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Mathematical Models Used for Brachytherapy Treatment Planning Dose Calculation Algorithms

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Abstract:

Brachytherapy treatment is primarily used for the certain handling kinds of cancerous tumors. Using radionuclides for the study of tumors has been studied for a very long time, but the introduction of mathematical models or radiobiological models has made treatment planning easy. Using mathematical models helps to compute the survival probabilities of irradiated tissues and cancer cells. With the expansion of using HDR-High dose rate Brachytherapy and LDR-low dose rate Brachytherapy for the treatment of cancer, it requires fractionated does treatment plan to irradiate the tumor. In this paper, authors have discussed dose calculation algorithms that are used in Brachytherapy treatment planning. Precise and less time-consuming calculations using 3D dose distribution for the patient is one of the important necessities in modern radiation oncology. For this it is required to have accurate algorithms which help in TPS. There are certain limitations with the algorithm which are used for calculating the dose. This work is done to evaluate the correctness of five algorithms that are presently employed for treatment planning, including pencil beam convolution (PBC), superposition (SP), anisotropic analytical algorithm (AAA), Monte Carlo (MC), Clarkson Method, Fast Fourier Transform, Convolution method. The algorithms used in radiotherapy treatment planning are categorized as correction-based and model-based.

Keywords: Algorithm, Brachytherapy, Models, Treatment planning, Tumors.

Introduction:

The use of mathematical models in radiotherapy[1, 2] helps to compute the survival probabilities of irradiated tissues[3, 4] and cancer cells[5]. The algorithms used in radiotherapy treatment planning are categorized as correction-based and model-based. It is also an essential tool in treatment planning in radiotherapy[6]. Treatment planning systems are the core of the radiation therapy and which help to improve the patient results. When the datasets were identified of images, and the tumors are recognized, the systems make a complex plan for each beamline and the radiation will be delivered to tumor with the therapy system. For radiotherapy, many treatment models and algorithms have been proposed. These computational algorithms help to create intelligence based TPS. For these accurate algorithms are required which help in TPS. The algorithms used in radiotherapy treatment planning are categorized as correction-based and model-based and the basic understanding of them supports in commissioning, detecting differences in different algorithms which in-turn helps in creating a uniformity in medical practices. Earlier, these studies were based on trial and error methods but presently with the advancement of these algorithms the optimization of treatment plan can be done in lesser time and with very high accuracy. Details of these studies have been discussed in the paper. These algorithms help in automating the dose optimization and help to give better treatment to patients suffering from cancer. In the present scenario with rising number of Covid-19 cases more refinement in algorithms are required to study tumors along-with infections caused by Covid-19[7,8] virus.

Clarkson Method:

This method is used for calculating the dose at open points which are present on irregular areas. In this method factors like non uniformity of surface and presence of wedge can be ignored. Generally, Clarkson[9] is implemented for calculation of point doses with irregular shapes of fields when it has shielded structures which are sensitive to radiation with main radiation or when this pitch spreads beyond the non-regularly fashioned contour of the patient's body. This technique can use compensator filters to do calculation of changes in dose. Another usage of Clarkson in IMRT QA [10], is it has to derive an intensity plan for each field in the practice of inverse planning in IMRT in treatment planning software[9]. If the compensator mode is being selected in IMRT, in that case the procedure to calculate the intensity will be converted into compensator wideness. In the treatment planning software, I/I_0 ratio is being calculated in existing intensity plans. According to this ratio in Clarkson's method dose of all the points can be measured by the values at each point. Since it's very difficult to calculate I/I_0 ratio from treatment planning software at every point therefore Mapcheck2 dosimeter was used. The principle behind Clarkson's method[10] is that it can calculate scattered component of the depth dose from primary component separately, which is does not dependent of the dimensions of the field.

Calculate scattered dose at each point (Q) of irregular field by dividing total field area into equal sectors having degrees $\Delta\Theta$. Calculating Scattered Maximum Ratio (SMR) for each sector of circular fields is done using SMR table[11]. Taking average of all SMR values is conducted to find average scatter maximum ratio.

$$\overline{SMR(}d,r_d) = \frac{1}{n}\sum_{i=1}^{n} SMR(d,r_i) \qquad 1$$

Where r_i is defined as the radius of the i-th sector at depth d, and n is the total number of sectors $(n = 2\pi/\Delta\theta)$ $SMR(d,r_d)$ is then changed into Tissue Maximum Ratio $TMR(d,r_d)$

$$\overline{TMR(}d,r_d) = \left[TMR(d,0) + \overline{SMR(}d,r_d)\right] \text{ x } \frac{s_p(0)}{\overline{s_p}(r_d)} \qquad 2$$

