence maps with such cohomology ( homology ) are nontrivial maps for which to compute the numbers $l(f^m, g^m)$ and apply theorem(1) to obtain information about their sets of periods .

***THE MAIN RESULTS***

Let $f,g : M \to M$ be a transversal maps and suppose that the rational cohomology (homology) of $M$ satisfies (1) . Let $(a_j)$ be the $1\times 1$ integer matrix defined by the homology endomorphism $\theta_j : H_j(M;Q) \to H_j(M;Q)$, where $\theta_j = D_j^{-1} g_{*n-j} D_j f^{*j}$, for each $j \in J \cap \{0\}$. Then

$$L_{f^m,g^m} = \sum_{j\in J\cup\{0\}} (-1)^j a_j^m \text{ for all } m \in N .$$

Therefore , if m > 1 the Lefschetz coincidence number of period $m$ will be

$$l(f^m, g^m) = \sum_{r/m} \mu(r) L_{f^{\frac{m}{r}}, g^{\frac{m}{r}}}$$

$$= \sum_{r/m} \mu(r) [\sum_{j\in J\cup\{0\}} (-1)^j a_j^{\frac{m}{r}}]$$

$$= \sum_{j\in J\cup\{0\}} (-1)^j \sum_{r/m} \mu(r) a_j^{\frac{m}{r}} .$$

For each $m \in N$ we define the polynomial

$$Q_m(x) = \sum_{r/m} \mu(r) x^{\frac{m}{r}} .$$

Then , if $m > 1$ we can write

$$l(f^m, g^m) = \sum_{j\in J\cup\{0\}} (-1)^j Q_m(a_j) \qquad (2)$$

Set $m = p_1^{\alpha_1} \ldots p_n^{\alpha_n}$ here $p_1 ,\ldots, p_n$ are distinct primes . The next proposition is proven in [2 ] .

***Proposition (2) :-***

***Let m*** $\in N$.

***a)If m is odd then*** $Q_m$ ***is an odd function , i. e.,*** $Q_m(x) = -Q_m(-x)$.

***b) If*** $4/m$ ***then*** $Q_m$ ***is an even function , i. e.*** $Q_m(x) = Q_m(-x)$.

***c) If 2 6 m and 4Φ m then*** $Q_m(x) = Q_{\frac{m}{2}}(x^2) - Q_{\frac{m}{2}}(x)$.

***d)*** $Q_m(0) = 0$.

***e) If m > 1 , then*** $Q_m(1) = 0$.

***f) If m > 2 , then*** $Q_m(-1) = 0$ .

***g) For all i*** $\in N$ ***we have*** $Q_m^{(i)}(1) \geq 0$ ***, where*** $Q_m^{(i)}(x)$ ***denote the*** $i-th$ ***derivative of*** $Q_m(x)$ ***with respect to the variable x .***

***h)*** $Q_m(x)$ ***is positive and increasing in in (1,∝) .***

***i) If m is even then the function*** $Q_m(x)$ ***is positive and decreasing in (-∝, -1) . Furthermore , if 4Φ m we have that*** $Q_m(x) \leq Q_m(-x)$ ***for all x*** $\in$ ***[1 ,∝) .***

***j) If m >2 , then*** $Q_m(1.6) > 2$.

The main results on the set of periods of periodic coincidence points of transversal coincidence maps following from the next two theorems :-

***Theorem (3) :-***

***Let f,g: $M \to M$ be a transversal coincidence maps. Suppose that the rational cohomology (homology) of M satisfies (1) with J = {p}. Denote by $(a_j)$ the 1×1 integer matrix defined by the induced homology endomorphism $\theta_j : H_j(M;Q) \to H_j(M;Q)$ for each $j \in J \cap \{0\}$ then the following statements hold.***

$l(f,g) = L_{f,g} = a_0 + (-1)^p a_p$.

***b)*** $l(f^2,g^2) = 0$ ***if and only if P is even and*** $\{a_0,a_p\} \subset \{0,1\}$***, or p is odd and*** $a_0 = a_p$ ***or*** $a_0 + a_p = 1$.

