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Estimation of Parameters for the Gumbel Type-I Distribution under Type-II Censoring Scheme

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Abstract:

This paper aims to decide the best parameter estimation methods for the parameters of the Gumbel type-I distribution under the type-II censorship scheme. For this purpose, classical and Bayesian parameter estimation procedures are considered. The maximum likelihood estimators are used for the classical parameter estimation procedure. The asymptotic distributions of these estimators are also derived. It is not possible to obtain explicit solutions of Bayesian estimators. Therefore, Markov Chain Monte Carlo, and Lindley techniques are taken into account to estimate the unknown parameters. In Bayesian analysis, it is very important to determine an appropriate combination of a prior distribution and a loss function. Therefore, two different prior distributions are used. Also, the Bayesian estimators concerning the parameters of interest under various loss functions are investigated. The Gibbs sampling algorithm is used to construct the Bayesian credible intervals. Then, the efficiencies of the maximum likelihood estimators are compared with Bayesian estimators via an extensive Monte Carlo simulation study. It has been shown that the Bayesian estimators are considerably more efficient than the maximum likelihood estimators. Finally, a real-life example is also presented for application purposes.

Keywords: Bayesian Methods, Gumbel Type-I Distribution, Simulation, Type -II Censoring.

Introduction:

The Gumbel distribution (GD) was first proposed by Gumbel in 1941[1]. It is widely used in meteorological phenomena, hydrology, and so on. GD is one of the important distributions in modeling extreme values such as maximum daily flood discharges and snowfalls, rainfalls, and extreme temperatures, see[2-4]. This distribution is called extreme value type- I distribution. It has two types. The first type is based on minimum order statistics and the second type is based on maximum order statistics. Here, the first type is discussed.

It is very important to obtain the model parameters of any distribution effectively and precisely. Therefore, the GD has been studied in the literature by numerous researchers. Abbas and Tang[5] obtained the Bayesian parameter estimation methods for the Gumbel type-II distribution. Malinowska and Szynal[6] discussed Bayesian estimators for GD on *kth* lower record values. Yılmaz et al.[7] compared different parameter estimation methods, including both classical and Bayesian for the two-parameter GD. Saleh[8] studied the unknown parameters of the Gumbel type-I distribution based on the moment and modification moment methods. Reyad and Ahmed[9] obtained E-Bayesian estimators of the GD under the type-II censored scheme. Saad et al.[10] introduced the Gumbel-Pareto distribution.

In survival analysis, the data sets are usually observed as censored samples. Type –II censoring type is well-known and widely used. In the type-II censoring, only the first failures $k \le m$ is observed among m units. In this censoring scheme, it is assumed that a set number of subjects or items are put on a test. The integer $k \le m$ is pre-fixed, and the experiment stops as soon as the $k-th$ failure is observed, see (Kundu and Ragab[11]). There are estimations of parameters of the different distributions using the type-II censoring in the literature. Altındağ et al. [12] studied maximum likelihood and maximum product spacing estimation methods for Burr-III distribution using type-II censored samples. Nassar et al.[13] discussed E-Bayesian estimation for the simple-stress model of exponential distribution based on the type-II censoring scheme. Okasha et al.[14] examined the properties and parameter estimation of Marshall-Olkin extended inverse Weibull distribution under type–II censoring. Xin et al. [15] obtained the reliability estimation of the three-parameter Burr-type-XII distribution under the type-II censoring scheme.

This paper focuses on the estimation of the unknown parameters of the Gumbel type-I distribution under the type-II censorship scheme with classical and Bayesian parameter estimation. The maximum likelihood (ML) estimation is used in the classical parameter estimation. The asymptotic confidence intervals (ACIs) of the unknown parameters are derived by using the observed Fisher information matrix. Since Bayesian estimators (BEs) cannot be obtained in explicit forms, Lindley (LD) and Markov Chain Monte Carlo (MCMC) methods are used for Bayesian calculations. The selection of a suitable loss function (LF) and prior distribution (PD) is of considerable importance in the Bayesian parameter estimation. Therefore, three different LFs are presented, namely the squared error loss function (SELF), the general entropy loss function (GELF), and the weighted squared error loss function (WSELF). Under these LFs, the normal and gamma PDs are used for the parameters μ and σ, respectively. The Bayesian credible intervals (BCIs) are also constructed based on the Gibbs sampling method. The performances of these estimation methods are compared through an extensive simulation study.

This paper aims to deal with the estimation of the unknown parameters of Gumbel type- I distribution based on the type-II censoring scheme under all these aforementioned estimation methods.

The rest of the paper is organized as follows: In section 2, the brief information about the Gumbel-type-I distribution is given and the ML estimators of the parameters μ and σ are presented. The limiting distributions of the ML estimators are also investigated. In Section 3, BEs of unknown model parameters are obtained by using LD, and MCMC methods. Some numerical comparisons between the ML and Bayes estimators are provided in Section 4. A real-life data set is used to illustrate the computations of the ML and Bayes estimators in Section 5. Concluding remarks are presented at the end of the paper.

Parameter Estimation:

Let Y be a random variable from the Gumbel type-I distribution with the location μ and the scale parameter σ. It's the cumulative distribution function (cdf) and the probability density functions (pdf) are given as:

$$F(y;\mu,\sigma)=1-e^{-e^{\left(\frac{y-\mu}{\sigma}\right)}},\quad -\infty<y<\infty;\ \ \mu\in\mathbb{R},\ \ \sigma>0, \qquad 1$$

$$f(y;\mu,\sigma)=\frac{1}{\sigma}\,e^{\left(\frac{y-\mu}{\sigma}\right)}e^{-e^{\left(\frac{y-\mu}{\sigma}\right)}}\quad -\infty<y<\infty;\ \ \mu\in\mathbb{R},\ \ \sigma>0, \qquad 2$$

respectively.