Where $\overline{s_p}(r_d)$, is the average phantom scatter factor for the uneven area and $S_p(0)$ is the phantom scatter factor for the 0×0 field area. $TMR\ (d,0)$ is the maximum tissue ratio for the 0×0 area field. This value for TMR is firmly effective firstly for the points along the central axis of a beam that is generally incidenting on an infinite phantom with flat surface. $\overline{TMR(}d,r_d)$ is changed in percent depth dose by using Eq. 2:

$TMR(d,r_d)$ can be changed into percent depth dose

$$P(d,r,f) = 100\left[K_P * TMR(d,0) + \overline{SMR(}\,\mathrm{d,rd})\right] \text{ x } \frac{s_p(0)}{s_p(r_d)} \text{ x } \frac{s_p(r_d)}{s_p(r_{10})} \text{ x } \left(\frac{f+t_0}{f+d}\right)^2 \qquad 3$$

$$\text{PDD}=100X\left[K_P * TMR(d,0) + \overline{SMR(}\,\mathrm{d,rd})\right] \text{ x } \frac{1}{1+\overline{SMR(}t_0, r_{r_0})}\left(\frac{f+t_0}{f+d}\right)^2 \qquad 4$$

In the general Clarkson's equation, the value of $TMR(d,0)$ is given as:

$$TMR(d,0) = e^{-\mu(d-t_0)} \qquad 5$$

The drawback of this method is that this technique is not practical for routine manual or computerized calculation since this technique is time consuming. This method needs a considerable amount of input data.

The alteration with planned and calculated physical doses at the middle of the spread-out Bragg peak SOBP varies when the middle angle of the sector segment is changed. Physical doses at center of the SOBP are within limit of ±1% for all irregularly shaped beams that are used to authenticate the calculation technique.

The correctness of this planned way depends on both the values of angular intervals used for Clarkson integration and for calculating the quality of the basic data which is: sampling numbers for the field size and the wideness of the range shifter. If these parameters are correctly chosen, the authors can obtain a calculated monitor unit number with high accuracy satisfactory for applicability in clinic[12-14].

Convolution Method:

Convolution or superposition method is one of the model-based computation method[15, 16]. This method has been successfully used for the calculations of external beam and being applied for dose distribution calculations[17]. This method monitors the calculation which is based on intensity of the beam not on the dose on phantom. Dose distribution using convolution method given by:

$$D(r) = \sum \psi(r')A(r-r') \qquad 6$$

Where $D(r)$ is define as the dose distribution, $\psi(r')$ is fluence in energy and $A(r-r')$ the Kernel from Monte Carlo simulation and r is the dose deposition site and r' is the primary interaction site.

Fast Fourier Transform:

Treatment planning system requires various algorithms. Authors have discussed few of them in the paper. The use of these algorithms gives us high level of accuracy in quality of treatment in radiotherapy. In TPS Dose volume histograms which are required for the dose optimization requires high level of correctness. In order to achieve that correctness these processes are time consuming and large volume implants require high number of assumptions and large input data. Using Fast Fourier Transform (FFT) [18] the time required for the dose calculations is independent of the number of sources which are used[19]. FFT algorithm has been applied in electron beam radiation therapy EBRT to integrate corrections in brachytherapy calculations. FFT with convolution method has been applied to improve the dose distribution calculations[20, 21]. Using convolution method Fast Fourier transform can be defined as:

$$F(t)G(t) = \int_{-\infty}^{+\infty} F(\tau)G(t-\tau)d\tau \qquad 7$$

Where F(t) and G(t) are two sample points[20] with Fourier transform F(F) and F(G). Dose distribution is expressed as:

$$D(X) = \int_{-\infty}^{+\infty} G(X")F(X-X") \qquad 8$$

The solution of dose distribution is can be calculated using inverse Fourier Transform[22] and given as:

$$D(X) = F^{-1}\left[\frac{1}{N}.F[G(X)].F[F(X)]\right] \qquad 9$$

This equation signifies the solution of dose distribution in one axis and for all the Cartesian coordinates it can be expressed as N_x, N_y and N_z.