***c) If*** $\{a_0,a_p\} \subset \{-1, 0, 1\}$***, then*** $l(f^m,g^m) = 0$ ***for every natural number m >2.***

***d) If*** $\{a_0,a_p\} \not\subset \{-1, 0, 1\}$ ***and m > 1 is odd, then*** $l(f^m,g^m) = 0$ ***if and only if*** $a_0 + (-1)^p a_p = 0$.

***e) if*** $\{a_0,a_p\} \not\subset \{-1,0,1\}$ ***and 4 6 m, then*** $l(f^m,g^m) = 0$ ***if and only if p is odd and*** $a_0 = \pm a_p$.

***f) If*** $\{a_0,a_p\} \not\subset \{-1,0,1\}$***, m >2 is even and 4 Φ m, then*** $l(f^m,g^m) = 0$ ***if and only if p is odd and*** $a_0 = a_p$.

***Proof :-***

From the definitions of $l(f,g)$ and $L_{f,g}$ it follows

$$l(f,g) = L_{f,g} = Trace(\theta_0) + (-1)^p Trace(\theta_p) = a_0 + (-1)^p a_p.$$

Which prove a). From (2) we get $l(f^2,g^2) = Q_2(a_0) + (-1)^p Q_2(a_p)$, and from proposition 2 (c) we get

$Q_2(x) = x^2 + x$. Therefore,

$$l(f^2,g^2) = (a_0^2 - a_0) + (-1)^p (a_p^2 - a_p).$$
$$= (a_0^2 + (-1)^p a_p^2) - (a_0 + (-1)^p a_p).$$

Assume $p$ is odd then

$$l(f^2,g^2) = (a_0 - a_p)(a_0 + a_p + 1).$$

$l(f^2,g^2) = 0$ if and only if $a_0 = a_p$ or $a_0 + a_p = 1$. Assume that $p$ is even then

$$l(f^2,g^2) = (a_0^2 + a_p^2) - (a_0 + a_P).$$

$l(f^2,g^2) = 0$ if and only if $a_0^2 + a_P^2 = a_0 + a_P$, or equivalently $\{a_0,a_P\} \subset$

***a)***

$\{0,1\}$. Therefore, b) is proven.

From (2) we get $l(f^m,g^m) = Q_m(a_0) + (-1)^P Q_m(a_P)$. If $a_0 = a_P = 0$ then from d), $Q_m(0) = 0$, so $l(f^m,g^m) = 0$ for every natural number $m$. Similar from statement e) and f) of proposition (2) we get $l(f^m,g^m) = 0$ for every natural number $m$. Therefore, c) is proven.

Let $m$ be odd from (2) we get

$$l(f^m,g^m) = Q_m(a_0) + (-1)^P Q_m(a_P).$$

From a) and h) of proposition (2) we have that $Q_m(x)$ is an increasing odd function in $(-\propto, -1) \cap (1,\propto)$. Assume that $\{a_0,a_p\} \not\subset \{-1, 0, 1\}$ then $l(f^m,g^m) = 0$ if and only if $Q_m(a_0) + (-1)^p Q_m(a_p) = 0$ or equivalently $a_0 + (-1)^p a_p = 0$. Hence d) is proven. Assume that 4 6 $m$. From b), h) and i) of proposition (2), we have $Q_m(x)$ is an even function, increasing in $(1, \propto)$, and $Q_m(x) = Q_m(-x)$ for all $x \in [1, \propto)$. Assume $\{a_0,a_p\} \not\subset \{-1, 0, 1\}$. Then

from (2) we get that

$l(f^m, g^m) = Q_m(a_0) + (-1)^p Q_m(a_p) = 0,$ if and only if $p$ is odd and $a_0 = \pm a_p$. Hence e) is proven.

Assume that $m > 2$ is even and $4 \Phi m$ . From (2) we get $l(f^m,g^m)=0$ if and if $Q_m(a_0)+(-1)^p Q_m(a_p)=0$.