Suppose that $Y_1,Y_2,\dots,Y_m$ be independent and identically Gumbel type -I distributed random variables representing the lifetimes of independent units. Based on type-II censoring scheme, a sample of m identical units is put on a life testing experiment and their lifetimes are recorded, and only the first k failure times are considered, i.e. $\left(y_{(1)}<y_{(2)}<\cdots<y_{(k)}\right)$.

In this study, ML and Bayesian parameter estimation methods for Eq. 2 under the type-II censorship scheme are discussed.

Maximum Likelihood Estimation:

The likelihood function of Eq. 2 using the type-II censoring scheme is as follows:

$$L(\mu,\sigma;y)=\frac{m!}{k!}\prod_{j=1}^{k}f\left(y_{(j)};\mu,\sigma\right)\left[1-F\left(y_{(k)};\mu,\sigma\right)\right]^{m-k}=\frac{m!}{(m-k)!}\frac{1}{\sigma^k}\left(\prod_{j=1}^{k}e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)}e^{-e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)}}\right)\left[e^{-e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)}}\right]^{m-k} \qquad 3$$

Then, the log-likelihood function is given by:

$$lnL(\mu,\sigma;y)=ln\left(\frac{m!}{(m-k)!}\right)-kln\sigma+\sum_{j=1}^{k}\frac{y_{(j)}-\mu}{\sigma}-\sum_{j=1}^{k}e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)}-(m-k)e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)} \qquad 4$$

By differentiating Eq. 4 with respect to the parameters μ and σ, the likelihood functions derived the following forms:

$$\frac{\partial lnL}{\partial\mu}=\frac{1}{\sigma}\sum_{j=1}^{k}e^{\frac{y_{(j)}-\mu}{\sigma}}+\frac{(m-k)}{\sigma}e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)}-\frac{k}{\sigma}=0, \qquad 5$$

$$\frac{\partial lnL}{\partial\sigma}=\frac{-k}{\sigma}-\frac{1}{\sigma^2}\sum_{j=1}^{k}\left(y_{(j)}-\mu\right)+\frac{1}{\sigma^2}\sum_{j=1}^{k}e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)}\left(y_{(j)}-\mu\right)+\frac{(m-k)}{\sigma^2}e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)}\left(y_{(k)}-\mu\right)=0 \qquad 6$$

The ML estimators of the parameters are obtained as the simultaneous solution of Eqs. 5 and 6.

However, these equations have no explicit solutions and they have to be found based on iterative solutions. Therefore, the Newton-Raphson method is used in this paper.

Let $\hat{\sigma}$ and $\hat{\mu}$ denote the MLEs of σ and μ, respectively.

Now, the asymptotic distributions of the ML estimators of the unknown parameters are

$$\begin{pmatrix}\hat{\mu}\\ \hat{\sigma}\end{pmatrix} \sim AN\left(\begin{pmatrix}\mu\\ \sigma\end{pmatrix}, I^{-1}(\mu,\sigma)\right) \quad 7$$

Here $I^{-1}(\mu,\sigma)$ is calculated from the inverse of the observed Fisher information matrix $I(\mu,\sigma)$ as given below.

$$I^{-1}(\mu,\sigma) = \begin{bmatrix} -\left(\frac{\partial^2 logL}{\partial^2\mu}\right) & -\left(\frac{\partial^2 logL}{\partial\mu\partial\sigma}\right) \\ -\left(\frac{\partial^2 logL}{\partial\mu\partial\sigma}\right) & -\left(\frac{\partial^2 logL}{\partial^2\sigma}\right) \end{bmatrix}^{-1}_{(\mu,\sigma)=(\hat{\mu},\hat{\sigma})} = \begin{pmatrix} var(\hat{\mu}) & cov(\hat{\mu},\hat{\sigma}) \\ cov(\hat{\mu},\hat{\sigma}) & var(\hat{\sigma}) \end{pmatrix}. \quad 8$$

The $I(\mu,\sigma) = [I_{ij}]_{2x2}$ are obtained as follows:

$$I_{11} = \left(-\frac{\partial^2 lnL}{\partial\mu^2}\right) = \frac{1}{\sigma^2}\sum_{j=1}^{k} e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)} + \frac{(m-k)}{\sigma^2} e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)},$$

$$I_{12} = \left(-\frac{\partial^2 lnL}{\partial\mu\partial\sigma}\right) = \frac{1}{\sigma^2}\sum_{j=1}^{k} e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)} + \frac{1}{\sigma^3}\sum_{j=1}^{k} e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)}\left(y_{(j)}-\mu\right) + \frac{(m-k)}{\sigma^2} e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)} + \frac{(m-k)}{\sigma^3} e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)}(y_{(k)}-\mu) - \frac{k}{\sigma^2},$$

and

$$I_{22} = \left(\frac{-\partial^2 lnL}{\partial\sigma^2}\right) = \frac{-2}{\sigma^3}\sum_{j=1}^{k}\left(y_{(j)}-\mu\right) + \frac{+2}{\sigma^3}\sum_{j=1}^{k} e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)}\left(y_{(j)}-\mu\right) + \frac{1}{\sigma^4}\sum_{j=1}^{k} e^{\left(\frac{y_{(j)}-\mu}{\sigma}\right)}\left(y_{(j)}-\mu\right)^2 + \frac{2(m-k)}{\sigma^3} e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)} + \frac{(m-k)}{\sigma^4} e^{\left(\frac{y_{(k)}-\mu}{\sigma}\right)}\left(y_{(k)}-\mu\right)^2 - \frac{k}{\sigma^2}.$$

