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Topological Indices Polynomials of Domination David Derived Networks

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Abstract

The chemical properties of chemical compounds and their molecular structures are intimately connected. Topological indices are numerical values associated with chemical molecular graphs that help in understanding the physicochemical properties, chemical reactivity and biological activity of a chemical compound. This study obtains some topological properties of second and third dominating David derived (DDD) networks and computes several K Banhatti polynomial of second and third type of DDD.

Keywords: Dominating David Derived networks, F-K Banhatti indices , harmonic K Banhatti indices, K-Banhatti polynomial, K-hyper Banhatti indices, symmetric division K Banhatti indices.

Introduction

Suppose that $G = (P, Q)$ is a finite, simple, connected graph [1, 2]. A chemical compound is represented by a simple graph called a molecular graph in chemical graph theory, which is a branch of graph theory, with the vertices representing the complex atoms and the edges representing the atomic bounds. If a graph contains at least one connection between its vertices, it is referred to be connected. A network is a graph with no loops or multiple edges. More information can be found in the book [3]. Another emerging field is chem-informatics, which improves the predictions of biological activities using the structure property and quantitative structure-activity relationships. If emphasized chemicals are used in these studies, topological indices and physicochemical properties are used to predict bioactivity [4, 5]. A cellular neural network, or cellular nonlinear network, is a computing paradigm used in computer science and machine learning that plays a major role in communicating between neighbouring units. CNN is used to solve partial differential equations (PDEs), perform image processing, analyze 3D surfaces, and resolve geodesic maps and sensory motor organ problems. CNN processors are systems that combine finite fixed topology, locally connected fixed location, and multiple inputs with a single output of nonlinear processing units. In the CNN processor, every cell (processor) has a single output, due to which it is communicated by other cells. The CNN processor was introduced in 1988 by Leon Chua and Lin Yang. In the original Chua Yang CNN processor (CYCNN), cells were weighted sums of different inputs, while output was a piecewise linear function [6]. A topological index is a number that describes a graphs topological. Mathematicians and chemists both studied this index [7]. Discusses how to construct [8] an n dimensional David derived and DDD.

In [9] V. R. Kulli introduced the I[st] & II[nd] K Banhatti indices (K B indices),

$B_1(G) = \sum_{r\,k} |E_{r\,k}| [d_G(r) + d_G(k)]$ $B_2(G) = \sum_{r\,k} |E_{r\,k}| [d_G(r) * d_G(k)]$

The I[st] & II[nd] K Banhatti Polynomials (K B Polynomial) are defined as follows using K Banhatti indices:

$$B_1(G, x, y) = \sum_{r\,k} |E_{r\,k}|\ x^{[d_G(r)+d_G(k)]}\ y^{[d_G(r)+d_G(k)]}$$

$$B_2(G, x, y) = \sum_{r\,k} |E_{r\,k}|\ x^{[d_G(r)*d_G(k)]}\ y^{[d_G(r)*d_G(k)]}$$

The I[st] & II[nd] K hyper Banhatti indices (K H B indices) [10] are defined as

$$HB_1(G)=\sum_{r\,k}|E_{r\,k}|\,[d_G(r)+d_G(k)]^2 \qquad HB_2(G)=\sum_{rk}|E_{rk}|[d_G(r)*d_G(k)]^2$$

The I[st] & II[nd] Banhatti polynomials of a graph can be calculated using the K hyper Banhatti indices as follows:

$$HB_1(G,x,y)=\sum_{r\,k}|E_{r\,k}|\ x^{[d_G(r)+d_G(k)]^2}\ y^{[d_G(r)+d_G(k)]^2}$$

$$HB_2(G,x,y)=\sum_{r\,k}|E_{r\,k}|x^{[d_G(r)*d_G(k)]^2}\ y^{[d_G(r)*d_G(k)]^2}$$

In [11], V. R. Kulli introduced the modified I[st] and II[nd] Banhatti indices,

$$^{M}B_1(G)=\sum_{r\,k}|E_{r\,k}|\,\frac{1}{d_G(r)+d_G(k)} \qquad ^{M}B_2(G)=\sum_{r\,k}|E_{r\,k}|\,\frac{1}{d_G(r)*d_G(k)}$$

The modified I[st] & II[nd] K Banhatti polynomials are defined as follow

$$^{M}B_1(G,x,y)=\sum_{r\,k}|E_{r\,k}|\,x^{\frac{1}{d_G(r)+d_G(k)}}\,y^{\frac{1}{d_G(r)+d_G(k)}}$$

$$^{M}B_2(G,x,y)=\sum_{r\,k}|E_{r\,k}|\,x^{\frac{1}{d_G(r)*d_G(k)}}\,y^{\frac{1}{d_G(r)*d_G(k)}}$$

The sum and product connectivity Banhatti indexes (S&P C B indices) [12] are calculated using the following formulas:

$$SB(G)=\sum_{r\,k}|E_{r\,k}|\,\frac{1}{\sqrt{d_G(r)+d_G(k)}} \qquad PB(G)=\sum_{r\,k}|E_{r\,k}|\,\frac{1}{\sqrt{d_G(r)*d_G(k)}}$$

The sum and product connectivity Banhatti polynomials(S&P C B Polynomial) of G are defined as:

$$SB(G,x,y)=\sum_{r\,k}|E_{r\,k}|\ x^{\frac{1}{\sqrt{d_G(r)+d_G(k)}}}\,y^{\frac{1}{\sqrt{d_G(r)+d_G(k)}}}$$

$$PB(G,x,y)=\sum_{r\,k}|E_{r\,k}|\ x^{\frac{1}{\sqrt{d_G(r)*d_G(k)}}}\,y^{\frac{1}{\sqrt{d_G(r)*d_G(k)}}}$$

The graph $G$is general I[st] & II[nd] K Banhatti indices [13] as:

$$B_1^a(G)=\sum_{r\,k}|E_{r\,k}|[d_G(r)+d_G(k)]^a \qquad B_2^a(G)=\sum_{r\,k}|E_{r\,k}|[d_G(r)*d_G(k)]^a$$

The general I[st] & II[nd] K Banhatti polynomial of a graph G using general I[st] & II[nd] K Banhatti indices as:

$$B_1^a(G,x,y)=\sum_{r\,k}|E_{r\,k}|\,x^{[d_G(r)+d_G(k)]^a}\ y^{[d_G(r)+d_G(k)]^a}$$

$$B_2^a(G,x,y)=\sum_{r\,k}|E_{r\,k}|\ x^{[d_G(r)*d_G(k)]^a}\ y^{[d_G(r)*d_G(k)]^a}$$

where a is a is a real number.