Assume that $\{a_0, a_p\} \not\subset \{-1,0,1\}$.From statements h) and i) of proposition (2) we have that $Q_m(a_0)$ and $Q_m(a_p)$ are positive.

So, if $p$ is even then $l(f^m, g^m) \neq 0$.

In the rest of the proof of statement f) suppose that $p$ is odd . Then $l(f^m,g^m)=0$ if and only if $Q_m(a_0)=Q_m(a_p)$. By proposition 2(h), $Q_m(x)$ is positive and increasing in $(1,\infty)$.So,if $a_0,a_p>0$ then $l(f^m,g^m)=0$ if and only if $a_0 = a_p$. Now assume that $a_0, a_p < 0$. By proposition 2( c) we have that $Q_m(x)= Q_{\frac{m}{2}}(x^2) - Q_{\frac{m}{2}}(x)$ with $m/2$ odd . Then $l(f^m,g^m)=0$ if and only if $Q_{\frac{m}{2}}(a_0^2) - Q_{\frac{m}{2}}(a_0) = Q_{\frac{m}{2}}(a_p^2) - Q_{\frac{m}{2}}(a_p)$ , or equivalently $Q_{\frac{m}{2}}(a_0^2)+Q_{\frac{m}{2}}(|a_0|)=Q_{\frac{m}{2}}(a_p^2)+ Q_{\frac{m}{2}}(|a_p|)$ ( see proposition 2 (a) ) , if $a_0, a_p < 0$ then from proposition 2 (h) $l(f^m,g^m)=0$ if and only if $a_0 = a_p$. So , in the rest of the proof of f) assume that $a_0 < 0 < a_p$.

Consider $0 < a_p < -a_0$. Since $l(f^m,g^m)=0$ if and only if $Q_{\frac{m}{2}}(a_0^2)+Q_{\frac{m}{2}}(|a_0|)=$ that only $Q_{\frac{m}{2}}(a_p^2) - Q_{\frac{m}{2}}(a_p)$ , and $0 < a_p < |a_0|$, by statements a) and h) of proposition (2) we obtain that $Q_{\frac{m}{2}}(a_0^2) + Q_{\frac{m}{2}}(|a_0|) > Q_{\frac{m}{2}}(a_p^2) - Q_{\frac{m}{2}}(a_p)$, hence $l(f^m,g^m)\neq 0$ .So we can assume that $0 < -a_0 < a_p$ .

By proposition 2 ( g ) we have that $Q_m(x) = \sum_{k=1}^{m} A_k (x-1)^k$ with $A_k = Q_m^{(k)}(1)/k! \geq 0$. Therefore , since $l(f^m, g^m) = 0$ if and only if $Q_m(a_p) = \sum_{k=1}^{m} A_k (a_p - 1)^k = \sum_{k=1}^{m} A_k (a_0 - 1)^k = Q_m(a_0)$ , it follows that if $|a_0| + 1 < a_p - 1$ , then $Q_m(a_p) > Q_m(a_0)$, and consequently $l(f^m, g^m) \neq 0$. Hence, since $0 < -a_0 < a_p$ the unique cases that remain to consider are $|a_0|+1=|a_p|-1$ and $|a_0| + 1 = a_p$.

Assume that $a_0 + 1 = a_p - 1 = b$ . So $a_p = b + 1$ and $a_0 = 1 - b$. Therefore , since

$\{a_q,a_p\} \not\subset \{-1,0,1\}$ we get that $b \geq 1$. Now $l(f^m,g^m)=0$ if and only if

$Q_m(b+1)=Q_m(-b+1)$ , or equivalently

$Q_{\frac{m}{2}}((b+1)^2)-Q_{\frac{m}{2}}(b+1)=Q_{\frac{m}{2}}((b-1)^2)-$

$Q_{\frac{m}{2}}(1-b)$ ( see proposition 2 (c)) .