Then, the approximate $100(1-\alpha)$confidence intervals for the μ and σ, the coverage probabilities (CPs) are:

$$\hat{\mu} \pm Z_{\alpha/2}\sqrt{Var(\hat{\mu})}, \quad CP_{\mu} = P\left[\left|\frac{\hat{\mu}-\mu}{\sqrt{Var(\hat{\mu})}}\right| \le Z_{1-\alpha/2}\right]$$

and

$$\hat{\sigma} \pm Z_{\alpha/2}\sqrt{Var(\hat{\sigma})}, \quad CP_{\sigma} = P\left[\left|\frac{\hat{\sigma}-\sigma}{\sqrt{Var(\hat{\sigma})}}\right| \le Z_{1-\alpha/2}\right]$$

respectively. Here, $Z_{\alpha/2}$ presents the $(\alpha/2)th$ percentile of the standard normal distribution.

Bayesian Inference:

In this subsection, the BEs of μ and σ parameters of Eq. 2 by using the type –II censored scheming under SELF, WSELF, and GELF are discussed. The loss function is very important in Bayesian parameter estimation, see [16-19]. One of the well-known loss functions is SELF. It is symmetrical so, it can be used when over and underestimates are equally serious. However, this is not a good criterion in most cases, see Helu and Samawi[16]. In such cases, asymmetric loss functions can be considered. Here, BEs of the parameters under GELF, and WSELF are discussed. These loss functions are asymmetrical. Suppose that the independent PDs of the parameters μ and σ are $Normal(a,b)$ and $Gamma\ (c,d)$ with probability density functions

$$\pi_1(\mu) \propto e^{\frac{-1}{2}\left(\frac{\mu-a}{b}\right)^2} \quad \mu \in \mathbb{R} \text{ and } \pi_2(\sigma) \propto \sigma^{c-1}e^{-d\sigma} \quad 9$$

Then, the joint prior density function of μ, and σ is

$$\pi(\mu,\sigma) = \pi_1(\mu)\pi_2(\sigma) \propto e^{\frac{-1}{2}\left(\frac{\mu-a}{b}\right)^2} \sigma^{c-1} e^{-d\sigma}. \quad 10$$

where assume that (a,b) and (c,d) are non-negative and known. They are also hyper-parameters of the PD.

Combining Eq. 10 with Eq. 3, the joint posterior distribution of μ and σ is obtained as

$$\pi(\mu,\sigma|y) = \frac{L(y|\mu,\sigma)v(\mu,\sigma)}{\int\int L(y|\mu,\sigma)v(\mu,\sigma)d\mu d\sigma} \propto \frac{m!}{(m-k)!} e^{\frac{-1}{2}\left(\frac{\mu-a}{b}\right)^2 - d\sigma} \sigma^{c-1-k}\left(\prod_{j=1}^{k} e^{\frac{y_{(j)}-\mu}{\sigma}} e^{-e^{\frac{y_{(j)}-\mu}{\sigma}}}\right)\left[e^{-e^{\frac{y_{(k)}-\mu}{\sigma}}}\right]^{m-k}. \quad 11$$

Then, from Eq. 11, the posteriors of μ and σ are obtained as follows:

$$\pi_1(\mu|y,\sigma) \propto e^{\frac{-1}{2}\left(\frac{\mu-a}{b}\right)^2}\left(\prod_{j=1}^{k} e^{\frac{y_{(j)}-\mu}{\sigma}} e^{-e^{\frac{y_{(j)}-\mu}{\sigma}}}\right)\left[e^{-e^{\frac{y_{(k)}-\mu}{\sigma}}}\right]^{m-k} \quad 12$$

and

$$\pi_2(\sigma|y,\mu) \propto e^{-d\sigma}\sigma^{c-1-k}\left(\prod_{j=1}^{k} e^{\frac{y_{(j)}-\mu}{\sigma}} e^{-e^{\frac{y_{(j)}-\mu}{\sigma}}}\right)\left[e^{-e^{\frac{y_{(k)}-\mu}{\sigma}}}\right]^{m-k} \quad 13$$

respectively. The conditional PDs of μ and σ from Eqs. 12, 13 are unknown. Therefore, two different approximations are used, namely LD and MCMC techniques. The details of them are briefly described in the following sections.

Lindley's Approximation:

Let $u(\mu,\sigma)$be a function of μ and σ, then by using Eq. 11, the posterior expectation of an arbitrary function $u(\mu,\sigma)$ is as follows:

$$\hat{u} = E(u(\mu,\sigma)|y) = \frac{\int_{-\infty}^{\infty}\int_0^{\infty} u(\mu,\sigma)v(\mu,\sigma)L(\mu,\sigma|y)d\sigma d\mu}{\int_{-\infty}^{\infty}\int_0^{\infty} v(\mu,\sigma)L(\mu,\sigma|y)d\sigma d\mu}, \quad 14$$

where $v(\mu,\sigma)$ is the joint prior density function and $L(\mu,\sigma|y)$ is the likelihood function.

The BE of $u(\mu,\sigma)$ is the solution of Eq. 14. However, the BEs cannot be evaluated analytically. Therefore, the Lindley approximation method is considered to obtain BEs of unknown parameters. This method is introduced by Lindley in 1980[20].