The $F-K$ Banhatti index of G is defined as follows:

$$FB(G)=\sum_{r\,k}|E_{r\,k}|\,[d_G(r)^2+d_G(k)^2]$$

Using the $F-K$ Banhatti index ($F-K$ B Indices), we define the $F-K$ Banhatti polynomial ($F-K$ B Polynomial) of G, as:

$$FB(G,x,y)=\sum_{r\,k}|E_{r\,k}|\ x^{[d_G(r)^2+d_G(k)^2]}\,y^{[d_G(r)^2+d_G(k)^2]}$$

A graph G is harmonic K Banhatti index (H K B index) [14] is defined as:

$$H_b(G)=\sum_{r\,k}|E_{r\,k}|\,\frac{2}{d_G(r)+d_G(k)}$$

The harmonic K Banhatti polynomial (H K B Polynomial) of a graph G is defined as:

$$H_b(G,x,y)=\sum_{r\,k}|E_{r\,k}|\ x^{\frac{2}{d_G(r)+d_G(k)}}\ y^{\frac{2}{d_G(r)+d_G(k)}}$$

The symmetric division K Banhatti index (S D K B index) of $G$is defined as:

$$SDB(G)=\sum_{r\,k}|E_{r\,k}|\,\left[\frac{d_G(r)}{d_G(k)}+\frac{d_G(k)}{d_G(r)}\right]$$

The symmetric division K Banhatti polynomial (S D K B Polynomial) of G is defined as:

$$SDB(G,x,y)=\sum_{r\,k}|E_{r\,k}|\,x^{\left[\frac{d_G(r)}{d_G(k)}+\frac{d_G(k)}{d_G(r)}\right]}\,y^{\left[\frac{d_G(r)}{d_G(k)}+\frac{d_G(k)}{d_G(r)}\right]}$$

The inverse sum index K Banhatti index (I S I K index) [15] of a graph Gas

$$ISB(G)=\sum_{r\,k}|E_{r\,k}|\,\left[\frac{d_G(r)d_G(k)}{d_G(r)+d_G(k)}\right]$$

The inverse sum K Banhatti polynomial (I S K B Polynomial) of a graph G as:

$$ISB(G,x,y)=\sum_{r\,k}|E_{r\,k}|\,x^{\left[\frac{d_G(r)d_G(k)}{d_G(r)+d_G(k)}\right]}\,y^{\left[\frac{d_G(r)d_G(k)}{d_G(r)+d_G(k)}\right]}$$

Result and Discussion

Second type of Domination David Derived Network

Consider the u dimensional David's star network. Add a new vertex to each edge and divide it in two parts. The David-derived network $DD(u)$ of dimension u will be obtained [14].

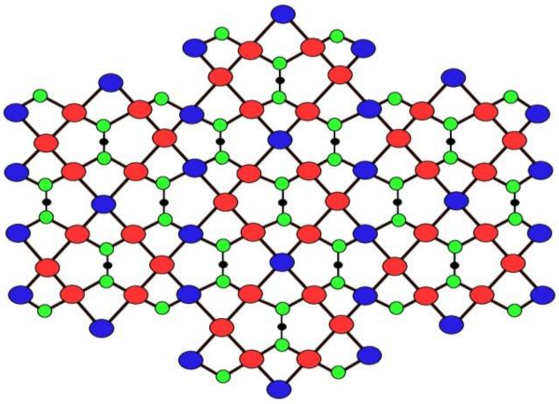

Figure 1. Domination David Derived network of the second type

The DDD network of the second type (as shown in Fig. 1) of dimension u can be obtained by connecting vertices of degree 2 of $DDD(u)$ by an edge that are not in the boundary and is denoted by $D_2(2)$.

In $G = D_2(u)$, the edge set of G can be divided into 5 partitions based on the degree of end vertices of each edge as given below.

Table 1. Edge partition of G

$d_G(r), d_G(s)$; $k = r\,s \in Q(G)$	(2, 2)	(2, 3)	(2, 4)	(3, 4)	(4, 4)
No of edges	$4u$	$18\,u^2 - 22\,u + 6$	$28u - 16$	$36u^2 - 56u + 24$	$36u^2 - 56u + 20$

Therefore the edge degree partition of $G = D_2(u)$ is given below.

Table 2. Edge degree partition of G

$d_G(r), d_G(s)$; $k = r\,s \in Q(G)$	(2, 2)	(2, 3)	(2, 4)	(3, 4)	(4, 4)
No of edges	$4u$	$18\,u^2 - 22\,u + 6$	$28u - 16$	$36u^2 - 56u + 24$	$36u^2 - 56u + 20$
$d_G(k)$	2	3	4	5	6

In the following Theorems, to compute the general first K Banhatti polynomial of DDD network.

Theorem 1.

If $G = D_2(u)$ is DDD network, then

$$B_1^a(G,x,y) = 4u\,x^{(4)^a}\,y^{(4)^a} + (18u^2 - 22u + 6)\,x^{(5)^a}\,y^{(6)^a} + (28u - 16)x^{(6)^a}\,y^{(8)^a} + (36u^2 - 56u + 24)x^{(8)^a}\,y^{(9)^a} + (36u^2 - 56u + 20)x^{(10)^a}\,y^{(10)^a}$$

Proof. As aforesaid table 2.

$$B_1^a(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{[d_G(r)+d_G(k)]^a}\, y^{[d_G(r)+d_G(k)]^a}$$

$$= 4u\,x^{(2+2)^a}\,y^{(2+2)^a} + (18u^2 - 22u + 6)\,x^{(2+3)^a}\,y^{(3+3)^a} + (28u - 16)x^{(2+4)^a}\,y^{(4+4)^a} + (36u^2 - 56u + 24)x^{(3+5)^a}\,y^{(4+5)^a} + (36u^2 - 56u + 20)x^{(4+6)^a}\,y^{(4+6)^a}$$

$$= 4u\,x^{(4)^a}\,y^{(4)^a} + (18u^2 - 22u + 6)\,x^{(5)^a}\,y^{(6)^a} + (28u - 16)x^{(6)^a}\,y^{(8)^a} + (36u^2 - 56u + 24)x^{(8)^a}\,y^{(9)^a} + (36u^2 - 56u + 20)x^{(10)^a}\,y^{(10)^a}$$

The following are the results of Theorem 1.

Result 1.
The I[st] K B polynomial of the $D_2(u)$ is (as shown in Fig. 2.)

$$B_1(G,x,y) = 4u\,x^4\,y^4 + (18u^2-22u+6)\,x^5\,y^6 + (28u-16)\,x^6\,y^8 + (36u^2-56u+24)\,x^8\,y^9 + (36u^2-56u+20)\,x^{10}\,y^{10}$$

Result 2.
The I[st] H K B polynomial of $D_2(u)$ is (as shown in Fig. 3.)

$$HB_1(G,x,y) = 4u\,x^{16}\,y^{16} + (18u^2-22u+6)\,x^{25}\,y^{36} + (28u-16)\,x^{36}\,y^{64} + (36u^2-56u+24)\,x^{64}\,y^{81} + (36u^2-56u+20)\,x^{100}\,y^{100}$$

Result 3.
The m K B polynomial of $D_2(u)$ is (as shown in Fig. 4.)

$$^{m}B_1(G,x,y) = 4u\,x^{\frac{1}{4}}\,y^{\frac{1}{4}} + (18u^2-22u+6)\,x^{\frac{1}{5}}\,y^{\frac{1}{6}} + (28u-16)x^{\frac{1}{6}}\,y^{\frac{1}{8}} + (36u^2-56u+24)x^{\frac{1}{8}}\,y^{\frac{1}{9}} + (36u^2-56u+20)x^{\frac{1}{10}}\,y^{\frac{1}{10}}$$

Result 4.
The S C K B polynomial of $D_2(u)$ is (as shown in Fig. 5.)

$$SB(G,x,y) = 4u\,x^{\frac{1}{2}}\,y^{\frac{1}{2}} + (18u^2-22u+6)\,x^{\frac{1}{\sqrt{5}}}\,y^{\frac{1}{\sqrt{6}}} + (28u-16)x^{\frac{1}{\sqrt{6}}}\,y^{\frac{1}{\sqrt{8}}} + (36u^2-56u+24)x^{\frac{1}{\sqrt{8}}}\,y^{\frac{1}{3}} + (36u^2-56u+20)x^{\frac{1}{\sqrt{10}}}\,y^{\frac{1}{\sqrt{10}}}$$