Since $m6$ is odd from proposition 2(a)

$l(f^m,g^m)=0$ if and only if $Q_{\frac{m}{2}}((b+1)^2)+$

$Q_{\frac{m}{2}}(b+1)=Q_{\frac{m}{2}}((b-1)^2)+Q_{\frac{m}{2}}(b-1)$,

or equivalently

$$\sum_{k=1}^{\frac{m}{2}} B_k[(b+1)^2-1]^k=\sum_{k=1}^{\frac{m}{2}} B_k[b^k+((b-1)^2-1)^k+(b-2)^k],$$

because $Q_{\frac{m}{2}}(x)=\sum_{k=1}^{m} B_k(x-1)^k$ , with

$B_k=Q_{\frac{m}{2}}^{(k)}(1)/k! \geq 0$. Therefore , since

$[(b+1)^2-1]^k=b^k(b+2)^k>b^k[2+(b-2)^k]>$
$b^k[1+(b-2)^k]+(b-2)^k=b^k+$
$((b-1)^2-1)^k+(b-2)^2$ if $b \geq 2$ ,
and $[(b+1)^2-1]^k>b^k+((b-1)^2-1)^k+(b-2)^k$ if $b=1$ , it follows that $l(f^m,g^m)\neq 0$.

Finally assume that $|a_0|+1=a_p=b+1$. So , $a_0=-b$ . Therefore since $\{a_0,a_p\}$ $\not\subset\{$ -1,0 1 we get that $b \geq 1$ . Now

$l(f^m,g^m)=0$ if and only if $Q_m(b+1)=Q_m(-b)$, or equivalently $Q_{\frac{m}{2}}((b+1)^2)-Q_{\frac{m}{2}}(b+1)=$

$Q_{\frac{m}{2}}(b^2)-Q_{\frac{m}{2}}(-b)$( see proposition 2(c)) .

Since $m62$ is odd, from proposition 2(a) $l(f^m,g^m)=0$ if and only if $Q_{\frac{m}{2}}((b+1)^2)=$

$Q_{\frac{m}{2}}((b+1)^2)=Q_{\frac{m}{2}}(b+1)+Q_{\frac{m}{2}}(b^2)+Q_{\frac{m}{2}}(b)$

or equivalently .

$$\sum_{k=1}^{\frac{m}{2}} B_k[(b+1)^2-1]^k=\sum_{k=1}^{\frac{m}{2}} B_k[b^k+(b^2-1)^k+(b-1)^k].$$

Therefore,since $[(b+1)^2-1]^k=b^k(b+2)^k>$
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$[(b-1)^k+1](b+2)^2>(b-1)^k(b+2)+$

$b^k=b^k+(b^2-1)^k+(b-1)^k$ if $k \geq 2$ and $b \geq 1$

, and $[(b+1)^2-1]^k>b^k+(b^2-1)^k+$

$(b-1)^k$ if $k=1$ ,and $b \geq 1$ ;it follows that

$l(f^m,g^m)\neq 0$. Hence f) is proven . □

From theorem (1) and theorem (3)

it follows easily the following corollary.

***Corollary (4) :-***

***In the assumptions of theorem (3) the following statements hold.***

***a) If*** $a_0+(-1)^p a_p \neq 0$, ***then 1*** $\in$ ***per (f ,g ).***

***b) If neither p is even and*** $\{a_0,a_p\}\subset\{0,1\}$ ***,nor p is odd and*** $a_0=a_p$ ***or*** $a_0+a_p=1$ ***then { 1 ,2 }*** $\cup$ ***per (f , g)*** $\neq \varnothing$.
}

***c) If*** $\{a_0,a_p\}\subset\{-1,0,1\}$, ***then the unique periods m that can be forced from the numbers*** $l(f^m,g^m)$ ***are 1 and 2.***

***d) If*** $\{a_0, a_p\} \not\subset \{-1,0,1\}$ ***and*** $a_0 + (-1)^p a_p \neq 0$ ***, then*** $\{3, 5, 7, \ldots\} \subset per(f, g)$.

***e) If*** $\{a_0, a_p\} \not\subset \{-1, 0, 1\}$ ***, p is odd,*** $a_0 \neq \pm a_p$ ***, 4 6 m, and*** $m \notin per(f, g)$ ***, then*** $\{m62, m\} \subset per(fg)$ .