By using Lindley's approximation method, $\hat{u}$ given in Eq. 14 can be approximated as:

$$\hat{u} \approx \left\{\begin{matrix} u(\mu,\sigma) + \\ 0.5[u_{11}\sigma_{11} + u_{22}\sigma_{22} + 2u_{12}\sigma_{12} + 2u_1(\sigma_{11}\rho_1 + \sigma_{21}\rho_2) + 2u_2(\sigma_{12}\rho_1 + \sigma_{22}\rho_2)] \\ +0.5[L_{111}(u_1\sigma_{11}^2 + u_2\sigma_{11}\sigma_{12}) + L_{112}\left(3u_1\sigma_{11}\sigma_{12} + u_2(\sigma_{11}\sigma_{22} + 2\sigma_{12}^2)\right) + \\ L_{122}(u_1(\sigma_{11}\sigma_{22} + 2\sigma_{12}^2) + 3u_2\sigma_{12}\sigma_{22}) + L_{222}(u_1\sigma_{12}\sigma_{22} + u_2\sigma_{22}^2)] \end{matrix}\right\}_{\hat{\mu},\hat{\sigma}} \quad 15$$

$u_1 = \frac{\partial u(\mu,\sigma)}{\partial\mu} =, u_{11} = \frac{\partial^2 u(\mu,\sigma)}{\partial\mu^2}, u_2 = \frac{\partial u(\mu,\sigma)}{\partial\sigma}, u_{22} = \frac{\partial^2 u(\mu,\sigma)}{\partial\sigma^2}, u_{12} = \frac{\partial^2 u(\mu,\sigma)}{\partial\mu\partial\sigma}$,

$\rho_1 = \frac{\partial lnv(\mu,\sigma)}{\partial\mu}, \rho_2 = \frac{\partial lnv(\mu,\sigma)}{\partial\sigma}, L_{111} = \frac{\partial^3 lnL}{\partial\mu^3}, L_{122} = \frac{\partial^3 lnL}{\partial\mu\partial\sigma^2}, L_{112} = \frac{\partial^3 lnL}{\partial\mu^2\partial\sigma}, L_{222} = \frac{\partial^3 lnL}{\partial\mu^3}$

and $\sigma_{ij}, i,j = 1,2$ are given in Eq. 8.

Bayesian Estimators under Symmetric Loss Function Using Lindley Approximation:

From Eq. 15, BEs of μ and σ based on SELF are

If $u(\mu,\sigma)=\mu,\ u_1 = 1, u_{11} = u_{12} = u_2 = u_{22} = 0$, then

$$\hat{\mu}_{SELF} = \hat{\mu} + \hat{\sigma}_{11}\hat{\rho}_1 + \hat{\sigma}_{21}\hat{\rho}_2 + 0.5\big[\hat{L}_{111}\hat{\sigma}_{11}^2 + 3\hat{L}_{112}\hat{\sigma}_{11}\hat{\sigma}_{12} + \hat{L}_{122}(\hat{\sigma}_{11}\hat{\sigma}_{22} + 2\hat{\sigma}_{12}^2) + \hat{L}_{222}\hat{\sigma}_{12}\hat{\sigma}_{22}\big],$$

and

if $u(\mu,\sigma) = \sigma, u_2 = 1, u_{22} = u_{12} = u_1 = u_{11} = 0$, then

$$\hat{\sigma}_{SELF} = \hat{\sigma} + \hat{\sigma}_{12}\hat{\rho}_1 + \hat{\sigma}_{22}\hat{\rho}_2 + 0.5\big[\hat{L}_{111}\hat{\sigma}_{11}\hat{\sigma}_{12} + \hat{L}_{112}(\hat{\sigma}_{11}\hat{\sigma}_{22} + 2\hat{\sigma}_{12}^2) + 3\hat{L}_{122}\hat{\sigma}_{12}\hat{\sigma}_{22} + \hat{L}_{222}\hat{\sigma}_{12}^2\big].$$

Here, $\hat{\mu}$ and $\hat{\sigma}$ are the ML estimators of μ and σ, respectively.

Bayesian Estimators under Asymmetric Loss Functions Using Lindley Approximation:

From Eq. 15, BEs of the model parameters under GELF are given below:

If $u(\mu,\sigma) = \mu^{-t}, u_1 = -t\mu^{-(t+1)}, u_{11} = t(t+1)\mu^{-(t+2)},\ u_2 = u_{22} = u_{12} = u_{21} = 0$, then

$$E(\mu^{-t}|y) = \hat{\mu}^{-t} + 0.5(\hat{u}_{11}\hat{\sigma}_{11}) + \hat{u}_1(\hat{\sigma}_{11}\hat{\rho}_1 + \hat{\sigma}_{21}\hat{\rho}_2) + 0.5\big[\hat{L}_{111}\hat{u}_1\hat{\sigma}_{11}^2 + 3\hat{L}_{112}\hat{u}_1\hat{\sigma}_{11}\hat{\sigma}_{12} + \hat{L}_{122}\hat{u}_1(\hat{\sigma}_{11}\hat{\sigma}_{22} + 2\hat{\sigma}_{12}^2) + \hat{L}_{222}\hat{u}_1\hat{\sigma}_{12}\hat{\sigma}_{22}\big]$$

Therefore, $\hat{\mu}_{GELF} = \big[E\big((\mu^{-t}|y)\big)\big]^{-1/t}$.