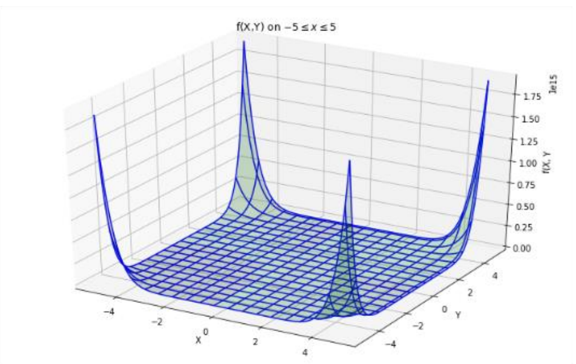

Figure 2. First K Banhatti Polynomial of Second Domination David Derived Network

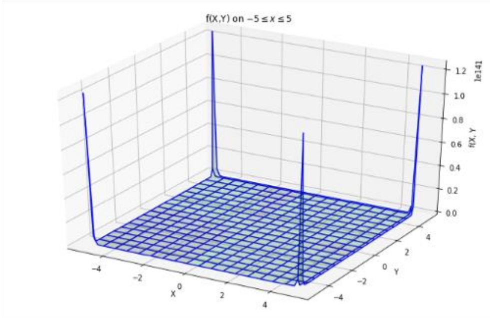

Figure 3. First Hyper K Banhatti polynomial of Second Domination David Derived Network

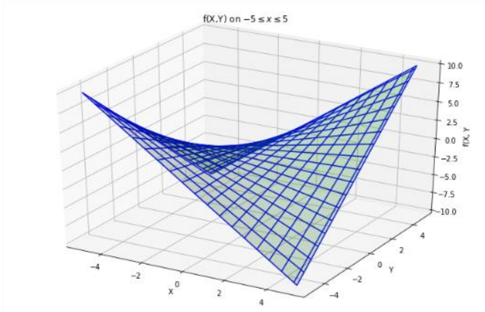

Figure 4. Modified K Banhatti polynomial of Second Domination David Derived Network

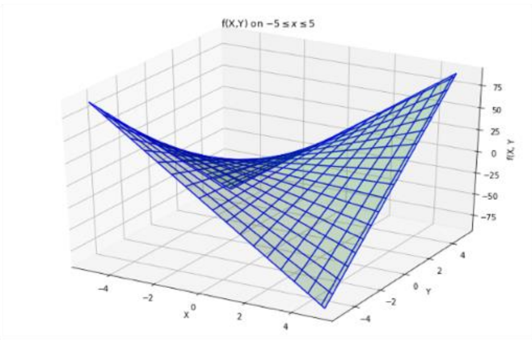

Figure 5. Sum Connectivity of K Banhatti polynomial of Second Domination David Derived Network

Theorem 2.
If $G = D_2(u)$ is DDD network, then

$$B_2^a(G,x,y) = 4u\,x^{(4)^a}\,y^{(4)^a} + (18u^2-22u+6)\,x^{(6)^a}\,y^{(9)^a} + (28u-16)x^{(8)^a}\,y^{(16)^a} + (36u^2-56u+24)x^{(15)^a}\,y^{(20)^a} + (36u^2-56u+20)x^{(24)^a}\,y^{(24)^a}$$

Proof. As aforesaid Table 2.

$$B_2^a(G,x,y) = \sum_{rk} E_{rk}\,x^{[d_G(r)*d_G(k)]^a}\,y^{[d_G(r)*d_G(k)]^a}$$

$$= 4u\,x^{(2*2)^a}\,y^{(2*2)^a} + (18u^2-22u+6)\,x^{(2*3)^a}\,y^{(3*3)^a} +$$

$$(28u-16)x^{(2*4)^a}\,y^{(4*4)^a} + (36u^2-56u+24)x^{(3*5)^a}\,y^{(4*5)^a} + (36u^2-56u+20)x^{(4*6)^a}\,y^{(4*6)^a}$$

$$B_2^a(G,x,y) = 4u\,x^{(4)^a}\,y^{(4)^a} + (18u^2-22u+6)\,x^{(6)^a}\,y^{(9)^a} + (28u-16)x^{(8)^a}\,y^{(16)^a} + (36u^2-56u+24)x^{(15)^a}\,y^{(20)^a} + (36u^2-56u+20)x^{(24)^a}\,y^{(24)^a}$$

The following are the results of Theorem 2.

Result 5.
The II[nd] K B polynomial of the $D_2(u)$ is (as shown in Fig. 6.)

$$B_2(G,x,y) = 4u\,x^4\,y^4 + (18u^2-22u+6)\,x^6\,y^9 + (28u-16)\,x^8\,y^{16} + (36u^2-56u+24)\,x^{15}\,y^{20} + (36u^2-56u+20)\,x^{24}\,y^{24}$$

Result 6.
The II[nd] H K B polynomial of $D_2(u)$ is (as shown in Fig. 7.)

$$HB_2(G,x,y) = 4u\,x^{16}\,y^{16} + (18u^2-22u+6)\,x^{36}\,y^{81} + (28u-16)\,x^{64}\,y^{256} + (36u^2-56u+24)\,x^{225}\,y^{400} + (36u^2-56u+20)\,x^{576}\,y^{576}$$

Result 7.
The II[nd] m K B polynomial of $D_2(u)$ is (as shown in Fig. 8.)

$$^{m}B_2(G,x,y) = 4u\,x^{\frac{1}{4}}\,y^{\frac{1}{4}} + (18u^2-22u+6)\,x^{\frac{1}{6}}\,y^{\frac{1}{9}} + (28u-16)x^{\frac{1}{8}}\,y^{\frac{1}{16}} + (36u^2-56u+24)x^{\frac{1}{15}}\,y^{\frac{1}{20}} + (36u^2-56u+20)x^{\frac{1}{24}}\,y^{\frac{1}{24}}$$

Result 8.
The P C K B polynomial of $D_2(u)$ is (as shown in Fig. 9.)

$$P\,B(G,x,y) = 4u\,x^{\frac{1}{2}}\,y^{\frac{1}{2}} + (18u^2-22u+6)\,x^{\frac{1}{\sqrt{6}}}\,y^{\frac{1}{\sqrt{6}}} + (28u-16)x^{\frac{1}{\sqrt{8}}}\,y^{\frac{1}{4}} + (36u^2-56u+24)\,x^{\frac{1}{\sqrt{15}}}\,y^{\frac{1}{\sqrt{20}}} + (36u^2-56u+20)x^{\frac{1}{\sqrt{24}}}\,y^{\frac{1}{\sqrt{24}}}$$

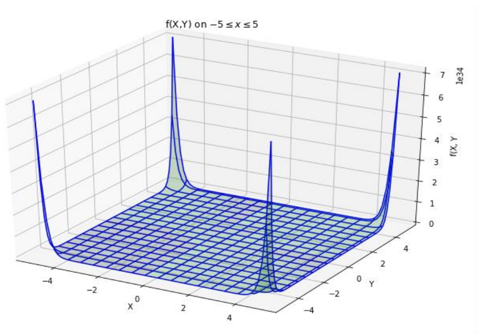

Figure 6. Second K Banhatti polynomial of Second Domination David Derived Network

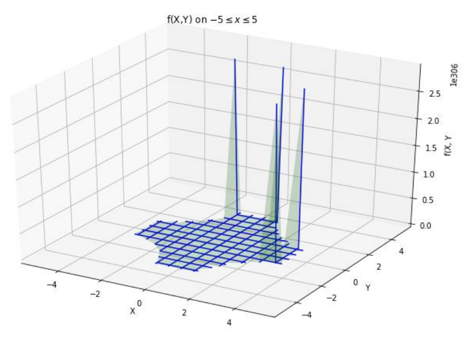

Figure 7. Second Hyper K B polynomial of Second Domination David Derived Network

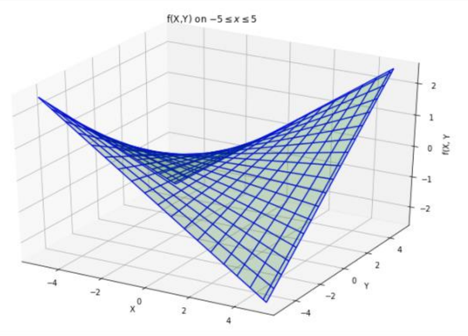

Figure 8. Second Modified K Banhatti polynomial of Second Domination David Derived Network