***f) If*** $\{a_0, a_p\} \not\subset \{-1,0,1\}$***, p and m>2 are even*** $m \notin per(f, g)$***, then*** $\{m62, 2m\} \subset per(f, g)$.

***g) If*** $\{a_0, a_p\} \not\subset \{-1,0,1\}$ ***, p is odd,*** $a_0 \neq a_p$***, m>2 is even and 4 Φ m, then*** $\{m62, m\} \cup per(f, g) \neq \varnothing$.

The following corollary is the special case of theorem (3) when $g$ the identity map .

***Corollary (5) :-***

***Let f, g :M→M be a transversal coincidence maps. Suppose that g is the identity map and the rational homology on M satisfies (1) with J =***

***{p}. Denote the (*** $a_j$ ***) the 1×1 integer matrix defined by the induced homology .*** $f_{*j} : H_j(M;Q) \to H_j(M;Q)$ ***for each*** $j \in N$***. Then the following statements holds.***

***a)*** $l(f, g) = L_{f,g} = 1 + (-1)^p a_p$ .

***b) If*** $l(f^2, g^2) = 0$ ***if and only if*** $a_p \in \{0,1\}$.

***c) If*** $m > 2$ $l(f^m, g^m) = 0$ ***and only if*** $a_p \in \{-1, 0, 1\}$.

***Proof :-***

Since $g$ is the identity map then for each $k \in N$ $g^{*k}$ is the identity map ,so $(a_i)$ the $1\times1$ integer matrices defined on the induced homology $\theta_i = f_{*i} : H_i(M;Q) \to H_i(M;Q)$ , were $i \in \{0, p\}$ and since $H_0(M;Q) \approx Q$, $M$ is connected ,then consequently $f^{*0}$ is the identity map ( see [5] for more details ), so $a_0 = 0$.

From the definition of $l(f, g)$ and $L_{f,g}$ we get that $l(f,g) = L_{f,g} = 1 + (-1)^p a_p$, this proves a ) .

From (2) we get $l(f^m, g^m) = (-1)^p Q_m(a_p)$ ,so $l(f^2, g^2) = (-1)^p Q_2(a_p)$ .From proposition (2) we get that, $Q_2(x) = x^2 - x$. So, $Q_2(a_p) = a_p^2 - a_p = a_p(a_p - 1)$, and consequently , $l(f^2, g^2) \neq 0$ if and only if $a_p \in \{0, 1\}$ . So , b) is proven .

If m>2 then from statements d) , e), and f) of proposition (2) we obtain $l(f^m, g^m) = 0$ if $a_p \in \{-1,0,1\}$. Now we assume that $a_p \notin \{-1,0,1\}$.From statements a) ,h) and i) of proposition (2) it follows that $l(f^m, g^m) = (-1)^p Q_m(a_p) \neq 0$. Hence c) follows . □

From theorem (1) and corollary (5) it follows the following corollary

***Corollary (6) :-***

***In the assumptions of corollary (5) if we assume that*** $a_p \notin \{-1,0,1\}$ ***then the following statements hold.***

***a)*** $\{1, 3, 5, 7, \ldots\} \subset per(f, g)$.

***b) If m is even and*** $m \notin per(f, g)$ ***, then*** $\{m\,62, 2m\} \subset per(f, g)$.

Let $S$ be the set of $(z_1, z_2, z_3) \in Z_0^3$ ***then*** with $Z_0^3 = Z\backslash\{-1,0,1\}$ satisfying at least one of the following conditions :

1) All the components have the same sign .
2) $|z_i| < \max_{j\neq i}\{z_j\}$ if $z_i$ is the component that has different sign .
3) $|z_i| > \sum_{j\neq i} |z_j|$ .