If $u(\mu,\sigma) = \sigma^{-t}, u_2 = -t\sigma^{-(t+1)}, u_{22} = t(t+1)\sigma^{-(t+2)},\ u_1 = u_{11} = u_{12} = u_{21} = 0$, then

$$E(\sigma^{-t}|y) = \hat{\sigma}^{-t} + \hat{u}_2(\hat{\sigma}_{12}\hat{\rho}_1 + \hat{\sigma}_{22}\hat{\rho}_2) + 0.5(\hat{u}_{22}\hat{\sigma}_{22}) + 0.5\big[\hat{L}_{111}\hat{u}_2\hat{\sigma}_{11}\hat{\sigma}_{12} + \hat{L}_{112}\hat{u}_2(\hat{\sigma}_{11}\hat{\sigma}_{22} + 2\hat{\sigma}_{12}^2) + 3\hat{L}_{122}\hat{u}_2\hat{\sigma}_{12}\hat{\sigma}_{22} + \hat{L}_{222}\hat{u}_2\hat{\sigma}_{22}^2\big]$$

So, $\sigma_{GELF} = \big[E\big((\sigma^{-t}|y)\big)\big]^{-1/t}$

Similarly, from Eq. 15, BEs of the parameters of interest (i.e. μ and σ) under WSELF are given by:

If f $u(\mu,\sigma) = \mu^{-1}, u_1 = -\mu^{-2}, u_{11} = 2\mu^{-3},\ u_2 = u_{22} = u_{12} = u_{21} = 0$, then

$$E(\mu^{-1}|y) = \hat{\mu}^{-1} + 0.5(\hat{u}_{11}\hat{\sigma}_{11}) + \hat{u}_1(\hat{\sigma}_{11}\hat{\rho}_1 + \hat{\sigma}_{21}\hat{\rho}_2) + 0.5\big[\hat{L}_{111}\hat{u}_1\hat{\sigma}_{11}^2 + 3\hat{L}_{112}\hat{u}_1\hat{\sigma}_{11}\hat{\sigma}_{12} + \hat{L}_{122}\hat{u}_1(\hat{\sigma}_{11}\hat{\sigma}_{22} + 2\hat{\sigma}_{12}^2) + \hat{L}_{222}\hat{u}_1\hat{\sigma}_{12}\hat{\sigma}_{22}\big]$$

The Bayesian estimator of μ is of the following form:

$$\hat{\mu}_{WSELF} = \big[E\big((\mu^{-1}|y)\big)\big]^{-1}$$

The WSELF with parameter σ is given by:
If $u(\mu,\sigma) = \sigma^{-1}, u_2 = -\sigma^{-2}, u_{22} = 2\sigma^{-3}, u_1 = u_{11} = u_{12} = u_{22} = u_{21} = 0$ then

$$E(\sigma^{-1}|y) = \hat{\sigma}^{-1} + \hat{u}_2(\hat{\sigma}_{12}\hat{\rho}_1 + \hat{\sigma}_{22}\hat{\rho}_2) + 0.5(\hat{u}_{11}\hat{\sigma}_{11}) + 0.5\big[\hat{L}_{111}\hat{u}_2\hat{\sigma}_{11}\hat{\sigma}_{12} + \hat{L}_{112}\hat{u}_2(\hat{\sigma}_{11}\hat{\sigma}_{22} + 2\hat{\sigma}_{12}^2) + 3\hat{L}_{122}\hat{u}_2\hat{\sigma}_{12}\hat{\sigma}_{22} + \hat{L}_{222}\hat{u}_2\hat{\sigma}_{22}^2\big]$$

Hence, the Bayesian estimator of σ is $\hat{\sigma}_{WSELF} = \big[E\big((\sigma^{-1}|y)\big)\big]^{-1}$,
where,

$$\hat{\rho}_1 = \frac{a-\hat{\mu}}{b^2},\ \hat{\rho}_2 = \frac{c-1}{\hat{\sigma}} - d,$$

$$\hat{L}_{111} = \frac{1}{\hat{\sigma}^3}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}} + \frac{m-k}{\hat{\sigma}^3} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}},$$

$$\hat{L}_{122} = \frac{-2k}{\hat{\sigma}^3} + \frac{2}{\hat{\sigma}^3}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}} + \frac{4}{\hat{\sigma}^4}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(j)-\hat{\mu}}\big) + \frac{1}{\hat{\sigma}^5}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(j)-\hat{\mu}}\big)^2 + \frac{2(m-k)}{\hat{\sigma}^3} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}} + \frac{4(m-k)}{\hat{\sigma}^4} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(k)-\hat{\mu}}\big) + \frac{(m-k)}{\hat{\sigma}^5} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(k)-\hat{\mu}}\big)^2,$$

$$\hat{L}_{112} = \frac{2}{\hat{\sigma}^3}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}} + \frac{1}{\hat{\sigma}^4}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(j)-\hat{\mu}}\big) + \frac{2(m-k)}{\hat{\sigma}^3} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}} + \frac{(m-k)}{\hat{\sigma}^4} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(k)} - \mu\big),$$

$$\hat{L}_{222} = \frac{-2k}{\hat{\sigma}^3} - \frac{6}{\hat{\sigma}^4}\sum_{j=1}^{k} \frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}} + \frac{6}{\hat{\sigma}^4}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(j)} - \hat{\mu}\big) + \frac{6}{\hat{\sigma}^5}\sum_{j=1}^{k} e^{\frac{y_{(j)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(j)-\hat{\mu}}\big)^2 + \frac{1}{\hat{\sigma}^6}\sum_{j=1}^{k} e^{\frac{(y_{(j)}-\hat{\mu})}{\hat{\sigma}}}\big(y_{(j)} - \hat{\mu}\big)^3 + \frac{6(m-k)}{\hat{\sigma}^4} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}} + \frac{6(m-k)}{\hat{\sigma}^5} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(k)} - \hat{\mu}\big)^2 + \frac{(m-k)}{\hat{\sigma}^6} e^{\frac{y_{(k)}-\hat{\mu}}{\hat{\sigma}}}\big(y_{(k)} - \hat{\mu}\big)^3.$$

Markov Chain Monte Carlo:
Here, the Gibbs sampling method is used to generate samples from posterior distributions of model parameters. This method is a sub-class of the MCMC method, see Smith and Roberts [21].