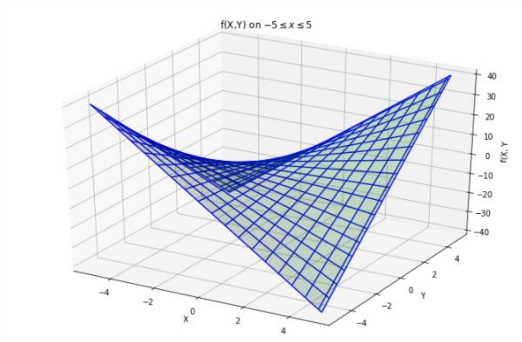

Figure 9. Product Connectivity K Banhatti polynomial of Second Domination David Derived Network

Theorem 3.

If $G = D_2(u)$ is DDD network, then

$$FB(G,x,y) = 4u\,x^{8}\,y^{8} + (18u^2 - 22u + 6)\,x^{13}\,y^{18} + (28u - 16)x^{20}\,y^{32} + (36u^2 - 56u + 24)\,x^{34}\,y^{41} + (36u^2 - 56u + 20)x^{52}\,y^{52}$$

Proof. Using Table 2.

$$FB(G,x,y) = \sum_{r\,k} E_{r\,k}\, x^{d_G(u)^2 + d_G(e)^2}\, y^{d_G(u)^2 + d_G(e)^2}$$

$$= 4u\,x^{2^2+2^2}\,y^{2^2+2^2} + (18u^2 - 22u + 6)\,x^{2^2+3^2}\,y^{3^2+3^2} + (28u - 16)x^{2^2+4^2}\,y^{4^2+4^2} + (36u^2 - 56u + 24)x^{3^2+5^2}\,y^{4^2+5^2} + (36u^2 - 56u + 20)x^{4^2+6^2}\,y^{4^2+6^2}$$

$$FB(G,x,y) = 4u\,x^{8}\,y^{8} + (18u^2 - 22u + 6)\,x^{13}\,y^{18} + (28u - 16)x^{20}\,y^{32} + (36u^2 - 56u + 24)x^{34}\,y^{41} + (36u^2 - 56u + 20)x^{52}\,y^{52}$$ (as shown in Fig. 10.)

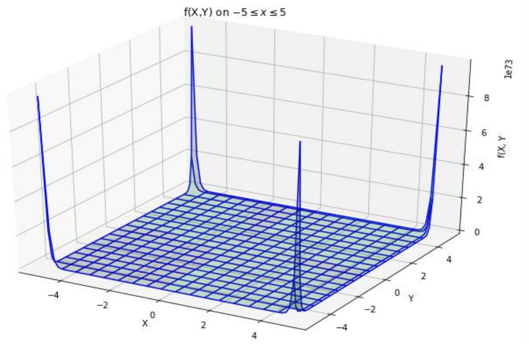

Figure 10. F-K Banhatti polynomial of Second Domination David Derived Network

Theorem 4.

If $G = D_2(u)$ is DDD network, then

$$H_b(G,x,y) = 4u\,x^{\frac{1}{2}}\,y^{\frac{1}{2}} + (18n^2 - 22u + 6)\,x^{\frac{2}{5}}\,y^{\frac{1}{3}} + (28u - 16)x^{\frac{1}{3}}\,y^{\frac{1}{4}} + (36u^2 - 56u + 24)\,x^{\frac{1}{4}}\,y^{\frac{2}{9}} + (36u^2 - 56u + 20)x^{\frac{1}{5}}\,y^{\frac{1}{5}}$$

Proof. As aforesaid Table 2.

$$H_b(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{\frac{2}{d_G(r)+d_G(k)}}\, y^{\frac{2}{d_G(r)+d_G(k)}}$$

$$= 4u\,x^{\frac{2}{2+2}}\,y^{\frac{2}{2+2}} + (18u^2 - 22u + 6)\,x^{\frac{2}{2+3}}\,y^{\frac{2}{3+3}} + (28u - 16)x^{\frac{2}{2+4}}\,y^{\frac{2}{4+4}} + (36u^2 - 56u + 24)x^{\frac{2}{3+5}}\,y^{\frac{2}{4+5}} + (36u^2 - 56u + 20)x^{\frac{2}{4+6}}\,y^{\frac{2}{4+6}}$$

$$= 4u\,x^{\frac{1}{2}}\,y^{\frac{1}{2}} + (18u^2 - 22u + 6)\,x^{\frac{2}{5}}\,y^{\frac{1}{3}} + (28u - 16)x^{\frac{1}{3}}\,y^{\frac{1}{4}} + (36u^2 - 56u + 24)x^{\frac{1}{4}}\,y^{\frac{2}{9}} + (36u^2 - 56u + 20)x^{\frac{1}{5}}\,y^{\frac{1}{5}}$$ (as shown in Fig. 11.)

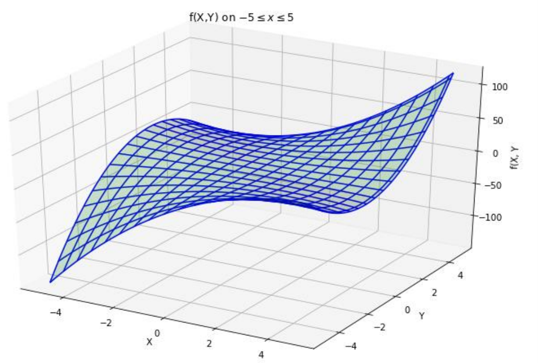

Figure 11. Harmonic K Banhatti polynomial of Second Domination David Derived Network

Theorem 5.

If $G = D_2(u)$ is DDD network, then

$$SDB(G,x,y) = 4u\,x^{2}\,y^{2} + (18u^2 - 22u + 6)\,x^{\frac{13}{6}}\,y^{2} + (28u - 16)x^{\frac{5}{2}}\,y^{2} + (36u^2 - 56u + 24)x^{\frac{34}{15}}\,y^{\frac{41}{20}} + (36u^2 - 56u + 20)x^{\frac{52}{24}}\,y^{\frac{52}{24}}$$

Proof. As aforesaid Table 2.

$$SDB(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{\frac{d_G(r)}{d_G(k)} + \frac{d_G(k)}{d_G(r)}}\, y^{\frac{d_G(r)}{d_G(k)} + \frac{d_G(k)}{d_G(r)}}$$

$$= 4u\,x^{\frac{2}{2}+\frac{2}{2}}\,y^{\frac{2}{2}+\frac{2}{2}} + (18u^2 - 22u + 6)\,x^{\frac{2}{3}+\frac{3}{2}}\,y^{\frac{3}{3}+\frac{3}{3}} + (28u - 16)x^{\frac{2}{4}+\frac{4}{2}}\,y^{\frac{4}{4}+\frac{4}{4}} + (36u^2 - 56u + 24)x^{\frac{3}{5}+\frac{5}{3}}\,y^{\frac{4}{5}+\frac{5}{4}} + (36u^2 - 56u + 20)x^{\frac{4}{6}+\frac{6}{4}}\,y^{\frac{4}{6}+\frac{6}{4}}$$

$$= 4u\,x^{2}\,y^{2} + (18u^2 - 22u + 6)\,x^{\frac{13}{6}}\,y^{2} + (28u - 16)x^{\frac{5}{2}}\,y^{2} + (36u^2 - 56u + 24)x^{\frac{34}{15}}\,y^{\frac{41}{20}} + (36u^2 - 56u + 20)x^{\frac{52}{24}}\,y^{\frac{52}{24}}$$ (as shown in Fig. 12.)