***Theorem (7) :-***

***Let f, g : $M \to M$ be a transversal coincidence maps . Suppose that the rational cohomology (homology) on M satisfies (1) with $J=\{p,q\}$. Denote by $(a_j)$ the $1\times 1$ integer matrix defined by the induced homology endomorphism $\theta_j : H_j(M;Q) \to H_j(M;Q)$ for each $j \in J \cap \{0\}$. Assume that p is even ( respectively odd ). Then the the following statements hold.***

***a) Let $m > 1$ be odd . If q is even (respectively odd) and $(a_0,a_p,a_q) \in S$ ,or q is odd (respectively even) and $(a_0,a_p,-a_q) \in S$ ,then $l(f^m,g^m) \neq 0$.***

***b) Let $m > 1$ be even. If q is even (respectively odd) , or q is odd (respectively even) and $(|a_0|,|a_p|,-|a_q|) \in S$, then $l(f^m,g^m) \neq 0$.***

***Proof :-***

Assume $p$ is even.The case $p$ is odd follows in a similar way .Also suppose that $m>1$ is odd , $q$ is even and $(a_0,a_p,a_q) \in S$.If $a_0,a_p$ and $a_q$ have the same sign, , then from (2) $l(f^m,g^m) = Q_m(a_0) + (-1)^p Q_m(a_p) + (-1)^q Q_m(a_q) = Q_m(a_0) + Q_m(a_p) + Q_m(a_q)$ have the same sign . From statements a) and h) of proposition (2) we have $Q_m(a_0), Q_m(a_p)$ and $Q_m(a_q)$ are an increasing odd functions in $(1,\infty)$, it follows that $l(f^m,g^m) = Q_m(a_0) + Q_m(a_p) + Q_m(a_q) \neq 0$.

Assume $a_q < -1$ and $1 < a_0 \le a_p$ , if $|a_q| < a_0$( i. e. $|a_q| < \max\{a_0,a_p\}$ ) it follows that $l(f^m,g^m) = Q_m(|a_0|) + Q_m(|a_p|) - Q_m(|a_q|) > Q_m(|a_p|) - Q_m(|a_q|) > 0$ . i .e ., $l(f^m,g^m) \neq 0$.

If $|a_q| > |a_0| + |a_p|$ we have that

$$Q_m(|a_q|) > Q_m(|a_0| + |a_p|) = \sum_{k=1}^{m} A_k(|a_0| + |a_p| - 1)^k,$$

because by proposition 2(g) we have that $Q_m(x) = \sum_{k=1}^{m} A_k (x-1)^k$ with $A_k = Q_m^{(k)}(1)/k! \ge 0$. Since $(|a_0| + |a_p| - 1)^k > [(|a_0| - 1) + (|a_p| - 1)]^k \ge (|a_0| - 1)^k + (|a_p| - 1)^k$, we obtain that

$$\sum_{k=1}^{m} A_k (|a_0| + |a_p| - 1)^k > \sum_{k=1}^{m} A_k (|a_q| - 1)^k + \sum_{k=1}^{m} A_k (|a_p| - 1)^k = Q_m(|a_0|) + Q_m(|a_p|) .$$

Hence

$$Q_m(|a_q|) > Q_m(|a_0| + |a_p|) > Q_m(|a_0|) + Q_m(|a_p|), \quad (3)$$

and consequently $l(f^m,g^m) \neq 0$.

Now a ssume that $q$ is odd .Then $l(f^m,g^m) = Q_m(a_0) + Q_m(a_p) - Q_m(a_q) = Q_m(a_0) + Q_m(a_p) + Q_m(-a_q)$, and since $(a_0,a_p,-a_q) \in S$ the argument of the above case can be again to obtain that $l(f^m,g^m) \neq 0$, so a) is proven .

Assume that $m > 1$ is even . By

statements b),h) and i)of proposition(2)

function $Q_m(x)$is positive in$(-\infty,-1)\cap$

$(1,\infty)$.Therefore,if $q$ is even $l(f^m,g^m)=$

$Q_m(a_0)+Q_m(a_p)-Q_m(a_q)$and consider two cases.

*case 1*: $4 \not| m$ .By proposition 2(b) the function $Q_m(x)$ is even . Therefore , $l(f^m,g^m)=Q_m(|a_0|)+Q_m(|a_p|)-Q_m(|-a_q|)$ . From the assumptions we have that $(|a_0|,|a_p|,-|a_q|)\in S$ .So we can repeat the the arguments of the proof of statement a) and obtain $l(f^m,g^m)\neq 0$.