It is clear from Eqs. 12, 13 that the BEs of the μ and σ cannot be found in closed form by using the conditional posterior distribution of these parameters. In this situation, the Metropolis-Hasting (M-H) algorithm can be considered, see Metropolis et al.[22] and Hasting[23].
The steps Gibbs sampling method are given below:
Steps1: Set $i = 1$ and let $\mu_0 = \hat{\mu}$ and $\sigma_0 = \hat{\sigma}$.
Steps2: Use the following M-H algorithm, generate μ_i and σ_i from $\pi_1(\mu_{i-1}|\sigma_{i-1}, y)$ and $\pi_2(\sigma_{i-1}|\mu_i, y)$with normal proposal distributions $N(\mu_{i-1}, I_{11}^{-1})$ and $(\sigma_{i-1}, I_{22}^{-1})$, where $I_{ij}^{-1}(i,j = 1,2)$ is given in Section 2.
Step3: Set $i = i + 1$.
Step4: Repeat steps 2-3 N times and obtain $(\mu_1, \sigma_1), \dots, (\mu_N, \sigma_N)$.
Now, the BEs of the μ and σ based on all these aforementioned loss functions are obtained as follows.

$$\hat{\mu}_{SELF} = \frac{1}{N}\sum_{i=1}^{N}\mu_i, \qquad \hat{\sigma}_{SELF} = \frac{1}{N}\sum_{i=1}^{N}\sigma_i,$$

$$\hat{\mu}_{GELF} = \Big(\frac{1}{N}\sum_{i=1}^{N}\mu_i^{-t}\Big)^{\frac{-1}{t}},\ \hat{\sigma}_{GELF} = \Big(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^{-t}\Big)^{\frac{-1}{t}},$$

and

$$\hat{\mu}_{WSELF} = \Big(\frac{1}{N}\sum_{i=1}^{N}\mu_i^{-1}\Big)^{-1},\ \hat{\sigma}_{WSELF} = \Big(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^{-1}\Big)^{-1}.$$

Then, the $100(1-\gamma)$ credible interval of μ and σ is obtained, see Chen and Shao[24].

Methodology:

In this Section, the performances of the ML estimators and BEs are investigated via Monte Carlo simulation. The performances of these estimators are compared in terms of absolute biases (ABias) and mean square errors (MSEs) under different sample sizes, censoring, and parameter values. The sample sizes provided are $m = 20(10)100$ and $\mu = -1, 0, 1$ and $\sigma = 0.5, 1, 2$ are considered. Two censoring proportions, %10 and %30, are evaluated for each sample size and parameter value. In the BEs, two different informative priors are considered for μ and σ parameters. Firstly the hyper-parameters $a = c = d = 0, b = 1$, are taken into account and are called Prior-I. Then, the hyper-parameters, $a = c = d = 4, b = 1$, are chosen and are called Prior- II. Based on Prior-I and Prio- II distributions, the BEs of μ and σ under SELF, GELF, and WSELF are calculated by using both LD and MCMC methods. MCMC samples of size 10000 are taken for the computation. In this study, the t value under GELF is taken as $t = 1.5$. Also, the maximum likelihood estimators are computed. All the computations are conducted in Matlab R 2013 with over 10000 replications for different cases. Since the simulation results are similar, only results based on $\mu = 0$ and $\sigma = 1$ are given in Tables 1-3. The simulated ABias and MSE values of ML estimators and BEs are presented in Tables 1, 3 respectively. Results of the ACIs, BCIs and the CPs are summarized in Table 2.

Table 1. The ABiases and the MSEs for the MLEs of μ and σ.

		$\hat{\mu}$		$\hat{\sigma}$	
m	k	ABias	MSE	ABias	MSE
30	27	0.0299	0.0416	0.0238	0.0258
	21	0.0291	0.0465	0.0355	0.0381
50	45	0.0180	0.0245	0.0162	0.0163
	35	0.0244	0.0290	0.0261	0.0238
100	90	0.0063	0.0112	0.0048	0.0081
	70	0.0119	0.0151	0.0114	0.0109

Table 2. The ACIs, BCIs and the CPs of μ and σ.

		$\hat{\mu}$			$\hat{\sigma}$		
			PRIOR- I	PRIOR- II		PRIOR- I	PRIOR -II
m	k	ACIs	BCIs	BCIs	ACIs	BCIs	BCIs
	27	(-0.3915; 0.3197) 0.9511	(-03941; 0.3436) 0.9628	(-0.4180; 0.3383) 0.9683	(0.6492;1.2489) 0.9516	(0.6812; 1.2522) 0.9567	(0.7196; 1.2854) 0.9660
30	21	(-0.447; 0.3895) 0.9487	(-0.4269;0.4215) 0.9533	(-0.4089; 0.3980) 0.9517	(0.5919; 1.3478) 0.9418	(0.6151; 1.3139) 0.9496	(0.5876; 1.3640) 0.9443
	45	(-0.3012; 0.2849) 0.9598	(-0.2870;0.2905) 0.9632	(-0.3225; 0.3320) 0.9676	(0.7357; 1.2320) 0.9536	(0.7481; 1.2390) 0.9592	(0.7501; 1.2192) 0.9560
50	35	(-0.3546; 0.3058) 0.9550	(-0.3317;0.3146) 0.9595	(-0.3421; 0.3226) 0.9582	(0.6759; 1.2791) 0.9522	(0.6858; 1.2793) 0.9538	(0.6811; 1.2522) 0.9558
	90	(-0.2061; 0.1981) 0.9792	(-0.1944;0.1894) 0.9817	(-02083; 0.1912) 0.9847	(0.8156; 1.1617) 0.9674	(0.8148; 1.554) 0.9692	(0.8332; 1.1752) 0.9683
100	70	(-0.2046; 0.2041) 0.9772	(-0.2344; 0.2245) 0.9812	(-0.2117; 0.2083) 0.9802	(0.8115; 1.1671) 0.9710	(0.7908; 1.2012) 0.9717	(0.8115; 1.1671) 0.9723

Table 3. The ABiases and the MSEs for the BEs of μ and σ.