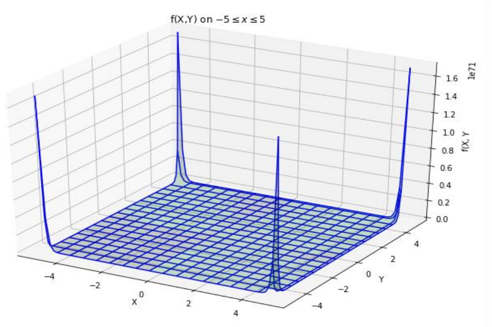

Figure 12. Symmetric Division K Banhatti polynomial of Second Domination David Derived Network

Theorem 6.
If $G = D_2(u)$ is DDD network, then

$$ISB(G,x,y) = 4u\,x\,y + (18u^2 - 22u + 6)\,x^{\frac{6}{5}}\,y^{\frac{3}{2}} + (28u - 16)x^{\frac{4}{5}}\,y^2 + (36u^2 - 56u + 24)x^{\frac{15}{8}}\,y^{\frac{20}{9}} + (36u^2 - 56u + 20)x^{\frac{12}{5}}\,y^{\frac{12}{5}}$$

Proof. Using Table 2.

$$ISB(G,x,y) = \sum_{r\,k} |E_{r\,k}|\; x^{\frac{d_G(r)*d_G(k)}{d_G(k)+d_G(r)}}\; y^{\frac{d_G(r)*d_G(k)}{d_G(k)+d_G(r)}}$$

$$= 4u\,x^{\frac{2*2}{2+2}}\,y^{\frac{2*2}{2+2}} + (18u^2 - 22u + 6)\,x^{\frac{2*3}{2+3}}\,y^{\frac{3*3}{3+3}} + (28u - 16)x^{\frac{2*4}{2+4}}\,y^{\frac{4*4}{4+4}} + (36u^2 - 56u + 24)x^{\frac{3*5}{3+5}}\,y^{\frac{4*5}{4+5}} + (36u^2 - 56u + 20)x^{\frac{4*6}{4+6}}\,y^{\frac{4*6}{4+6}}$$

$$= 4u\,x\,y + (18u^2 - 22u + 6)\,x^{\frac{6}{5}}\,y^{\frac{3}{2}} + (28u - 16)x^{\frac{4}{5}}\,y^2 + (36u^2 - 56u + 24)\,x^{\frac{15}{9}}\,y^{\frac{20}{9}} + (36u^2 - 56u + 20)x^{\frac{12}{5}}\,y^{\frac{12}{5}}$$ (as shown in Fig. 13.)

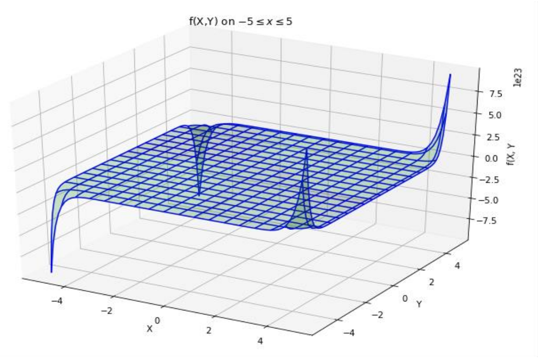

Figure 13. Inverse Sum Index K Banhatti polynomial of Second Domination David Derived Network

Third type of Domination David Derived Network

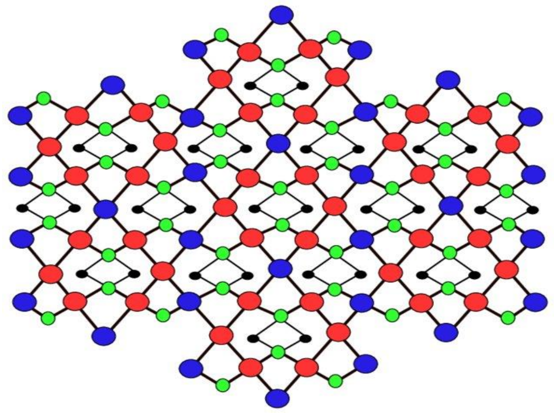

Figure 14. Domination David Derived network of the third type

The DDD network of the third type (as shown in Fig. 14) of dimension u can be obtained by connecting vertices of degree 2 of $DDD(u)$ by an edge that are not in the boundary and is denoted by $D_3(u)$.
In $G = D_3(u)$, the edge set of G can be divided into 3 partitions based on the degree of end vertices of each edge as given in Table 3.

Table 3. Edge partition of G

$d_G(r), d_G(s)$; $k = r\,s \in Q(G)$	**(2, 2)**	**(2, 4)**	**(4, 4)**
No of edges	$4u$	$36u^2 - 20u$	$72u^2 - 108u + 44$

There fore the edge degree partition of $G = D_3(u)$ is given in Table 4.

Table 4. Edge degree partition of G

$d_G(r), d_G(s)$; $k = r\,s \in Q(G)$	**(2, 2)**	**(2, 4)**	**(4, 4)**
No of edges	$4u$	$36u^2 - 20u$	$72u^2 - 108u + 44$
$d_G(k)$	2	4	6

To compute the general I[st] K B polynomial DDD network using the following Theorem.

Theorem 7.
If $G = D_3(u)$ is DDD network, then
$B_1^a = 4u\, x^{4^a}\, y^{4^a} + (36u^2 - 20u)\, x^{6^a}\, y^{8^a} + (72u^2 - 108u + 44)\, x^{10^a}\, y^{10^a}$

Proof. The aforementioned data Table 4.

$$B_1^a(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{[d_G(r)+d_G(k)]^a}\, y^{[d_G(r)+d_G(k)]^a}$$
$$= 4u\, x^{(2+2)^a}\, y^{(2+2)^a} + (36u^2 - 20u) x^{(2+4)^a}\, y^{(4+4)^a} + (72u^2 - 108u + 44) x^{(4+6)^a}\, y^{(4+6)^a}$$
$$= 4u\, x^{4^a}\, y^{4^a} + (36u^2 - 20u)\, x^{6^a}\, y^{8^a} + (72u^2 - 108u + 44)\, x^{10^a}\, y^{10^a}$$

The following are the results of Theorem 7.