Inverse Planning Algorithm:

The application of an inverse planning algorithm[23] is to find the minimum value of a combined objective function based on a group of predefined objectives of the dose. As compared to forward planning, inverse planning has few benefits like less time required for treatment planning, good reproducibility, more target coverage, and less dose to organs at high risk. Presently two main types of inverse planning algorithm[24] are being used i.e. IPSA (inverse planning simulated annealing) and HIPO (hybrid inverse planning optimization).

IPSA allows high fast dose optimization due to less computation time which helps to deliver high dose of radiation and achieve high conformity in radiation[25]. HIPO is an algorithm used for optimization which takes the sum of stochastic algorithm and limited memory Broyden-Fletcher-Goldfarb-Shanno LBFGS and then it finds the three-dimensional optimization of dose. With the combination of IPSA and HIPO can limit the dwell time variance and which can be used to calculate acceptable treatment plan. Gradient Based Planning Optimization [26- 28] is based on Task Group 43 protocol. Dose for nth voxel can be calculated by using the formula:

$$D_n = \sum_{m=1}^{N_M}\sum_{N=1}^{N_N} d_{m,n}t_{m,n} \qquad 10$$

Where N_M is the number of channels, N_N is the number of dwell position in m^th channel and $d_{m,n}$, $t_{m,n}$ are the rate for dose contribution and the dwell weight for two positions i.e. m^{th} and n^{th}.

Conclusion:

Treatment planning is very crucial part of cancer treatment. This TPS cannot be completed without the use of proper mathematical models. In this paper the use of all the mathematical models which are being widely used in medical centers. Using these models and algorithms the exact dose value can be calculated. As in superposition method, a modified of the convolution method, and it predicts dose in a range of a few percent of the Monte Carlo method and about an order of magnitude faster for few calculation fields. Also in Monte Carlo simulations dose convolution reduces the statistical noise while doing the calculations. As we cannot allow dose higher then recommended dose so accurate input values can be calculated using above models. As all the models have different applicability in cancer treatment or in treatment planning systems but out of all Monte Carlo dose convolution is the best possible option as it has impeccable accuracies in dose calculations.

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Authors' declaration:

- Conflicts of Interest: None.

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Authors' contributions statement:

Sh. K.: Conception, design, acquisition of data, analysis, interpretation, Literature work and collected data for all the models analysis, interpretation. S. G.: Drafting the MS, revision and proofreading, Supervised and investigate all the findings

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النماذج الرياضية المستخدمة لتخطيط العلاج الموضعي خوارزميات حساب الجرعة

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الخلاصة:

تستخدم المعالجة الكثبية بشكل أساسي في معالجة أنواع معينة من الأورام السرطانية. تمت دراسة استخدام النويدات المشعة لدراسة الأورام لفترة طويلة جدًا ، إدخال النماذج الرياضية أو النماذج الإشعاعية الحيوية جعل تخطيط العلاج أمرًا سهلاً. يساعد استخدام النماذج الرياضية في حساب احتمالات بقاء الأنسجة المشععة والخلايا السرطانية. مع التوسع في استخدام المعالجة الكثبية ذات معدل الجرعات العالية HDRوالمعالجة الكثبية ذات معدل الجرعات المنخفضة LDR لعلاج السرطان ، فإنه يتطلب خطة علاج مجزأة لإشعاع الورم. في هذه الورقة ، ناقش المؤلفون خوارزميات حساب الجرعات المستخدمة في تخطيط العلاج الموضعي. تعد الحسابات الدقيقة التي تستغرق وقتًا أقل باستخدام توزيع الجرعات ثلاثية الأبعاد للمريض أحد الضروريات المهمة في علاج الأورام بالإشعاع الحديث. لهذا يجب أن يكون لديك خوارزميات دقيقة تساعد في .TPS هناك قيود معينة مع الخوارزمية المستخدمة لحساب الجرعة. تم إجراء هذا العمل لتقييم صحة خمس خوارزميات يتم استخدامها حاليًا لتخطيط العلاج ، بما في ذلك التفاف الحزمة بالقلم الرصاص (PBC) ، التراكب (SP) ، الخوارزمية التحليلية متباينة الخواص (AAA) ، مونت كارلو (MC) ، طريقة كلاركسون ، فاست فورييه طريقة التحويل. يتم تصنيف الخوارزميات المستخدمة في تخطيط العلاج الإشعاعي على أنها قائمة على التصحيح وقائمة على النموذج.

الكلمات المفتاحية: الخوارزمية، المعالجة الكثبية، النماذج، التخطيط العلاجي، الأورام.