				Lindley				MCMC			
				$\hat{\mu}$		$\hat{\sigma}$		$\hat{\mu}$		$\hat{\sigma}$	
	m	k	Method	ABias	MSE	ABias	MSE	ABias	MSE	ABias	MSE
Prior -I (a=0, b=1 ; c=d=0)	30		SELF	0.0130	0.0357	0.0280	0.0314	0.0097	0.0355	0.0265	0.0296
		27	GELF	0.0085	0.0190	0.0251	0.0285	0.0078	0.0107	0.0240	0.0265
			WSELF	0.0110	0.0176	0.0260	0.0275	0.0091	0.0146	0.0248	0.0262
			SELF	0.0205	0.0382	0.0296	0.0363	0.0184	0.0369	0.0255	0.0330
		21	GELF	0.0126	0.0261	0.0287	0.0339	0.0106	0.0166	0.0257	0.0335
			WSELF	0.0113	0.0243	0.0282	0.0329	0.0112	0.0167	0.0260	0.0296
			SELF	0.0134	0.0251	0.0126	0.0128	0.0108	0.0233	0.0120	0.0164
	50	45	GELF	0.0102	0.0136	0.0119	0.0105	0.0099	0.0078	0.0109	0.0160
			WSELF	0.0111	0.0125	0.0114	0.0100	0.0106	0.0115	0.0119	0.0158
			SELF	0.0144	0.0288	0.0194	0.0186	0.0121	0.0273	0.0177	0.0180
		35	GELF	0.0088	0.0144	0.0182	0.0182	0.0075	0.0084	0.0176	0.0172
			WSELF	0.0104	0.0132	0.0187	0.0180	0.0108	0.0199	0.0178	0.0164
	100		SELF	0.0058	0.0096	0.0031	0.0060	0.0031	0.0098	0.0026	0.0056
		90	GELF	0.0031	0.0057	0.0028	0.0056	0.0020	0.0029	0.0021	0.0045
			WSELF	0.0030	0.0052	0.0024	0.0055	0.0033	0.0055	0.0016	0.0042
		70	SELF	0.0097	0.0131	0.0084	0.0107	0.0071	0.0124	0.0086	0.0100
			GELF	0.0059	0.0076	0.0071	0.0086	0.0064	0.0044	0.0064	0.0082
			WSELF	0.0037	0.0069	0.0074	0.0089	0.0032	0.0055	0.0070	0.0090
Prior -II (a=0, b=1 ; c=d=4)	30		SELF	0.0257	0.0310	0.0228	0.0251	0.0232	0.0305	0.0257	0.0230
		27	GELF	0.0221	0.0235	0.0200	0.0227	0.0171	0.0208	0.0242	0.0196
			WSELF	0.0204	0.0218	0.0212	0.0218	0.0116	0.0201	0.0240	0.0210
			SELF	0.0270	0.0336	0.0267	0.0378	0.0254	0.0224	0.0242	0.0358
		21	GELF	0.0183	0.0256	0.0250	0.0315	0.0143	0.0221	0.0238	0.0307
			WSELF	0.0155	0.0239	0.0262	0.0308	0.0119	0.0274	0.0239	0.0305
	50		SELF	0.0120	0.0231	0.0145	0.0122	0.0106	0.0227	0.0124	0.0116
		45	GELF	0.0102	0.0134	0.0128	0.0102	0.0099	0.0096	0.0116	0.0100
			WSELF	0.0110	0.0123	0.0131	0.0118	0.0101	0.0097	0.0100	0.0110
		35	SELF	0.0188	0.0281	0.0191	0.0191	0.0175	0.0276	0.0196	0.0176
			GELF	0.0108	0.0147	0.0185	0.0153	0.0103	0.0125	0.0094	0.0164
			WSELF	0.0104	0.0136	0.0181	0.0166	0.0092	0.0132	0.0112	0.0160
	100		SELF	0.0058	0.0105	0.0153	0.0140	0.0044	0.0113	0.0116	0.0080
		90	GELF	0.0042	0.0063	0.0132	0.0137	0.0039	0.0033	0.0095	0.0082
			WSELF	0.0049	0.0056	0.0123	0.0136	0.0048	0.0065	0.0112	0.0079
			SELF	0.0092	0.0139	0.0217	0.0219	0.0082	0.0148	0.0184	0.0176

70	GELF	0.0077	0.0082	0.0194	0.0199	0.0060	0.0048	0.0119	0.0144
	WSELF	0.0102	0.0074	0.0204	0.0153	0.0078	0.0096	0.0145	0.0153

*The CPs are the values presented in the second row.

From Table 1-3;

- First of all, as the sample size increases, both the ABias and MSE values of all the estimators decrease in all cases. It implies that all the estimators are asymptotically efficient.
- Similarly, in almost all cases, as the censoring proportion increases, the ABias and MSE values of all the estimators increase.
- When Bayesian methods are compared with the maximum likelihood method according to ABias and MSE values, it is seen that Bayesian methods demonstrate better performance than the maximum likelihood estimation method under both Prior-I and Prior-II.
- Among the loss functions, it is evident that as far as ABiases and MSEs are concerned, Bayesian estimation under the GELF works the best in most cases. It is followed by Bayesian estimation under the WSELF.
- It is also clear from Table 2 that the width of the ACIs is wider than the BCIs in all cases (ie, Prior-I and Prior-II). Moreover, the BCI and the CP values based on Prior-I and Prior-II distributions are closer to a nominal value of 95% than the ACIs.