Result 9.
The I[st] K B polynomial of the $D_3(u)$ is (as shown in Fig. 15.)

$$B_1(G,x,y) = 4u\, x^4\, y^4 + (36u^2 - 20u)\, x^6\, y^8 + (72u^2 - 108u + 44)\, x^{10}\, y^{10}$$

Result 10.
The I[st] H K B polynomial of $D_3(u)$ is (as shown in Fig. 16.)

$$HB_1(G,x,y) = 4u\, x^{16}\, y^{16} + (36u^2 - 20u)\, x^{36}\, y^{64} + (72u^2 - 108u + 44)\, x^{100}\, y^{100}$$

Result 11.
The m K B polynomial of $D_3(u)$ is (as shown in Fig. 17.)

$$^m B_1(G,x,y) = 4u\, x^{\frac{1}{4}}\, y^{\frac{1}{4}} + (36u^2 - 20u) x^{\frac{1}{6}}\, y^{\frac{1}{8}} + (72u^2 - 108u + 44) x^{\frac{1}{10}}\, y^{\frac{1}{10}}$$

Result 12.
The S C K B polynomial of $D_3(u)$ is (as shown in Fig. 18.)

$$SB(G,x,y) = 4u\, x^{\frac{1}{2}}\, y^{\frac{1}{2}} + (36u^2 - 20u) x^{\frac{1}{\sqrt{6}}}\, y^{\frac{1}{\sqrt{8}}} + +(72u^2 - 108u + 44) x^{\frac{1}{\sqrt{10}}}\, y^{\frac{1}{\sqrt{10}}}$$

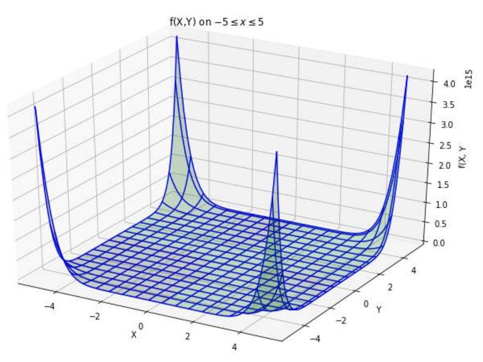

Figure 15. First K Banhatti polynomial of third Domination David Derived Network

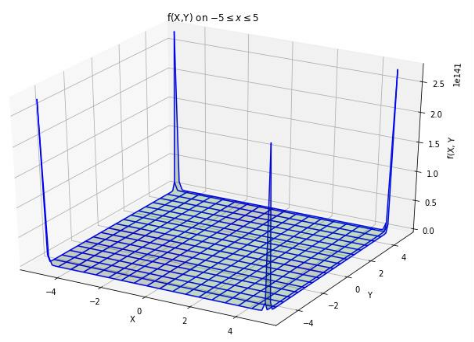

Figure 16. First Hyper K Banhatti polynomial of third Domination David Derived Network

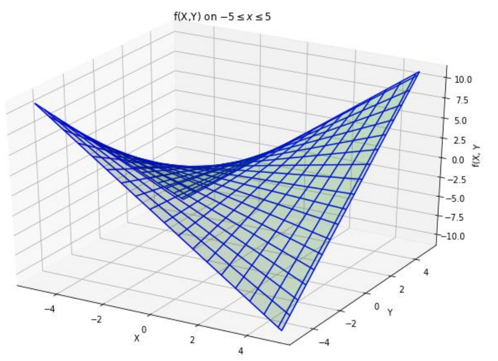

Figure 17. Modified K Banhatti polynomial of third Domination David Derived Network

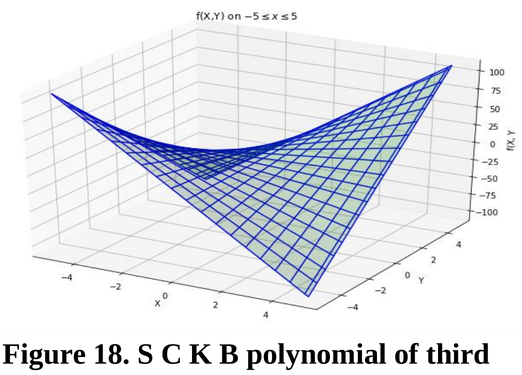

Figure 18. S C K B polynomial of third Domination David Derived Network

To compute the general II[nd] K B polynomial of the $D_3(u)$

Theorem 8.
If $G = D_3(u)$ is DDD network, then

$$B_2^a(G,x,y) = 4u\, x^{(4)^a}\, y^{(4)^a} + (36u^2 - 20u)x^{(8)^a}\, y^{(16)^a} + (72u^2 - 108u + 44)x^{(24)^a}\, y^{(24)^a}$$

Proof. The aforementioned data Table 4.

$$B_2^a(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{[d_G(r) * d_G(k)]^a}\, y^{[d_G(r) * d_G(k)]^a}$$

$$= 4u\, x^{(2*2)^a}\, y^{(2*2)^a} + (36u^2 - 20u)x^{(2*4)^a}\, y^{(4*4)^a} + (72u^2 - 108u + 44)x^{(4*6)^a}\, y^{(4*6)^a}$$

$$B_2^a(G,x,y) = 4u\, x^{(4)^a}\, y^{(4)^a} + (36u^2 - 20u)x^{(8)^a}\, y^{(16)^a} + (72u^2 - 108u + 44)x^{(24)^a}\, y^{(24)^a}$$

The following are the results of Theorem 8.

Result 13.
The II[nd] K B polynomial of the $D_3(u)$ is (as shown in Fig. 19.)

$$B_2(G,x,y) = 4u\, x^4\, y^4 + (36u^2 - 20u)\, x^8\, y^{16} + (72u^2 - 108u + 44)\, x^{24}\, y^{24}$$

Result 14.
The II[nd] H K B polynomial of $D_3(u)$ is (as shown in Fig. 20.)

$$HB_2(G,x,y) = 4u\, x^{16}\, y^{16} + (36u^2 - 20u)\, x^{64}\, y^{256} + +(72u^2 - 108u + 44)\, x^{576}\, y^{576}$$

Result 15.
The m K B polynomial of $D_3(u)$ is (as shown in Fig. 21.)

$${}^{m}B_2(G,x,y) = 4u\, x^{\frac{1}{4}}\, y^{\frac{1}{4}} + (36u^2 - 20u)x^{\frac{1}{8}}\, y^{\frac{1}{16}} + (72u^2 - 108u + 44)x^{\frac{1}{24}}\, y^{\frac{1}{24}}$$

Result 16.
The P C K B polynomial of $D_3(u)$ is (as shown in Fig. 22.)

$$P\,B(G,x,y) = 4u\, x^{\frac{1}{2}}\, y^{\frac{1}{2}} + (36u^2 - 20u)x^{\frac{1}{\sqrt{8}}}\, y^{\frac{1}{4}} + +(72u^2 - 108u + 44)x^{\frac{1}{\sqrt{24}}}\, y^{\frac{1}{\sqrt{24}}}$$

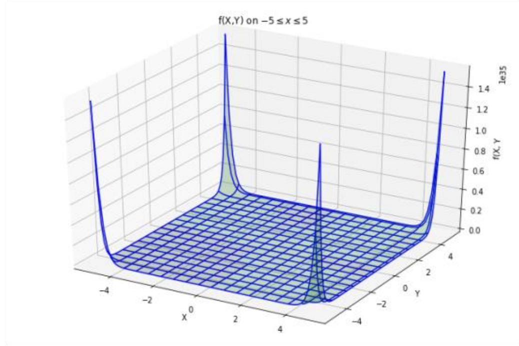

Figure 19. Second K Banhatti polynomial of third Domination David Derived Network

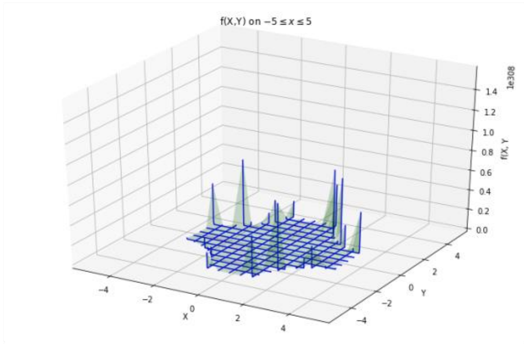

Figure 20. Second Hyper K Banhatti polynomial of third Domination David Derived Network