*case 2* : $4 \mid m$ . We have $|a_q|<\max\{|a_0|,|a_p|\}$ or $|a_q|>|a_0|+|a_p|$, because $(|a_0|,|a_p|,-|a_q|)\in S$ we assume that the first inequality holds .Then from statements h) and i) of proposition (2) , $l(f^m,g^m)=$ $Q_m(|a_p|)-Q_m(-|a_q|)$ . Since $(|a_0|,|a_p|,-|a_q|)$ $\in S$ we can repeat the arguments of the proof of statements a) , and obtain $l(f^m,g^m)\neq 0$.

Now assume that the second inequality holds.Then by statements c) and a) of proposition (2) ,and by (3) $Q_m(-|a_0|+|a_p|)=Q_{\frac{m}{2}}((|a_0|+|a_p|)^2)+Q_{\frac{m}{2}}(|a_0|+|a_p|)\geq$

$Q_{\frac{m}{2}}((|a_0|^2+|a_p|^2)+Q_{\frac{m}{2}}(|a_0|+|a_p|)>$

$Q_{\frac{m}{2}}(|a_0|^2)+Q_{\frac{m}{2}}(|a_p|^2)+Q_{\frac{m}{2}}(|a_0|)+Q_{\frac{m}{2}}(|a_p|)=$

$Q_m(-|a_0|)+Q_m(-|a_p|)$ .Therefore,from statements h) and i) of proposition (2) and by (3)$l(f^m,g^m)=Q_m(a_0)+Q_m(a_p)-Q_m(a_q)$ $\leq Q_m(-|a_0|)+Q_m(-|a_p|)-Q_m(-|a_q|)<$ $Q_m(-(|a_0|+|a_p|))-Q_m(|a_q|)\leq Q_m(|a_0|+|a_p|)-Q_m(a_q)<0$ . Hence b) is proven . □

From theorem (1) and theorem (7) ,the then it follows immediately the following corollary

***Corollary (8) :-***

***In the assumption of theorem (7) the following statements hold***

***a) Let m > 1 be odd . If q is even (respectively odd ) and*** $(a_0,a_p,a_q)\in$ ***S ,or q is odd ( respectively even )and*** $(a_0,a_p,-a_q)\in$ ***S, then { 3,5,7,... }*** $\subset$ ***per (f , g) .***

***b) Let m > 1 be even . If q is even ( respectively odd ) , or q is odd ( respectively even ) and*** $(|a_0|,|a_p|,-|a_q|)\in$ ***S, then { m 6 2,m }*** $\cup$ ***per (f ,g)*** $\neq\varnothing$.

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# الدوريات لدوال متطابقة مستعرضة على مطوي مرصوص معطى له الكوهومولوجية ( الهومولوجية )

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**المستخلص**

ليكن $M$ مطوي مرصوص مترابط املس بحيث ان الكوهومولوجية ( الهومولوجية ) له تحقق $H^j(M;Q) \approx Q(H_j(M;Q) \approx Q)$ اذا كان $j \in J \cap \{0\}$ و $H^j(M;Q) \approx \{0\}(H_j(M;Q) \approx \{0\})$ خلافا لذلك حيث $J$ مجموعة جزئية من مجموعة الاعداد الطبيعية وعدد عناصرها 1 أو 2 .

الدوال $f, g : M \to M$ من النوع $C^1$ يقال بأنها متطابقة مستعرضة اذا كان لكل $m \in N$ الرسم البياني ل $f^m$ يتقاطع مستعرضا مع الرسم البياني ل $g^m$ عند كل نقطة $(x , f^m(x) = g^m(x))$ بحيث ان $x$ نقطة تطابق ل $f^m$ و $g^m$ .

هذا البحث يتناول الدوريات ل $f$ و $g$ بأستخدام اعدا د لبشز للتطابق لنقاط متطابقة دورية .