Application:

This section considers a real dataset given by Dodson[25] to illustrate the estimation procedure developed in the previous sections. This data set was also studied by Balakrishan and Kateri[26]. They reported this dataset for Weibull distribution under type-II censorship. The dataset contains 12 failure times of identical grinders of total 20 items. The censoring scheme is type-II censoring with $k=12$ and $m=20$. Here, the logarithm of Y, (i.e., $U=\ln Y$) is taken. Thus, the distribution of U becomes Gumbel type-I distribution. The data set corresponding to the log-failure times is given below.

{2.5227 3.1946 4.0639 4.2195 4.2356 4.5591 4.5706 4.5747 4.7380 4.8133 4.8331 5.0285}

The estimators of the Gumbel type-I distribution using MLE and Lindley and MCMC methods are presented in Table 4. Furthermore, the ACIs and BCIs are presented in Table 5.

Table 4. Estimation of parameters for the real data set.

	Prior-I				Prior-II			
	Lindley		MCMC		Lindley		MCMC	
Method	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
MLE	51.94	154.24	51.94	154.24	51.94	154.24	51.94	154.24
SELF	52.56	154.68	52.18	154.35	52.71	154.55	52.17	154.36
GELF	52.35	154.92	52.54	154.18	52.60	154.87	52.38	154.45
WSELF	52.98	155.11	52.30	154.36	52.74	154.96	52.46	154.72

Table 5. Interval estimators for the real data set.

	Prior-I		Prior-II	
	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
ACI	(-19.24; 104.65)	(101.53; 206.94)	(-19.24; 104.65)	(101.53;206.94)
BCI	(-17.35; 102.18)	(105.86; 208.47)	(-18.97; 103.27)	(104.97; 207.65)

Although all the estimators are close to each other, there are some differences among them. Based on the simulation results given in Tables 1-3, the most suitable estimators were selected. It has been observed from the simulation results that the estimators obtained by Bayesian methods outperform the ML estimators. Therefore, it is recommended to use Bayesian estimators in these examples.

Conclusions:

Here, the classical and Bayesian estimators are investigated for Gumbel type-I distribution under the type-II censoring scheme based on various censoring rates and sample sizes. In view of classical parameter estimation, the ML estimation method is used. For the Bayesian calculations, LD and MCMC methods are obtained. Then, the Bayesian estimators under different loss functions and different prior distributions are considered. Also, the ACIs are constructed by using the ML estimators. The BCIs are also derived based on the

MCMC method. The performances of all the methods discussed in the study are compared with the Monte Carlo simulation study. When the maximum likelihood estimators are compared to Bayesian estimators, it is observed that Bayesian estimators have higher efficiencies than the maximum likelihood estimators. It is also observed that the MCMC method performs slightly better than the LM. Finally, the real-life examination has illustrated the modeling capacity of the Gumbel type-I distribution. For more efficient estimators of the Gumbel type-I distribution, it is recommended to use Bayesian estimation methods with prior-I distribution and the general entropy loss function.

Considering both the simulation results and these points, we may suggest that this study can be further extended considering the Gumbel type-I distribution for different censoring schemes with a Bayesian framework under symmetric and asymmetric loss functions.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Van Yüzüncü Yıl University.

Authors' contributions statement:

YILMAZ A. conceived the presented idea. YILMAZ A. and KARA M. developed the theory and performed the computations and verified the analytical methods. All authors discussed the results and contributed to the final manuscript.

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تقدير معلمات لتوزيع كامبل من النوع الاول في ظل نظام الرقابة من النوع الثاني

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الخلاصة:

تهدف هذه الورقة إلى تحديد أفضل طرق تقدير المعلمات لمعاملات توزيع كامبل من النوع الأول تحت نظام الرقابة من النوع الثاني. لهذا الغرض، تم الاخذ في الحسبان إجراءات تقدير المعلمات الكلاسيكية و معلم بايسون. كما تم استخدام تقديرات الاحتمالية القصوى لإجراء تقدير المعلمة الكلاسيكي. كذلك تم اشتقاق التوزيعات المقاربة لهذه المقدرات. والتي ليس من الممكن الحصول على حلول صريحة لمقدرات بايسون. ولذلك ، تم أخذ تقنيات سلسلة ماركوف و مونت كارلو ولندلي في الاعتبار لتقدير المعلمات غير المعروفة. في تحليل بايسون ، من المهم جدًا تحديد تركيبة مناسبة من التوزيعات السابقة ودالة الخسارة. ولهذا، تم استخدام توزيعين مختلفين سابقين. أيضًا ، تم التحقق من تقديرات بايسون فيما يتعلق بالمعلمات ذات الأهمية في ظل وظائف الخسارة المختلفة. وتم أستخدام خوارزمية جبز لأخذ العينات لإنشاء فترات بايسون ذات مصداقية. بعد ذلك ، تتم مقارنة كفاءات مقدرات الاحتمالية القصوى بمقدرات بايز من خلال دراسة محاكاة مونت كارلو واسعة النطاق. لقد ثبت أن مقدر بايسون أكثر كفاءة بكثير من مقدر الاحتمالية القصوى. أخيرًا ، تم أيضًا تقديم مثال واقعي لأغراض التطبيق.

الكلمات المفتاحية: طرق بايسون، توزيع كامبل من النوع الأول، المحاكاة، الرقابة من النوع الثاني.