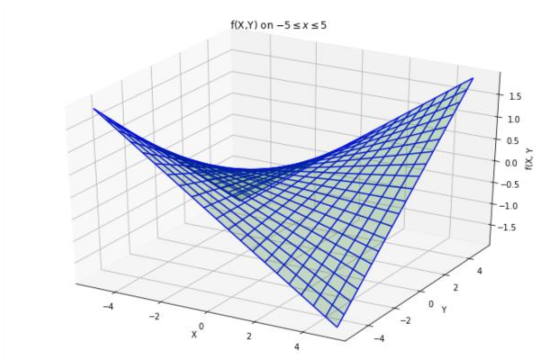

Figure 21. Modified K Banhatti polynomial of third Domination David Derived Network

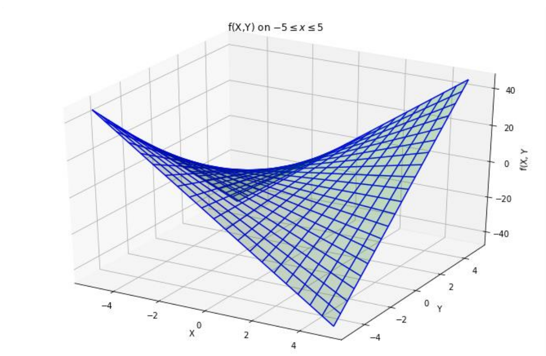

Figure 22. Product Connectivity K Banhatti polynomial of Domination David Derived Network

Theorem 9.
If $G = D_3(u)$ is DDD network, then
$FB(G,x,y) = 4u\, x^8\, y^8 + (36u^2 - 20u)x^{20}\, y^{32} + + (72u^2 - 108u + 44)x^{52}\, y^{52}$
Proof. The aforementioned data Table 4.

$$FB(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{d_G(r)^2 + d_G(k)^2}\, y^{d_G(r)^2 + d_G(k)^2}$$

$$= 4u\, x^{2^2+2^2}\, y^{2^2+2^2} + (36u^2 - 20u)x^{2^2+4^2}\, y^{4^2+4^2} + (72u^2 - 108u + 44)x^{4^2+6^2}\, y^{4^2+6^2}$$

$$FB(G,x,y) = 4u\, x^8\, y^8 + (36u^2 - 20u)x^{20}\, y^{32} + (72u^2 - 108u + 44)x^{52}\, y^{52}$$

(as shown in Fig. 23.)

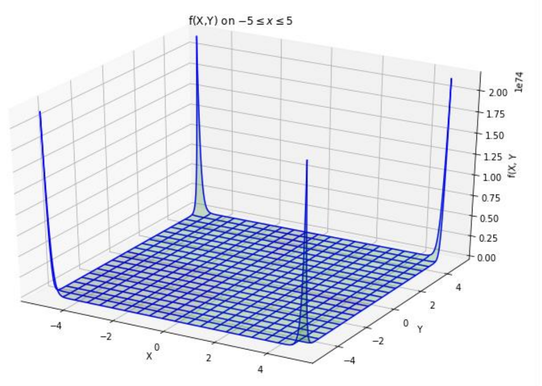

Figure 23. F-K Banhatti polynomial of third Domination David Derived Network

Theorem 10.
If $G = D_3(u)$ is DDD network, then

$$H_b(G,x,y) = 4u\, x^{\frac{1}{2}}\, y^{\frac{1}{2}} + (36u^2 - 20u)x^{\frac{1}{3}}\, y^{\frac{1}{4}} + (72u^2 - 108u + 44)x^{\frac{1}{5}}\, y^{\frac{1}{5}}$$

Proof. By using Table 4.

$$H_b(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{\frac{2}{d_G(r)+d_G(k)}}\, y^{\frac{2}{d_G(r)+d_G(k)}}$$

$$= 4u\, x^{\frac{2}{2+2}}\, y^{\frac{2}{2+2}} + (36u^2 - 20u)x^{\frac{2}{2+4}}\, y^{\frac{2}{4+4}} + (72u^2 - 108u + 44)x^{\frac{2}{4+6}}\, y^{\frac{2}{4+6}}$$

$$H_b(G,x,y) = 4u\, x^{\frac{1}{2}}\, y^{\frac{1}{2}} + (36u^2 - 20u)x^{\frac{1}{3}}\, y^{\frac{1}{4}} + (72u^2 - 108u + 44)x^{\frac{1}{5}}\, y^{\frac{1}{5}}$$

(as shown in Fig. 24.)

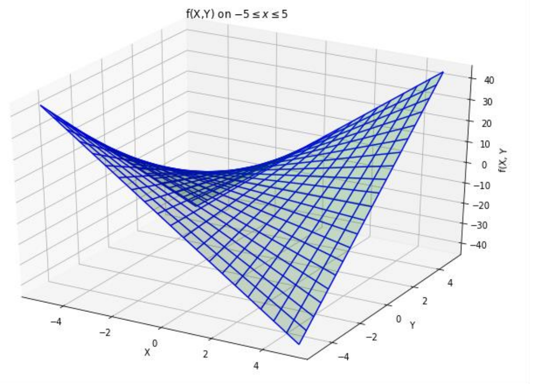

Figure 24. Hyper K Banhatti polynomial of third Domination David Derived Network

Theorem 11.
If $G = D_3(u)$ is DDD network, then

$$SDB(G,x,y) = 4u\, x^2\, y^2 + (36u^2 - 20)x^{\frac{5}{2}}\, y^2 + (72u^2 - 108u + 44)x^{\frac{52}{24}}\, y^{\frac{52}{24}}$$

Proof. The aforementioned data Table 4.

$$SDB(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{\frac{d_G(r)}{d_G(k)} + \frac{d_G(k)}{d_G(r)}}\, y^{\frac{d_G(r)}{d_G(k)} + \frac{d_G(k)}{d_G(r)}}$$

$$= 4u\, x^{\frac{2}{2}+\frac{2}{2}}\, y^{\frac{2}{2}+\frac{2}{2}} + (36u^2 - 20u)x^{\frac{2}{4}+\frac{4}{2}}\, y^{\frac{4}{4}+\frac{4}{4}} + (72u^2 - 108u + 44)x^{\frac{4}{6}+\frac{6}{4}}\, y^{\frac{4}{6}+\frac{6}{4}}$$

$$SDB(G,x,y) = 4u\, x^2\, y^2 + (36u^2 - 20)x^{\frac{5}{2}}\, y^2 + (72u^2 - 108u + 44)x^{\frac{52}{24}}\, y^{\frac{52}{24}}$$

(as shown in Fig. 25.)

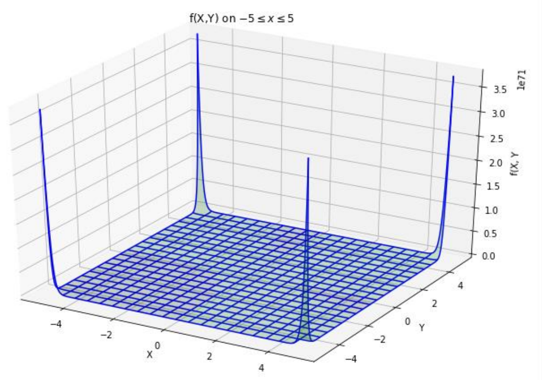

Figure 25. Symmetric Division K Banhatti polynomial of third Domination David Derived Network

Theorem 12.
If $G = D_3(u)$ is DDD network, then

$$ISB(G,x,y) = 4u\, x\, y + (36u^2 - 20u)x^{\frac{4}{3}}\, y^2 + (72u^2 - 108u + 44)x^{\frac{12}{5}}\, y^{\frac{12}{5}}$$

Proof. By using Table 4.

$$ISB(G,x,y) = \sum_{r\,k} |E_{r\,k}|\, x^{\frac{d_G(r) * d_G(k)}{d_G(k) + d_G(r)}}\, y^{\frac{d_G(r) * d_G(k)}{d_G(k) + d_G(r)}}$$

$$= 4u\, x^{\frac{2*2}{2+2}}\, y^{\frac{2*2}{2+2}} + (36u^2 - 20u)x^{\frac{2*4}{2+4}}\, y^{\frac{4*4}{4+4}} + (72u^2 - 108u + 44)x^{\frac{4*6}{4+6}}\, y^{\frac{4*6}{4+6}}$$

$$ISB(G,x,y) = 4u\,x\,y + (36u^2 - 20u)x^{\frac{4}{3}}\, y^2 + (72u^2 - 108u + 44)x^{\frac{12}{5}}\, y^{\frac{12}{5}}$$ (as shown in Fig. 26.)

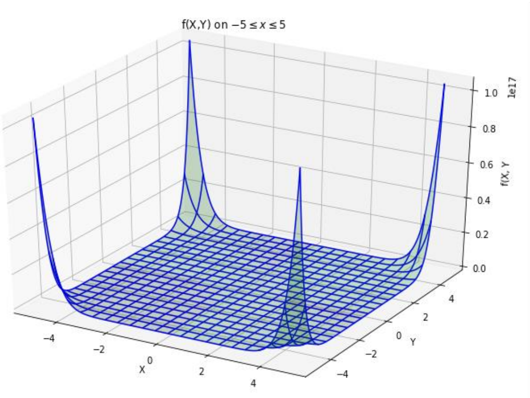

Figure 26. Inverse Sum Index K Banhatti polynomial of third Domination David Derived Network

All k-Banhatti polynomial of $G_1 = D_2(u)$ and $G_2 = D_3(u)$ (as shown in Fig. 27. and Table.5)

Table 5. Exact values of $G_1 = D_2(u)$and $G_2 = D_3(u)$

n	1	2	3	4	5	6	7	8	9	10
$G_1 = D_2(u)$	22	190	538	1066	1774	2662	3730	4978	6406	8014
$G_2 = D_3(u)$	28	228	644	1276	2124	3188	4468	5964	7676	9604

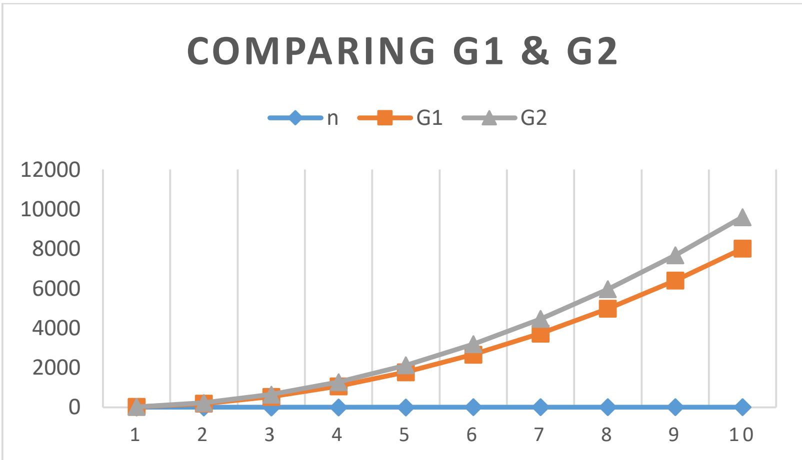

Figure 27. Comparing the graphs second and third type of Domination David Derived Network

Conclusion:

This paper has studied and computed the second and third type of DDD network through topological indices and polynomials.

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Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.

- Ethical Clearance: The project was approved by the local ethical committee in Guru Nanak Institutions Technical Campus University.

Authors' contribution statement:

AM; Conception, Design, Acquisition of data, analysis, drafting the MS, interpreting the results and design the figures

U VCK, R M; interpretation , revision and proofreading. All the authors discussed the results and commented on the manuscript.

References:

1. Ahmed A. Omran, Haneen Hamed Oda. H_n-Domination in Graphs. Baghdad Sci. J. 2019; 16(1(Suppl.)): 242-247.
2. Al-Harere M N, Khuda Bakhash P A. Tadpole Domination in Graphs. Baghdad Sci. J. 2018 Dec 9; 15(4): 466-471.
3. Huang C, Wang Y, Li X, Ren L, Zhao J, Hu Y. Clinical features of patients infected with 2019 novel coronavirus in Wuhan, China. The Lancet. 2020; 395(10223): 497-506.
4. Lung J, Lin YS, Yang YH, Chou YL. Shu LH, Cheng YC. The potential chemical structure of anti SARS CoV-2 RNA dependent RNA polymerase. J Med Virol. 2020; 92(6):693-697.
5. Morse JS, Lalonde T, Xu S, Liu WR. Learning from the past: Possible urgent prevention and treatment options for severe acute respiratory infections caused by 2019 nCoV. Chem Bio Chem. 2020; 21(5): 730-738.
6. Corinto F, Gilli M. Comparison between the dynamic behaviour of Chua-Yang and full-range cellular neural networks. Int. J Circuit Theory Appl. 2003; 31(5): 423–441.
7. Yumei Hu, Xueliang Li, Yongtang Shi, Tianyi Xu. On molecular graphs with smallest and greatest zeroth- order general Randi´c index. MATCH Commun. Math. Comput. Chem. 2005; 54(2): 425-434.
8. Imran M, Baig AQ, Ali H. On topological properties of dominating David derived networks. Can J Chem. 2015; 94(2): 137-148.
9. Kulli VR. On K Banhatti indices of graphs. J Math Comput Sci. 2016; 7: 213-218.
10. Kulli VR. On K hyper-Banhatti indices and coindices of graphs. IRJPA. 2016; 6(5): 300-304.
11. Kulli VR. New K Banhatti topological indices. Int J Fuzzy Math. 2017; 12(1): 29-37.
12. Kulli VR. B. Chaluvaraju and H. S. Boregowda. Connectivity Banhatti indices for certain families of benzenoid systems. J Ultra Chem. 2017; 13(4): 81-87.
13. Kulli VR. Computing Banhatti indices of networks. Int j adv Math. 2018; 2018(1): 31-40.
14. Kulli VR. K Banhatti Indices of Chloroquine and Hydroxy chloroquine: Research Applied for the Treatment and Prevention of COVID-19. Int J Appl Chem. 2020; 7(1): 63-68.
15. Kulli VR. Revan indices of chloroquine and hydroxyl chloroquine, remdesivir: Research advances for the treatment of COVID-19. Int J Eng Sci Res Technol. 2020; 9(5): 73-84.

المؤشرات الطوبولوجية للشبكات المشتقة من متعددة حدود ديفيد المهيمنة

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الخلاصة:

ترتبط الخصائص الكيميائية للمركبات الكيميائية وبنيتها الجزيئية ارتباطًا وثيقًا. المؤشرات الطوبولوجية هي قيم عددية مرتبطة بالرسوم البيانية الجزيئية الكيميائية التي تساعد في فهم الخصائص الفيزيائية والكيميائية والتفاعل الكيميائي والنشاط البيولوجي للمركب الكيميائي .
يتطرق هذا البحث على بعض الخصائص الطوبولوجية للشبكات المشتقة من dominating David derived وتحصي العديد من K Banhatti متعدد الحدود من النوع الثاني والثالث منDDD .

الكلمات المفتاحية: شبكات ديفيد المشتقة المهيمنة، مؤشرات F-K Banhatti ، مؤشرات K Banhatti التوافقية، متعددة حدود مؤشرات K Banhatti، مؤشرات K Banhatti الفوقية، مؤشرات K Banhatti المقسومة المتناظرة